Distributed Time-varying Kalman Filter Design and Estimation over Wireless Sensor Networks Using OWA Sensor Fusion Technique

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***Abstract*— In this paper, a novel estimation procedure is proposed, which consists of designing a distributed class of time-varying Kalman filter based on wireless sensor networks topology along with a new sensor fusion method. The proposed technique is employed to estimate the states and outputs of a linear time-varying system with a high level of accuracy. Both the dynamics of the system and the measurements are assumed to be contaminated by external noises. The notion of Orness and Ordered Weighted Averaging (OWA) operator technique are utilized to fuse the estimation of the sensors. O'Hagan method, along with the gradient descent method, is employed to find the optimal weights. In the introduced approach, OWA weights are learned for each observation such that they efficiently minimize the estimation error for that particular observation. This will result in an outstanding high accurate sensor fusion outcome. In addition, two optimistic and pessimistic exponential OWA operators are used and compared together to achieve a pre-specified level of Orness. The simulation results are shown on a given linear time-varying system to verify the effectiveness of the proposed sensor fusion distributed filtering design method.**

I. Introduction

A. Motivation

One of the most interested scopes of the control community is designing and implementing different types of filters to estimate the states of a target plant. In simple cases, it is assumed that there exist no disturbances involved in the dynamics of the system under study and also an ideal measurement with no measurement noises is considered. Although it simplifies the design and analysis of the filter in a great deal, it is not being confirmed by practical aspects in real-world applications anymore. Wireless sensor networks are great candidates to be adopted by their sparseness topology to design filters with high practical capabilities of estimating the states of the system in the presence of disturbances and measurement noises. Due to the limitation of the transmission bandwidth and power transmission of each sensor, it is preferable to take transmission actions as conservative as possible over the network.

Decentralized filtering is one of the ways that could address the above challenge by benefitting from the base structure of wireless sensor networks, which is shown in Fig. 1. In this manner, a set of intelligent sensors (nodes) try to estimate the states of the system locally and cooperatively. In some class of filters, the data will be sent over the network only when a certain triggering condition is satisfied [1]. In fact, by utilizing certain conditions of triggering, they will send their data to their neighbors only in a set of certain time instances. The triggering rules could be designed based on different methods and regarding user/application requirements. These class of event-triggered filters, reduce the use of transmission bandwidth and energy resources in a great amount in comparison to time-triggered counterparts. A general view of the event-based filtering procedure is shown in Fig. 2. The cases in which the considered plant is assumed to have disturbances and there exist measurement noises could be addressed by the decentralized approach over a wireless sensor network as well by developing stochastic filters [2], [3]. In particular, in the research works such as [4] and [5], the authors design the stochastic version of the Kalman filter for multi-sensor networks in which there exist inevitable unreliabilities. In the latter, stochastic stability conditions are also derived for the extended version of the Kalman filter.

Besides, in some recent works, an upper bound for the estimation error covariance matrix has been proposed, which is minimized by properly designing the filter parameters [1]. In some cases, the limitation on the transmission bandwidth is a highly demanding requirement that requires the filtering procedure to have the only valuable data transmitted over the network [6].

B. Literature Review

In [1], a distributed recursive filter based on the structure of a wireless sensor network has been designed. This filter is an event-triggered one, which makes it quite distinguishable from typical time-driven counterparts. In particular, in that work, a linear time-varying plant has been considered. Based on a wireless sensor network, each intelligent sensor (node) tries to estimate the states of the system locally. Each sensor sends its data to its neighbors only when a certain threshold called SoD (Send-on-Delta) has been satisfied. This will have two main advantages in comparison to time-driven filters. Firstly, it reduces a significant number of unnecessary data transmissions over the network. Secondly, the use of energy resources will be reduced, which is an essential feature due to the limited power transmission of sensor nodes.

In [7], a consensus-based approach but relying on Lyapunov stability theorem to improve the estimation performance has been proposed. Furthermore, not only a sensing system but also actuating devices have been studied there. It has shown that using that consensus-based design, one can achieve a faster convergence to the system states. Fur-

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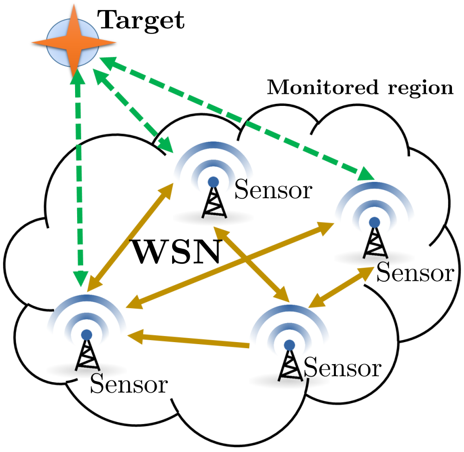

Fig. 1. Base structure of wireless sensor networks [6]

thermore, the hybrid model derived in [8] could be utilized to enhance the state estimation performance of the systems composed of both event and time-driven components.

