ECE 686: Filtering and Stochastic Control
Winter 2023

Instructors: *Professors Andrew Heunis and Ravi R. Mazumdar*, , e-mail: heunis@uwaterloo.ca and mazum@uwaterloo.ca

Schedule: [redacted]

There will be no lectures during the week of March 20. Therefore 3 additional lectures on Wed March 15, Wed. March 29 and Wed. April 5 have been scheduled in [redacted]

Course Website: LEARN (Announcements, notes and problem sets will be posted)

Pre-requisite: Probability ECE 203 (or equivalent), ECE 604 (ideal), and ECE682 (Multivariable Control ideal).

Aims: The principal aim of this course is to introduce the student to to the basic mathematical theory of filtering and control of linear stochastic systems, and to Markov Decision Processes that are playing an important role in modern machine learning. The course is divided into two parts:

Part I: Filtering and Control of Linear Stochastic Systems:

The focus will be on state estimation and control in stochastic linear dynamical systems, in both discrete and continuous-time settings. State estimation in linear systems is shown to be equivalent to projection onto a closed subspace, generated by an observation process, in a Hilbert space of random variables. This formulation of state estimation allows one to obtain representations for state estimators in the form of the Kalman filter equations under very general conditions. The discrete-time Kalman filter and dynamic programming are used to study the discrete linear stochastic optimal control problem with quadratic cost function.

Part II: Introduction to Markov Decision Processes.

We will first introduce the framework for markov decision theory. Here the state of the system is assumed to be discrete and evolves according to a markov chain whose transition probabilities can be altered by decisions or actions. We will first study the optimality equations that allow us to choose optimal decisions to optimize cost functions. We will consider the discounted problem and the ergodic setting or the long-term average problem. We will see how optimal policies can be obtained. The applications focus will be on an important set of problems referred to as multi-armed bandit ptoblems and we will see how the optimal decisions can be characterized through the Gittins Index policies. Finally we will discuss the notion of regret in terms of policies. Applications to randomized testing and Machine Learning problems will be given.

COURSE OUTLINE

Part I

1. Elements of metric space theory and Hilbert spaces.

2. The geometric theory of linear filtering.

3. Discrete-time linear filtering and the discrete-time Kalman filter.

4. Discrete-time stochastic optimal control with complete and partial observations.

Part II

5. Markov Decision Processes; Optimality of Markov controls. Dynamic programming and the markov operator.

6 Discounted costs and infinite horizon dynamic programming

7. Average-cost dynamic programming

8. Multi-armed Bandit problems: Gittins Index, Indexable systems. Regret.

TEXT AND REFERENCES

There is no required text for the course. Course notes will be provided. The necessary real analysis, stochastic processes background, dynamic programming etc will be covered as part of the course.

Suggested Texts and References:

For Part I

M.H. A. Davis and R.B. Vinter: Stochastic Modelling and Control, Chapman and Hall, London, 1984

For Part II

S. M. Ross : Introduction to Stochastic Dynamic Programming, Academic Press, N.Y., 1982

Course Evaluation

- Problem sets will be handed out. The onus is on you all to attempt them. Solutions will be posted.
- There will be a midterm that will count for 50% of the grade and a final exam that will count for 50%.
- The midterm will be in-class on Tuesday February 28, 2023. The date for the final exam will be announced later. The final exam will also be in-class.

Additional remarks

- All in term exams will be open notes.
- If you miss the midterm exam no make-up exam will be given. If you have a valid reason then your final grade will be based on your performance in the rest of the course.
- Dishonesty will be dealt with by the rules of the university.