ECE 686 Filtering and Control of Stochastic Linear Systems Winter 2027

Tentative Course Overview

Please note: This document is a *tentative* overview of ECE 686, provided to help graduate students with course selection. The official course outline will be published on outline.uwaterloo.ca at the start of the Winter 2027 term, and it will supersede this document.

Instructor

Prof. Stephen L. Smith (stephen.smith@uwaterloo.ca)

- Please put "ECE686" at the front of your subject line for all course-related email.
- Class meetings are currently scheduled for Tuesdays and Fridays, 11:30 AM–12:50 PM; see Quest for the current schedule and room.

Course Description

Course Motivation: Broadly speaking, this is a course on decision making under uncertainty. Specifically, the course focuses on the study of stochastic linear systems where the dynamics have certain components of a stochastic nature. The course will cover both estimation and control of such systems. The first half of the course establishes the fundamentals of the estimation problem, culminating in the derivation of the fact that state estimation in linear systems is equivalent to projection onto a closed linear subspace generated by an observation process in a Hilbert space of random variables. This leads to the Kalman filter, which finds use in many applications ranging from aerospace to finance. The course will then cover the issues of stochastic optimal control (based on dynamic programming), control of Markov chains, which naturally leads to an introduction to reinforcement learning as a method to perform approximate dynamic programming.

Calendar Description for ECE 686:

This is a course on continuous-parameter state estimation and control for stochastic linear systems. It is based on a single unifying theme, namely that state estimation in linear systems is equivalent to projection onto a closed linear subspace generated by an observation process in a Hilbert space of random variables. This formulation of state estimation leads to the innovations theorem of Kailath, and this in turn has a number of corollaries of considerable practical importance, such as the Kalman-Bucy filtering formulae and the Rauch-Tung-Striebel prediction formulae which are much used for example in problems of inertial guidance and control in aerospace, in stochastic optimal control, and (more recently) in econometrics.

Anticipated Learning Outcomes

By the end of this course, students should be able to

- Apply probability theory to model stochastic systems
- Understand the projection theorem and its application in estimation and filtering
- Develop estimators using least squares optimization
- Apply the Kalman filter for state estimation of linear systems
- Utilize dynamic programming to formulate optimal control problems
- Apply linear quadratic regulators (LQR) to control stochastic linear systems
- Understand Markov chains and their corresponding optimal control problems

Tentative List of Topics

The following is a tentative list of topics that are expected to be covered in the course.

1. Introduction and Mathematical Background
    - Motivation and overview
    - Review of probability theory and stochastic processes
2. Derivation of the Projection Theorem
    - Metric and linear spaces
    - Hilbert spaces and the projection theorem for linear estimation
3. Estimation of Discrete-Time Linear Systems
    - Least-squares and recursive estimation
    - The discrete-time Kalman filter
4. Stochastic Optimal Control
    - Dynamic programming and the principle of optimality
    - Optimal control with complete and partial observations
    - Linear quadratic regulators
5. Controlled Markov Chains
    - Markov decision processes
    - Value and policy iteration
    - Q-learning and introduction to reinforcement learning

Course Materials

There is no required textbook; notes will be provided each week with blanks that should be filled in during lecture. Parts of the course are based on notes by Prof. Andrew Heunis, and the following book:

1. P. R. Kumar and P. Varaiya, *Stochastic Systems: Estimation, Identification and Adaptive Control*, 1986.

The following textbooks may also be useful for additional information:

1. J. L. Speyer and W. H. Chung, *Stochastic Processes, Estimation and Control*, 2008.
2. D. P. Bertsekas, *Dynamic Programming and Optimal Control*, 2005.

Anticipated Course Structure and Assessment

The course is expected to include the following components. Final assessment details, weights, and policies will be specified in the official course outline.

- **Assignments** (approx. 25%): three or four problem-set assignments.
- **Course project** (approx. 25%, including a proposal and a final presentation with code): you may either (1) apply estimation and/or optimal control to a problem motivated by your own research, or (2) select a research paper or topic that uses estimation or optimal control. In both cases, the main deliverable is a working implementation of the method in MATLAB or Python, with results on your research problem or a simulation reproducing the main result of the paper.
- **Final exam** (approx. 50%): an in-person exam during the final exam period.