While in [1] a linear model has been considered for the plant under study, in [6] a nonlinear continuous-time stochastic system has been considered for the target plant. The main goal of that study is to design filter parameters so that the filtering error $e_i(t) = \hat{x}_i(t) - x(t)$ is exponentially mean square stable. Theorem 1 of that paper proposes a sufficient condition to determine filter parameters such that the exponentially stability of the filtering error is guaranteed. Besides it introduces an upper bound for the mean of the filtering error. Some other studies, like [9], [10], focus on consensus Kalman filter design aiming at different applications such as tracking in sensor networks.

C. Contributions

To the knowledge of the authors, there is not a suitable study in which time-varying Kalman filter design is incorporated with OWA sensor fusion technique that can minimize the estimation errors corresponding to each of the observation data. In the current study, we will design a distributed class of time-varying Kalman filter for a class of linear time-varying systems based on a sensor network topology. Then, OWA sensor fusion technique will be employed to fuse the sensor estimations to achieve an accurate estimation of the states of the system for a pre-specified degree of Orness. In the proposed method, the OWA sensor fusion technique is modified such that the resulting optimal weights minimize the estimation error for each of the observations separately. Besides, exponential OWA operators will also be utilized and compared together so that a pre-specified degree of Orness is achieved.

D. Paper Organization

The remainder of this paper is organized as follows. A description of the general structure of the plant considered in this study is represented in Sec. II. Sec. III and Sec. IV are devoted to describe the proposed structure method and the fusion method, respectively. In Sec. V, the simulation results

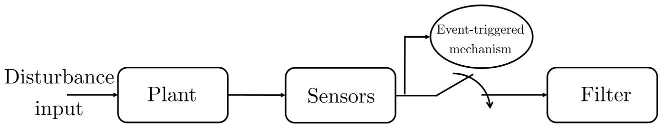

Fig. 2. A general view of event-based filtering procedure [11]

of a numerical example are stated. Finally, a conclusion is represented in Sec. VI.

II. Plant Description

The general structure that we are going to consider has the following form which is a linear time-varying system,

$$x(k+1) = A(k)x(k) + B(k)u(k) + Gw(k) \quad (1)$$
$$y_v(k) = C(k)x(k) + Dv(k) \quad (2)$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system states, $w(k) \in \mathbb{R}^{n_w}$ is the process noise and $v(k) \in \mathbb{R}^{n_v}$ is the measurement noise with covariance Q and R respectively. In this study both $w(k)$ and $v(k)$ are assumed to be white Gaussian noise. A, B, C and D are matrices with appropriate dimensions.

III. Proposed Structure Method

Here we will have a brief review on two main Kalman filter designs, steady-state and time-varying design.

A. Steady-State Kalman Filter

The equations of the steady-state Kalman filter for the above problem are given as follows (in this case without loss of generality assume $B = G$),

- Measurement update:

$$\hat{x}(k|k) = \hat{x}(k|k-1) + M(y_v(k) - C\hat{x}(k|k-1)) \quad (3)$$

- Time update:

$$\hat{x}(k+1|k) = A\hat{x}(k|k) + Bu(k) \quad (4)$$

where

$\hat{x}(k|k-1)$ is the estimate of $x(k)$, given past measurements up to $y_v(k-1)$. Besides, $\hat{x}(k|k)$ is the updated estimate based on the last measurement $y_v(k)$. Subscript v in $y_v(k)$ denotes to the measurement which is contaminated by an external noise $v(k)$.

