ECE 780 Model Predictive Control Winter 2022
M-F/1130-1250/ EIT 3153

Update Jan-9-2022: Classes are online till at least January 25th 2022, please see LEARN for the zoom link. Upon resumption of in-person classes, the lectures are scheduled to be held in EIT 3153.

Calendar Description

This course will tackle the problem of optimal control of dynamical systems with constraints. This is done through an optimization-based method called Model Predictive Control, or MPC. The course covers: 1) basic concepts of system theory, including state-estimation and hybrid systems, 2) convex optimization, constrained and unconstrained optimal control, 3) concepts of stability, reachability, invariant sets, 4) Model Predictive Control formulations, and associated mathematical guarantees on robustness, optimality and recursive feasibility, 5) numerical methods for MPC.

Instructor

Dr. Yash Vardhan Pant (yash.pant@uwaterloo.ca)
Office: [redacted]. Virtual Office hours will be held [redacted] Zoom. See LEARN for the zoom link.

Course Outline

Increased system complexity and more demanding performance requirements have rendered traditional control laws inadequate whether simple PID loops are considered or robust feedback controllers are designed according to some H2/infinity criterion. Applications ranging from the process industries to the automotive and the communications sector are making increased use of Model Predictive Control (MPC) where a fixed control law is replaced by on-line optimization performed over a receding horizon. The advantage is that MPC can deal with almost any time-varying process and specifications, limited only by the availability of real-time computation power. In the last few years we have seen tremendous progress in this interdisciplinary area where fundamentals of systems theory, computation and optimization interact. For example, methods have emerged to handle hybrid systems, i.e. systems comprising both continuous and discrete components. Also, it is now possible to perform most of the computations off-line thus reducing the control law to a simple look-up table online.

The first part of the course is an overview of basic concepts of system theory and optimization, including hybrid systems and multi-parametric programming. In the second part we will show how these concepts are utilized to derive MPC algorithms and to establish their properties. Based on the makeup of the class, domain specific examples will be formulated and analyzed as Model Predictive Control algorithms.

See table 1 for a detailed timeline of the course.

Grading

The course will consist of 4 homework assignments (weighted equally) and a final exam. The grading scheme is:

- Homeworks: 60%
- Final exam: 40%

The final exam will be an open-book take home exam. See table 1 for dates.

Late Turn-in Policy

Homework will released on Mondays and due on Fridays the following week. Homework received by the Monday after the due date (i.e., 2 weeks after release) will be accepted with a 20$\%$ late penalty. Homework will not be accepted after the late submission date (see table 1).

Intended Learning Outcomes

By the end of this course, students will be able to:

1. Recognize control problems where Model Predictive Control (MPC) offers advantages over classical control methods (e.g., PID and pole-placement) and modern optimal control methods (e.g., LQR).

2. Formulate constrained optimal control problems (e.g., motion planning of robotic systems, control of chemical plants etc.) as Model Predictive Control optimizations, and deploy the correct solvers to obtain sequences of control signals.

3. Verify that closed-loop control with the designed MPC has guarantees on stability, optimality (or bounded sub-optimality), robust constraint satisfaction (state and input constraints) and recursive feasibility of the underlying optimization.

4. Implement MPC algorithms using the Multi-Parametric Optimization (MPT 3.0+) toolbox in MATLAB.

Table 1: Course timeline, based on 24 lectures, each of 1.5 hours in duration. MPC is an abbreviation for Model Predictive Control, which is the focus of this course.

