University of Waterloo
Department of Electrical and Computer Engineering

ECE 602 – Introduction to Optimization
Winter, 2023

Classes: [redacted]
Course website: `www.ece.uwaterloo.ca/~ece602`
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Course Outline

Control systems, communication and networking, electronic circuit design, data analysis and modelling, statistics, signal and image processing, and finance are only a few among numerous areas of modern engineering and science, which routinely depend on the methods of numerical optimization. Whether one is prototyping an electronic circuit or developing an investment portfolio, the desired solution can often be associated with an extremum of a specially designed function, known as a cost or objective function. Consequently, the task of finding an optimal solution can often be cast in the form of finding a point, at which such function reaches its minimum (or, alternatively, maximum) value. Naturally, how tractable and realizable the above task is depends on the properties of the cost function at hand as well as its domain and range, both of which can be either continuous or discrete. Accordingly, the fundamental objective of this course is to introduce some principal concepts of optimization theory along with its key numerical techniques. Throughout the course, students will gain valuable background in optimization methods applicable to a wide range of engineering problems along with experience in solving optimization problems of their own choice.

Syllabus

Part I: Continuous optimization

1. *Mathematical preliminaries* (3 hours)
   Vectors, norms, distances, open and closed sets, continuous functions, closed functions, derivative and gradient, Hessian, definiteness and matrix inequalities, singular value and eigenvalue decompositions, pseudo-inverse, algorithm complexity, solution of linear equations.

2. *Convex optimization problems* (3 hours)
   Basic terminology, standard form of convex optimization problems, local and global optima, first and second order optimality conditions, linear optimization problems, quadratic optimization problems, second-order cone programming, semi-definite programming, Lagrange duality, saddle-point theorem, primal vs dual problems.

3. *Non-smooth optimization* (3 hours)
   Smooth vs non-smooth optimization, subdifferential and subgradient, optimality conditions for non-differentiable costs, proximal operators and their properties, Moreau decomposition, proximal gradient methods.

4. *Algorithms: Unconstrained minimization* (3 hours)
   Quadratic minimization and least squares, descent methods, line search, backtracking, gradient descent method, steepest descent method, Newton's method, non-linear least squares, Gauss-Newton method, implementation and examples.

5. *Algorithms: Constrained minimization* (3 hours)
   Primal and dual feasibility conditions, Newton's method with equality constraints, convex-concave games, barrier method, primal-dual interior point methods, practical considerations and examples.

6. *Proximal algorithms* (3 hours)
   Proximal gradient methods, proximal gradient methods with line search, acceleration, dual decomposition, dual proximal gradient method, Douglas-Rachford splitting, alternating directions method of multipliers, implementation and examples.

7. *Applications of convex optimization* (6 hours)
   Approximation and fitting, least-norm problems, regularized approximation, robust approximation, maximum-likelihood estimation, logistic regression, maximum-a-posteriori probability estimation, optimal detection and hypothesis testing, robust detectors, experiment design, extremal volume ellipsoids, centring, classification, location problems with path constraints.

Part II: Discrete optimization

8. *Graphs and network flow problems* (3 hours)
   Graphs and flows, examples of network flow models, an overview of network flow algorithms, algorithm complexity (good, bad, and polynomial algorithms).

9. *Shortest path problems* (3 hours)
   Problem formulation and applications, a generic shortest path algorithm, label setting (Dijkstra) methods, label correcting methods, single origin/single destination methods, auction algorithms, dynamic programming.

10. *The max-flow and the min-flow cost problems* (3 hours)
    Cuts in a graph, the max-flow/min-cut theorem, the maximal and minimal saturated cuts, decomposition of infeasible network problems, the Ford-Fulkerson algorithm, transformations and equivalences, duality.

11. *Network problems with integer constraints* (3 hours)
    Basic formulation, branch-and-bound, Lagrangian relaxation, cutting plane methods, local search methods, genetic algorithm, simulated annealing, rollout algorithms.

Course Textbooks

The students will be provided with copies of lecture slides developed based on:

1. S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004 (freely available online at `http://stanford.edu/~boyd/cvxbook/`)

2. D. P. Bertsekas, *Network Optimization: Continuous and Discrete Models*, Athena Scientific, 1998 (freely available online at `http://web.mit.edu/dimitrib/www/net.html`)

Marking Scheme

1. Home Assignments (5 assignments $\times$ 5 points): 25%

2. Course project: 35%

3. Final Exam: 40%