To have a one-step-ahead predictor, the time update tries to predict the state of the system at the next time instance $k+1$ by having the current estimate $\hat{x}(k|k)$. This prediction will then be adjusted by the measurement update based on the new measurement $y_v(k+1)$. The correction term is a function of the innovation, which is the difference between the measured and predicted values of $y(k+1)$. This difference is given by:

$$y_v(k+1) - C\hat{x}(k+1|k) \quad (5)$$

The innovation gain M is chosen to minimize the steady-state covariance of the estimation error, by having the noise covariances,

$$\mathbb{E}\left(w(k)w(k)^T\right) = Q \quad (6)$$

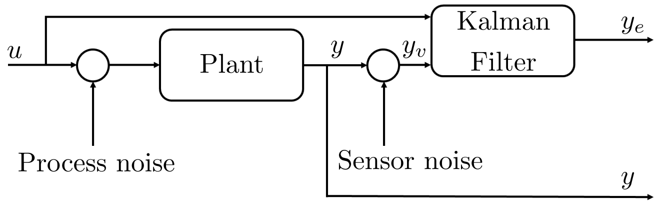

Fig. 3. A general schematic view of the plant and Kalman filter [13]

$$\mathbb{E}\left(v(k)v(k)^T\right) = R \tag{7}$$
$$N = \mathbb{E}\left(w(k)v(k)^T\right) = 0 \tag{8}$$

It is also possible to combine the time and measurement update equations into one state-space model, the Kalman filter,

$$\begin{aligned}\hat{x}(k+1|k) &= A(I-MC)\hat{x}(k|k-1) \\ &+ [B \quad AM]\begin{bmatrix} u(k) \\ y_v(k)\end{bmatrix}\end{aligned} \tag{9}$$
$$\hat{y}(k|k) = C(I-MC)\hat{x}(k|k-1) + CMy_v(k) \tag{10}$$

This filter generates an optimal estimate $\hat{y}(k|k)$ of y_v. It is notable that the filter state is $\hat{x}(k|k-1)$. A general schematic view of the plant and Kalman filter is shown in Fig. 3 [12], [13].

B. Time-Varying Kalman Filter

A generalized version of the steady-state filter for time-varying systems (or LTI systems with nonstationary noise covariance) is time-varying Kalman filter.

The time-varying Kalman filter for system model defined by (1)-(2) is given by the following recursive equations:

- Measurement update:

$$\hat{x}(k|k) = \hat{x}(k|k-1) + M(k)(y_v(k) - C\hat{x}(k|k-1)) \tag{11}$$
$$M(k) = P(k|k-1)C^T(R(k) + CP(k|k-1)C^T)^{-1} \tag{12}$$
$$P(k|k) = (I - M(k)C)P(k|k-1) \tag{13}$$

- Time update

$$\hat{x}(k+1|k) = A(k)\hat{x}(k|k) + Bu(k) \tag{14}$$
$$P(k+1|k) = A(k)P(k|k)A^T + GQ(k)G^T \tag{15}$$

where $\hat{x}(k|k-1)$ and $\hat{x}(k|k)$ are defined as before. The following definitions should be used in the measurement and time update steps [12], [13]:

$$Q(k) = \mathbb{E}(w(k)w(k)^T) \tag{16}$$
$$R(k) = \mathbb{E}(v(k)v(k)^T) \tag{17}$$
$$P(k|k) = \mathbb{E}(\{x(k) - \hat{x}(k|k)\}\{x(k) - \hat{x}(k|k)\}^T) \tag{18}$$
$$P(k|k-1) = \mathbb{E}(\{x(k) - \hat{x}(k|k-1)\}\{x(k) - \hat{x}(k|k-1)\}^T) \tag{19}$$

In order to implement time-varying Kalman filter, we assume the initial conditions as $\hat{x}(1|0) = 0$ and $P(1|0) = BQB^T$.

IV. Sensor Fusion Based on OWA Operators

Now we intend to perform a procedure in which we fuse our sensor estimations together and achieve a united estimation for each of the two states and the output of the system. In this respect, first, we will see how to learn the optimal weights of the OWA operator. Then we will employ our modified sensor fusion technique. In our contribution, we will learn the OWA weights such that they minimize the estimation error for each particular observation dataset. To learn the weights of the OWA operator, we define the fused value in sample k for n different attributes as follows,

$$F(a_{k1}, a_{k2}, \ldots, a_{kn}) = d_k \tag{20}$$
$$b_{k1}w_1 + b_{k2}w_2 + \ldots + b_{kn}w_n = d_k \tag{21}$$

where, d_k is the desired aggregated value. We would use the Hurwicz method to define the desired aggregated value d_k,

$$d_k = \rho \max_i a_i + (1-\rho)\min_i a_i \tag{22}$$

where, ρ is the optimism of the decision maker and belongs to $[0, 1]$. Based on the O'Hagan method a constrained minimization problem should be solved [14],

$$\min e(k) = \frac{1}{2}(b_{k1}w_1 + b_{k2}w_2 + \ldots + b_{kn}w_n - d_k)^2 \tag{23}$$
$$\sum_{i=1}^{n} w_i = 1, \quad w_i \in [0, 1] \tag{24}$$