Class	**Date**	**General Topic**	**Specific Content**	**Assignments**
#01	Fri, Jan 7	Introduction and overview	Limitations of classical control, Optimization-based Control, Origins of MPC, applications	
#02	Mon, Jan 10	System Theory Basics (1)	Models of dynamic systems, Analysis of Discrete-time Linear Systems	
#03	Fri, Jan 14	System Theory Basics (2)	Analysis of Discrete-time Non-linear systems	
#04	Mon, Jan 17	Model Uncertainty and State Estimation	Uncertainty modeling (stochastic and worst-case disturbances), Linear State Estimation	Homework 1 released
#05	Fri, Jan 21	Convex Optimization (1)	Convex sets, functions and optimization problems	
#06	Mon, Jan 24	Convex Optimization (2)	Duality, Generalized Inequalities, connection to optimal control	
#07	Fri, Jan 28	Unconstrained Linear Optimal Control (1)	Finite horizon, Receding Horizon Control problems	Homework 1 due
#08	Mon, Jan 31	Unconstrained Linear Optimal Control (2)	Solutions via dynamic programming, Infinite Horizon Control	Homework 1 late
#09	Fri, Feb 4	Constrained Finite Time Optimal Control (1)	State/input constraints, Predictive Control basics	
#10	Mon, Feb 7	Constrained Finite Time Optimal Control (2)	Constrained Optimal control $(1, 2, \infty)$-norm, Quadratic Program Formulations	Homework 2 released
#11	Fri, Feb 11	Feasibility and Stability of MPC	Receding horizon MPC, Terminal Conditions, Stability guarantees	
#12	Mon, Feb 14	Invariance	Recursive feasibility of MPC, Controlled Invariance, set representations	
#13	Fri, Feb 18	Reachability and set invariance (1)	Reachable & Invariant sets, set computations	Homework 2 due
#14	Mon, Feb 28	Reachability and set invariance (2)	Reachability & Controllabillity, Robust MPC	Homework 3 released, Homework 2 late
#15	Fri, Mar 4	Practical issues in MPC	Reference tracking, Soft constraints, Generalizing MPC	
#16	Mon, Mar 7	Explicit MPC (1)	Offline-online control, Multi-Parametric Programming (mpQP, mpLP)	
#17	Fri, Mar 11	Explicit MPC (2)	Real-time MPC via explicit feedback laws, Computation tool	Homework 3 due
#18	Mon, Mar 14	Robust MPC (1)	Uncertainty models, bounded additive noise, Robust open-loop MPC	Homework 3 late
#19	Fri, Mar 18	Robust MPC (2)	MPC as a game, closed-loop MPC, Tube-MPC	
#20	Mon, Mar 21	Hybrid MPC (1)	Hybrid Systems, Optimal Control of Hybrid Systems	Homework 4 released
#21	Fri, Mar 25	Hybrid MPC (2)	MPC and Explicit MPC for Hybrid Sytems	
#22	Mon, Mar 28	Numerical methods (1)	Gradient and Newton methods, Preconditioning and convergence	
#23	Fri, Apr 1	Numerical methods (2)	Alternating minimization, Interior point methods, Software	Homework 4 due
#24	Mon, Apr 4	MPC applications in autonomous systems	Recent research outcomes - guest lecturer (TBD)	Final exam released (Due Apr 15), Homework 4 late

Recommended Background

While not required, it is recommended that students have a background in linear systems, control theory and convex optimization, e.g., have taken courses such as ECE 488, ECE/CO 602, ECE 682 (or equivalent) prior to enrolling in this course. The basics in linear systems and convex optimization will be reviewed in first few lectures. We will make use of the Multi-Parametric Toolbox (MPT3) for MATLAB which was developed by the automatic control group at ETH Zurich, and other universities. The student should be comfortable writing MATLAB code.

Recommended reading

- Stanford Engineering's course on Convex Optimization.
- Stanford Engineering's course on Introduction to Linear Dynamical Systems.

Textbook

F. Borrelli, A. Bemporad and M. Morari, "Predictive Control for Linear and Hybrid Systems", Cambridge University Press.
Note: The book (in pdf form) and other related material are available on Professor Francesco Borrelli's MPC course web page.

Fair Contingencies for Emergency Remote Teaching

We are facing unusual and challenging times. The course outline presents the instructor's intentions for course assessments, their weights, and due dates in Winter 2022. As best as possible, we will keep to the specified assessments, weights, and dates. To provide contingency for unforeseen circumstances, the instructor reserves the right to modify course topics and/or assessments and/or weight and/or deadlines with due and fair notice to students. In the event of such challenges, the instructor will work with the Department/Faculty to find reasonable and fair solutions that respect rights and workloads of students, staff, and faculty.
Note: In case of a short or long term cancellation of in-person classes, the course will switch to online lectures held at the same times as the scheduled in-person classes. These online lectures will be recorded and made available to students in different time zones.

Acknowledgement

The course instructor would like to thank Professor Manfred Morari, whose slides are a basis for most of the lectures in this course.