ECE 780 T12: Distributed Optimization

Spring 2024

Department of Electrical and Computer Engineering University of Waterloo

Cyber-physical systems often involve large numbers of agents whose physics, sensing, communication, and computation are coupled over a network. This course provides an introduction to the design, analysis, and implementation of algorithms for distributed optimization of networked systems with limited or no centralized coordination. It is intended primarily for graduate students in engineering, mathematics, or related areas with an interest in distributed optimization and its applications to engineered systems. The course focuses on the powerful modern framework of monotone operator theory, which presents a unified approach for developing seemingly distinct algorithms from a common meta-algorithm. The theory will be illustrated throughout the course on practical examples from control, planning, estimation, and machine learning, with applications such as mobile robots, electric power systems, wireless sensor networks, autonomous vehicles, and others, depending on student interest. Students will gain experience with numerical implementation of the associated algorithms on these practical examples.

Instructor

Prof. Michael W. Fisher Office: EIT-3112 Website: https://michaelwfisher.github.io Email: michael.fisher@uwaterloo.ca (Please place "ECE 780" in the subject header of all email correspondence.)

Course Website and Credit

Units: 0.5

Website: Hosted via LEARN. For access issues please contact learnhelp@uwaterloo.ca.

(Note: Course website will be available from the first day of class until the final exam is given.)

- 1. Convex Optimization Background (5 hours) Convex functions and optimization, subgradients and subdifferentials, Fermat's rule, Lagrangian duality, conjugate functions, proximal operators.
- 2. Monotone Operators and Fixed Point Iteration (6 hours) Monotone, contractive, and averaged operators; monotone inclusions, fixed point iteration, convergence of fixed point iteration for averaged operators, resolvents, algorithms: gradient descent, dual ascent, proximal point method, method of multipliers.
- 3. Base Operator Splitting Methods (3 hours) Operator splitting, forward-backward splitting, backward-forward splitting, Douglas-Rachford splitting, algorithms: proximal gradient, projected gradient, fixed point iteration with Douglas-Rachford splitting, variable metric forward-backward splitting.
- 4. *Primal-Dual Splitting Methods* (3 hours) Primal-dual splitting, dualization technique, variable metric technique, algorithms: alternating direction method of multipliers, dual proximal gradient, Condat-Vu.
- 5. Partially Distributed Optimization (3 hours) Consensus problem, algorithms: partially distributed proximal gradient, partially distributed alternating direction method of multipliers.
- Distributed Graph Consensus (3 hours) Mixing matrices, graph adjacency and Laplacian matrices, Perron-Frobenius theorem, distributed averaging, distributed graph consensus.
- Fully Distributed Optimization (3 hours)
 Dynamic tracking, algorithms: distributed inexact gradient method with gradient tracking, distributed alternating direction method of multipliers with graph consensus.
- 8. Applications of Distributed Optimization (6 hours) Distributed algorithms for model predictive control, trajectory optimization, regression, classification, optimal power flow, resource allocation, target estimation and localization.

Prerequisites

Undergraduate calculus and linear algebra. A course in optimization theory (e.g., CO/ECE 602 or equivalent) is strongly recommended.

Schedule

Lectures:

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Instructor Office Hours:

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Textbooks

The main textbook for the course is

• E. K. Ryu and W. Yin. Large-Scale Convex Optimization: Algorithms and Analyses via Monotone Operators, Cambridge University Press, 2023.

It is available for free as a pdf at this website, along with supplementary material. Other references that may be useful background material but are not required include

- F. Bullo. Lectures on Network Systems, Kindle Direct Publishing, 2022.
- E. K. Ryu and S. Boyd. "A Primer on Monotone Operator Methods," *Applied and Computational Mathematics*, 15(1): 3-43, 2016.
- G. Notarstefano, I. Notarnicola, and A. Camisa. Distributed Optimization for Smart Cyber-Physical Networks, 2020.
- S. Boyd and L. Vandenberghe. *Convex Optimization*, Cambridge University Press, 2009.

Assignments, Final Exam, and Evaluation

There will be approximately three to four substantial homework assignments. Students may discuss general strategies for assignments with one another, but they may not look at the assignments or programming code of any of their classmates and must submit their own individual assignments. Assignments will be graded based on completeness, accuracy, and clarity of presentation, which includes clear and well-commented code.

The course will have an in-person final exam.

Your final grade in the course is given by the formula

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Final Grade = (0.6 \times \text{Assignments}) + (0.4 \times \text{Final})
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Continuity of Education Plan

In the case of either

- a short-term (for example, one week) cancellation of in-person meetings, whether for this particular course or University-wide;
- a longer-term cancellation of in-person meetings, whether for this particular course or University-wide;

all lectures and office hours during that time will be transitioned to remote.

Academic Integrity

In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. For an overview of academic integrity policies and sanctions, see here.

Grievances

A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline

A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about rules for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate Associate Dean. For information on categories of offences and types of penalties, students should refer to Policy 71, Student Discipline. For typical penalties check Guidelines for the Assessment of Penalties.

Appeals

A decision made or penalty imposed under Policy 70 (Student Petitions and Grievances) (other than a petition) or Policy 71 (Student Discipline) may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72 (Student Appeals).

Student Wellness and Mental Health

Many students experience anxiety or other mental health related distress at some point in their academic careers; the University of Waterloo is committed to supporting student wellness and mental health. Resources are available to you, including this overview of wellness resources, and this repository of seminars and helpful workshops. The Student Wellness Coordinator is Kora Sevo, and can be reached at ENGWellness@uwaterloo.ca if you would like someone to talk to and to help arrange the support you need.

Disability Policy

AccessAbility Services, located in Needles Hall 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.