ECE 780 Model Predictive Control Spring 2023

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Calendar Description

This course will tackle the problem of optimal control of dynamical systems with constraints. This is done through an optimization-based method called Model Predictive Control, or MPC. The course covers: 1) basic concepts of system theory, including state-estimation and hybrid systems, 2) convex optimization, constrained and unconstrained optimal control, 3) concepts of stability, reachability, invariant sets, 4) Model Predictive Control formulations (such as robust and hybrid MPC) and associated mathematical guarantees on robustness, optimality and recursive feasibility, 5) numerical methods for MPC. While the primary focus is on the underlying theory, the course will also cover modern software tools for implementing MPC.

Instructor

Dr. Yash Vardhan Pant (yash.pant@uwaterloo.ca)
Office: [redacted]. Office hours will be announced at the beginning of the term.

Course Outline

Increased system complexity and more demanding performance requirements have rendered traditional control laws inadequate whether simple PID loops are considered or robust feedback controllers are designed according to some H2/infinity criterion. Applications ranging from the process industries to the automotive and the communications sector are making increased use of Model Predictive Control (MPC) where a fixed control law is replaced by on-line optimization performed over a receding horizon. The advantage is that MPC can deal with almost any time-varying process and specifications, limited only by the availability of real-time computation power. In the last few years we have seen tremendous progress in this interdisciplinary area where fundamentals of systems theory, computation and optimization interact. For example, methods have emerged to handle hybrid systems, i.e. systems comprising both continuous and discrete components. Also, it is now possible to perform most of the computations off-line thus reducing the control law to a simple look-up table online.

The first part of the course is an overview of basic concepts of system theory and optimization, including hybrid systems and multi-parametric programming. In the second part we will show how these concepts are utilized to derive MPC algorithms and to establish their properties. Based on the makeup of the class, domain specific examples will be formulated and analyzed as Model Predictive Control algorithms.

See table 1 for a detailed timeline of the course.

Grading

The course will consist of 4 homework assignments (weighted equally) and a project. The grading scheme is:

- Homeworks: 80%
- Project: 20%

The course project, based on the interests of the student can be either: a) formulating and solving a challenging problem in their domain of interest via a MPC-based approach, b) a survey of existing MPC-like approaches for a particular problem (such as risk-based MPC for vehicle motion planning). The course project deliverables are:

- a (no more than) 6-page 2 column IEEE-conference like research paper that formulates the problem, covers related work, discusses the solution and has numerical simulation results via the student's own implementation.
- a brief presentation that covers the most important points of the project.

Note, for the survey-based project, the student is expected to implement at least one of the surveyed methods on their own.

Late Turn-in Policy

Homeworks will released on Mondays and due on Wednesdays the following week. Homeworks received by the Monday after the due date (i.e., 2 weeks after release) will be accepted with a 20% late penalty. Homeworks will not be accepted after the late submission date.

Intended Learning Outcomes

By the end of this course, students will be able to:

1. Recognize control problems where Model Predictive Control (MPC) offers advantages over classical control methods (e.g., PID and pole-placement) and modern optimal control methods (e.g., LQR).

Table 1: Course timeline. Each lecture is 1.5 hours in duration. MPC is an abbreviation for Model Predictive Control, which is the focus of this course.