It is common to convert the above-constrained problem to an unconstrained one by defining,

$$w_i = \frac{e^{\lambda_i}}{\sum_{j=1}^{n} e^{\lambda_j}}, \quad i = 1, 2, \ldots, n \tag{25}$$

Here we use the gradient descent method with learning rate $\beta \in [0, 1]$ to learn the weights,

$$\lambda_i(l+1) = \lambda_i(l) - \beta w_i(l)(b_{ki} - \hat{d}_k)(\hat{d}_k - d_k) \tag{26}$$

The estimate of the desired aggregated value based on the updated weights is obtained as follows for each observation,

$$\hat{d}_k = b_{k1}w_1(l) + b_{k2}w_2(l) + \ldots + b_{kn}w_n(l), \quad k = 1, \ldots, m \tag{27}$$

where the reordered objects of the kth sample are denoted by b_{k1}, b_{k2}, …, b_{kn} where b_{kj} is the jth largest element of the argument collection a_{k1}, a_{k2}, …, a_{kn}. It is notable that, as was mentioned before, in our approach, we learn OWA weights for each of the observations such that they minimize the error defined by (23) for that particular observation. Although at first glance it may come to mind that it might not be a fully efficient method for estimating the desired weights, it will result in a high accurate fusion outcome. Besides, these days due to the availability of powerful computers, the computational complexity of our method will definitely not be a big deal especially for a sufficiently small set of data, and its outstanding result would clearly justify its complexity.

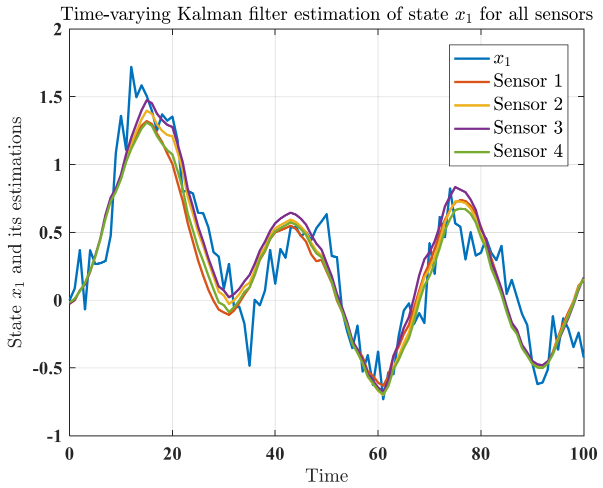

Fig. 4. Actual state x_1 of the system and four estimation corresponding to each of the sensors

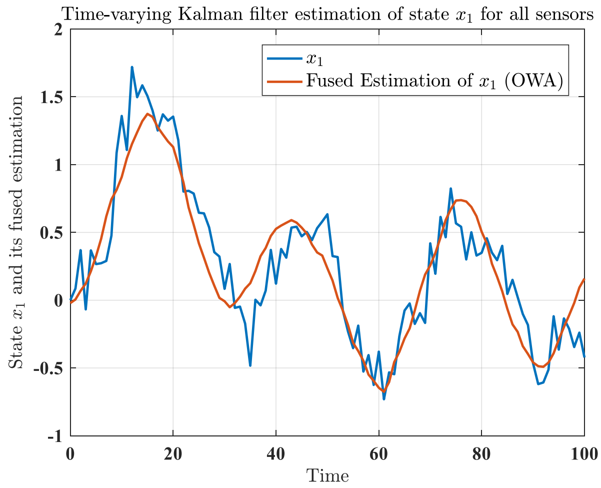

Fig. 6. Actual state x_1 of the system and fused estimation of the sensors using OWA operators

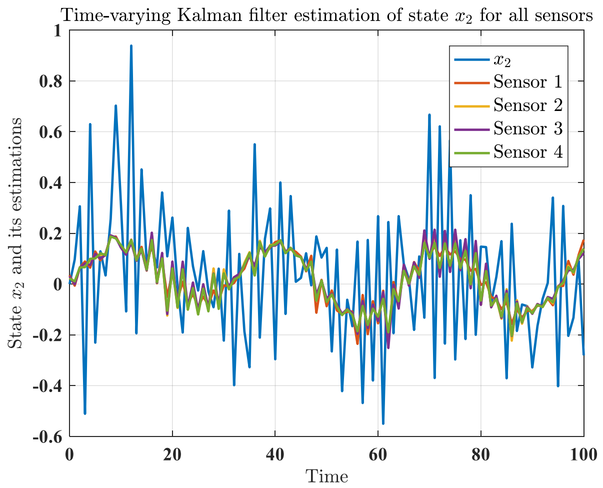

Fig. 5. Actual state x_2 of the system and four estimation corresponding to each of the sensors

