University of Waterloo
ECE 682: Multivariable Control Systems
Fall 2025
(tentative)

Lectures: Tuesday, Thursday 4:00 pm to 5:20 pm. EIT-3151.

Instructor: Prof. Christopher Nielsen.
Office hours: Monday 4:00pm to 5:20pm, EIT-4106.
Contact: cnielsen@uwaterloo.ca.
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Course description: The principle aim of this course is to introduce students to a rigorous treatment of control theory for linear systems. We study linear control systems in the state-space paradigm where we use vector differential equations. The advantage is that it allows us to treat multi-input multi-output systems, also called multivariable systems, in the same way as single-input single-output systems.

We study the structural properties of linear systems, e.g., controllability and observability, that are invariant under linear changes of coordinates to solve some fundamental control and estimation problems. This includes a comprehensive treatment of the most central issue of control theory: tracking and regulation with internal stability. Throughout the course, our approach is to first characterize solvability of a given control design problem as a verifiable structural properties of the plant model. Then, if all is well, we present simple, constructive procedures to synthesize controllers.

Calendar description: An introduction to control theory for linear time-invariant finite-dimensional systems from both the state-space and input-output viewpoints. State-space theory: the concepts of controllability, observability, stabilizability, and detectability; the pole-assignment theorem; observers and dynamic compensation; L.Q.R. regulators. Input-output theory: the ring of polynomials and the field of rational functions; the algebra of polynomial and rational matrices; coprime factorization of transfer matrices; Youla parametrization. Introduction to optimal control.

Intended learning outcomes: At the end of the course it is hoped that you are able to:

- Find minimal state-space realizations of linear time-invariant control systems
- Apply linear algebra in an abstract (geometric) style to study structural properties of linear systems
- Design stabilizing state feedback using pole placement and linear quadratic optimal control
- Design controllers that solve the output feedback stabilization problem and the output regulation problem
- Apply $\mathcal{H}_2$-optimal control to design output feedback controllers.

Recommended background: Undergraduate linear algebra; introductory course on feedback control (ECE380 or equivalent).

Text: Course notes are available on the course website. The optional suggested textbook is

Linear System Theory and Design, 3rd edition, C.T. Chen.

Additional references

- Linear System Theory, F.M. Callier and C.A. Desoer.
- Control Theory for Linear Systems, H.L. Trentelman, A.A. Stoorvogel, M. Hautus.
- Linear Systems Theory, J.P. Hespanha.

Student Assessment:

50% Final exam. Open book.

30% Assignments. Three assignments spread over the term.

20% Course project. Due on 2025/12/02 at 4:30pm. Details on course website.

Tentative Topics List:

1. **Introduction to linear multivariable systems**
   Motivation, examples.
2. **Linear state-space models**
   Deriving state models, Solution of state equation, Realizations, Poles and zeros of a multivariable system.
3. **Linear algebra**
   Vector spaces, Linear transformations, Quotient spaces, Invariant subspaces.
4. **Controllability**
   Reachable states, Properties of controllability, PBH test, Equivalence of pole placement and controllability, Stabilizability.
5. **Observability**
   The Kalman decomposition, Detectability, Observers, Observer based controllers.
6. **Quadratic optimal control**
   Lyapunov equation, Riccati equation, The LQR problem and its solution.
7. **Stability of feedback systems**
   Well-posedness, Feedback stability, Output feedback stabilization.
8. **Regulation and tracking**
   Output regulation problem, Solution in the case of full information, Solution in the case of measurement feedback, Structurally stable synthesis.

9. **The $\mathcal{H}_2$-optimal control problem**
   Problem setup and motivation, Standard form of the problem and its solution, Regular form of the problem, Relationship to the LQG problem.

Academic integrity, grievance, discipline, appeals and note for students with disabilities:

Academic integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check the Office of Academic Integrity for more information.]

Grievance: A student who believes that a decision affecting some aspect of their university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4. When in doubt, please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for their actions. [Check the Office of Academic Integrity for more information.] A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate associate dean. For information on categories of offences and types of penalties, students should refer to Policy 71, Student Discipline. For typical penalties, check Guidelines for the Assessment of Penalties.

Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes they have a ground for an appeal should refer to Policy 72, Student Appeals.

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