Lecture	Date	General Topic	Specific Content	Assignments
#01	Mon, May 8	Introduction and overview	Limitations of classical control, Optimization-based Control, Origins of MPC, applications	
#02	Wed, May 10	System Theory Basics (1)	Models of dynamic systems, Analysis of Discrete-time Linear Systems	
#03	Mon, May 15	System Theory Basics (2)	Analysis of Discrete-time Non-linear systems	Homework 1 released
#04	Wed, May 17	Model Uncertainty and State Estimation	Uncertainty modeling (stochastic and worst-case disturbances), Linear State Estimation	
#05	Tue, May 23[1]	Convex Optimization (1)	Convex sets, functions and optimization problems	
#06	Wed, May 24	Convex Optimization (2)	Duality, Generalized Inequalities, connection to optimal control	Homework 1 due
#07	Mon, May 29	Unconstrained Linear Optimal Control (1)	Finite horizon, Receding Horizon Control problems	
#08	Wed, May 31	Unconstrained Linear Optimal Control (2)	Solutions via dynamic programming, Infinite Horizon Control	
#09	Mon, June 5	Constrained Finite Time Optimal Control (1)	State/input constraints, Predictive Control basics	Homework 2 released
#10	Wed, June 7	Constrained Finite Time Optimal Control (2)	Constrained Optimal control $(1, 2, \infty)$-norm, Quadratic Program Formulations	
#11	Mon, June 12	Feasibility and Stability of MPC	Receding horizon MPC, Terminal Conditions, Stability guarantees	
#12	Wed, June 14	Invariance	Recursive feasibility of MPC, Controlled Invariance, set representations	Homework 2 due
#13	Mon, June 19	Reachability and set invariance (1)	Reachable & Invariant sets, set computations	Homework 3 released
#14	Wed, June 21	Reachability and set invariance (2)	Reachability & Controllabillity, Robust MPC	
#15	Mon, June 26	Practical issues in MPC	Reference tracking, Soft constraints, Generalizing MPC	
#16	Wed, June 28	Explicit MPC (1)	Offline-online control, Multi-Parametric Programming (mpQP, mpLP)	Homework 3 due
#17	Mon, July 3	Explicit MPC (2)	Real-time MPC via explicit feedback laws, Computation tool	
#18	Wed, July 5	Robust MPC (1)	Uncertainty models, bounded additive noise, Robust open-loop MPC	
#19	Mon, July 10	Robust MPC (2)	MPC as a game, closed-loop MPC, Tube-MPC	Homework 4 released
#20	Wed, July 12	Hybrid MPC (1)	Hybrid Systems, Optimal Control of Hybrid Systems	
#21	Mon, July 17	Hybrid MPC (2)	MPC and Explicit MPC for Hybrid Sytems	
#22	Wed, July 19	Numerical methods	Gradient and Newton methods, Interior point methods, Software	Homework 4 due
#23	Mon, July 24	Data-driven MPC for systems with partially known dynamics	Gaussian Process-based MPC, Bayesian Linear Regression-based MPC.	
#24	Wed, July 26	Project presentations		
#25	Mon, July 31	Project presentations		Project paper due.

2. Formulate constrained optimal control problems (e.g., motion planning of robotic systems, control of chemical plants etc.) as Model Predictive Control optimizations, and deploy the correct solvers to obtain sequences of control signals.

3. Verify that closed-loop control with the designed MPC has guarantees on stability, optimality (or bounded sub-optimality), robust constraint satisfaction (state and input constraints) and recursive feasibility of the underlying optimization.

4. Implement MPC algorithms using the Multi-Parametric Optimization (MPT 3.0+) toolbox in MATLAB.

Recommended Background

While not required, it is recommended that students have a background in linear systems, control theory and convex optimization, e.g., have taken courses such as ECE 488, ECE/CO 602, ECE 682 (or equivalent) prior to enrolling in this course. The basics in linear systems and convex optimization will be reviewed in first few lectures. We will make use of the Multi-Parametric Toolbox (MPT3) for MATLAB which was developed by the automatic control group at ETH Zurich, and other universities. The student should be comfortable writing MATLAB code.

Recommended reading

- Stanford Engineering's course on Convex Optimization.
- Stanford Engineering's course on Introduction to Linear Dynamical Systems.

Textbook

F. Borrelli, A. Bemporad and M. Morari, "Predictive Control for Linear and Hybrid Systems", Cambridge University Press.
Note: The book (in pdf form) and other related material are available on Professor Francesco Borrelli's MPC course web page.

Acknowledgement

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