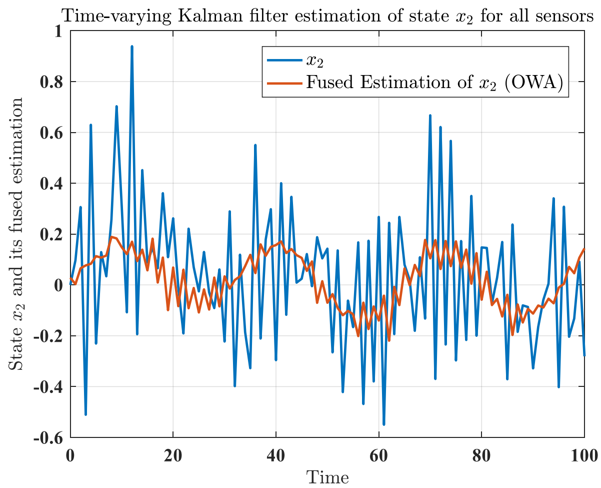

Fig. 7. Actual state x_2 of the system and fused estimation of the sensors using OWA operators

V. Simulation Results

The plant model which is a linear time-varying one that is considered in this study has the form defined by (1)-(2) with the following matrices,

$$A(k) = \begin{bmatrix} 0.98 + 0.05\sin(0.12k) & -0.4 \\ 0.15 & -0.75 \end{bmatrix} \quad (28)$$

$$B(k) = [0.16 \quad 0.18]^T \quad (29)$$

that is a modified version of the system introduced in [1]. In this plant, 4 sensors have been considered for the sake of output measurement. The dynamics of the sensors are as follows,

$$C_1 = [0.82 \quad 0.62 + \cos(0.12k)] \quad (30)$$

$$C_2 = [0.75 + 0.25\sin(0.1k) \quad 0.80] \quad (31)$$

$$C_3 = [0.74 + 0.5\sin(0.1k) \quad 0.75 + 0.5\cos(0.1k)] \quad (32)$$

$$C_4 = [0.75 \quad 0.65] \quad (33)$$

with process and measurement covariances $Q = 1$ and $R = 1$, respectively. We excite the system with the input signal $u(t) = \sin(\frac{t}{5})$. This will provide the opportunity to benefit from the *persistence of excitation (PE) property in our estimations, which is pointed out in Remark 1 [15].* Since our plant is a time-varying one, we design a time-varying Kalman filter for each of the sensors. The estimation of each of the sensors for states of the system is calculated. Fig. 4 shows state x_1 of the system and its estimation by four sensors. The actual state and estimations of x_2 is also shown in Fig. 5.

Now we use the sensor fusion technique, OWA operator, to fuse the sensor state estimations. We will also use the exponential OWA operators and compare the sensor fusion procedure in two cases, an optimistic exponential OWA operator and a pessimistic exponential OWA operator. In addition, we will see the output error of the system using

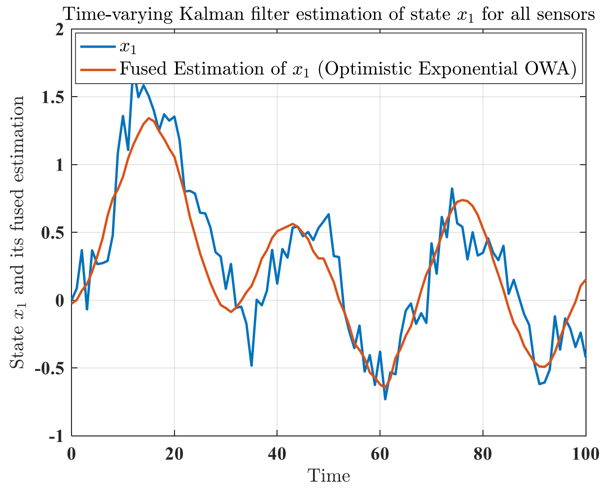

Fig. 8. Actual state x_1 of the system and fused estimation of the sensors using optimistic exponential OWA operator

