University of Waterloo
ECE 688: Nonlinear Systems
Winter 2025 (tentative)

Lectures: Tuesday, Thursday 14:30 pm to 16:00. EIT-3141.

Instructor: Prof. Christopher Nielsen.
Office hours: Monday 4:00pm to 5:20pm, EIT-4106.
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Course description:

Virtually all systems are nonlinear in nature. While it is sometimes possible to approximate a system's behaviour using a linear model, this approach is only valid when the system stays close to a "nominal" set of operating conditions. When the system deviates significantly from these conditions, a linearized model becomes inadequate for describing the underlying phenomena. This is where the material covered in this course becomes valuable.

In this course we cover classical and modern approaches to the analysis of finite-dimensional, deterministic, nonlinear systems modeled by ordinary differential equations with an emphasis on stability, robustness and the effect of interconnecting dynamical system and provide an introduction to nonlinear stabilization. The material offers a rigorous foundation for engineers interested in an in-depth understanding of nonlinear systems, with applications across all branches of engineering.

Calendar description: Equilibrium points, linearization; second order systems; contraction mapping principle; existence and uniqueness of solutions to nonlinear differential equations; periodic solutions; Lyapunov stability; the Lure problem; introduction to input-output stability, introduction to nonlinear control techniques.

Intended learning outcomes: At the end of the course it is hoped that you are able to:

- Explain the relationship between a dynamical system and its associated vector field
- Investigate the stability of a nonlinear system using Lyapunov's direct and indirect methods and the invariance principle
- Design stabilizing state feedback using linearization and control Lyapunov functions
- Determine if a system is dissipative for a given storage function and supply rate
- Design passivity-based stabilizing controllers for simple mechanical systems

Recommended background: Undergraduate calculus and linear algebra; some exposure to state-space models.

Text: Course notes are available on the course website. The optional suggested textbook is

Nonlinear Systems, 3rd edition, H.K. Khalil.

Additional references

- Nonlinear Systems Analysis, 2nd edition. M. Vidyasagar (2002).
- Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach. W. Haddad and V. Chellaboina (2008).
- $\mathcal{L}_2$-Gain and Passivity Techniques in Nonlinear Control, A. van der Schaft (1996).
- Differential Equations, Dynamical Systems, and Linear Algebra. M. Hirsch and S. Smale (1974).

Evaluation:

50% Final exam: open book.

50% Assignments: Four (4) assignments spread over the term.

Tentative Topics List:

1. **Introduction to nonlinear models and phenomena**
   Examples.
2. **Mathematical preliminaries**
   Norms, basic topology, continuity and differentiation.
3. **Dynamical systems and differential equations**
   Dynamical systems, vector fields and local flows, existence and uniqueness.
4. **Key concepts in dynamics**
   Equilibria and closed orbits, invariant sets, Nagumo's theorem, limit sets, linearization of nonlinear systems.
5. **Stability theory**
   Notions of stability, Lyapunov's direct method, the invariance principle, exponential stability and linearization, converse theorems.
6. **Introduction to nonlinear stabilization**
   Stabilization using linearization, control Lyapunov functions, Artstein-Sontag theorem, Brockett's necessary conditions for continuous stabilizability.
7. **Dissipative systems**
   Dissipative systems and Lyapunov stability, finite-gain stability, interconnected dissipative systems, application to mechanical control systems.

Academic integrity, grievance, discipline, appeals and note for students with disabilities:

Academic integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check the Office of Academic Integrity for more information.]

Grievance: A student who believes that a decision affecting some aspect of their university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4. When in doubt, please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for their actions. [Check the Office of Academic Integrity for more information.] A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate associate dean. For information on categories of offences and types of penalties, students should refer to Policy 71, Student Discipline. For typical penalties, check Guidelines for the Assessment of Penalties.

Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes they have a ground for an appeal should refer to Policy 72, Student Appeals.

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