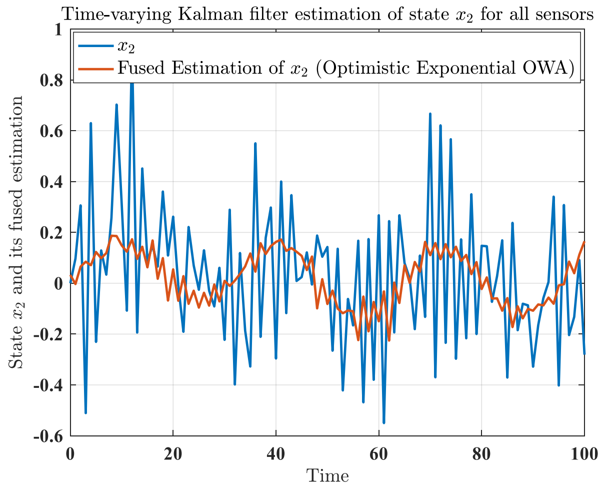

Fig. 9. Actual state x_2 of the system and fused estimation of the sensors using optimistic exponential OWA operator

the fused sensor outputs.

To learn the OWA weights we update the weights in a recursive manner based on gradient descent method described in (26). In each time sample, we solve the minimization problem (23) using gradient descent method with learning rate $\beta = 0.35$ and optimism of the decision maker $\rho = 0.2$. As one can easily see from Fig. 6 and 7, the fused estimation of both states x_1 and x_2 closely represent the true values.

For the sake of sensor fusion using exponential OWA operator to achieve a pre-specified degree of Orness, first, we need to introduce the definition of Orness. The definition of Orness(W) is a follows,

$$\text{Orness}(W) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i \tag{34}$$

We choose Orness Orness$(W) = 0.9$ and try both the optimistic and the pessimistic exponential OWA operators.

1) Optimistic exponential OWA operator: In this case, after setting Orness Orness$(W) = 0.9$ and $n = 4$, by using the optimistic exponential OWA operators curve [14] we select $\alpha = 0.75$. The corresponding weights for the each of the sensors are calculated,

$$w_1 = \alpha = 0.75, w_2 = \alpha(1-\alpha) = 0.1875,$$
$$w_3 = \alpha(1-\alpha)^2 = 0.0469, w_4 = \alpha(1-\alpha)^3 = 0.0156,$$
$$\sum_{i=1}^{n=4} w_i = 1$$

With the above weights, the actual Orness would be Orness$(W) = 0.8906$.

2) Pessimistic exponential OWA operator: In this case, after setting Orness Orness$(W) = 0.9$ and $n = 4$, by using the pessimistic exponential OWA operators curve [14] we select $\alpha = 0.93$. The corresponding weights for the each of the sensors are calculated,

$$w_1 = \alpha^3 = 0.8044, w_2 = (1-\alpha)\alpha^2 = 0.0605,$$
$$w_3 = (1-\alpha)\alpha = 0.0651, w_4 = (1-\alpha) = 0.0700,$$
$$\sum_{i=1}^{n=4} w_i = 1$$

With the above weights, the actual Orness would be Orness$(W) = 0.8664$.

Based on the above results, since the actual Orness in the optimistic case is closer to the considered Orness, we use the weights calculated in this case for our sensor fusion purpose.

By utilizing the derived weights for the optimistic exponential OWA operator, the fused estimation of the sensors for state x_1 and x_2 of the system are shown in Fig. 8 and 9. As is seen from these figures, the fused estimation of four sensors can finally estimate both states x_1 and x_2 accurately.

Since in most of the applications, the output of the system is of high importance (for tracking or regulation objectives), we are also interested in fusing output estimation of the sensors. In this respect, we fuse four sensor output estimations and compare it with the actual output of the system. Moreover, it would also be helpful to compare the fused measurement error of the sensors with their fused estimation error. Fig. 10 shows these quantities. We see from the top figure that the fused estimations of output resulting from the four sensors can track the actual output. Besides, it can be easily seen from the bottom figure that the fused estimation error is significantly reduced compared to the fused measurement error for almost all of the time instances.

Remark 1: It is remarkable that similar to a notion called the "*certainty equivalence*" design approach in adaptive state estimation, the estimated states may not fully converge to their true values (x_2); however, due to the chosen PE input, the persistence of excitation condition is satisfied that guarantees the convergence of the estimated output to the true value [16]. □

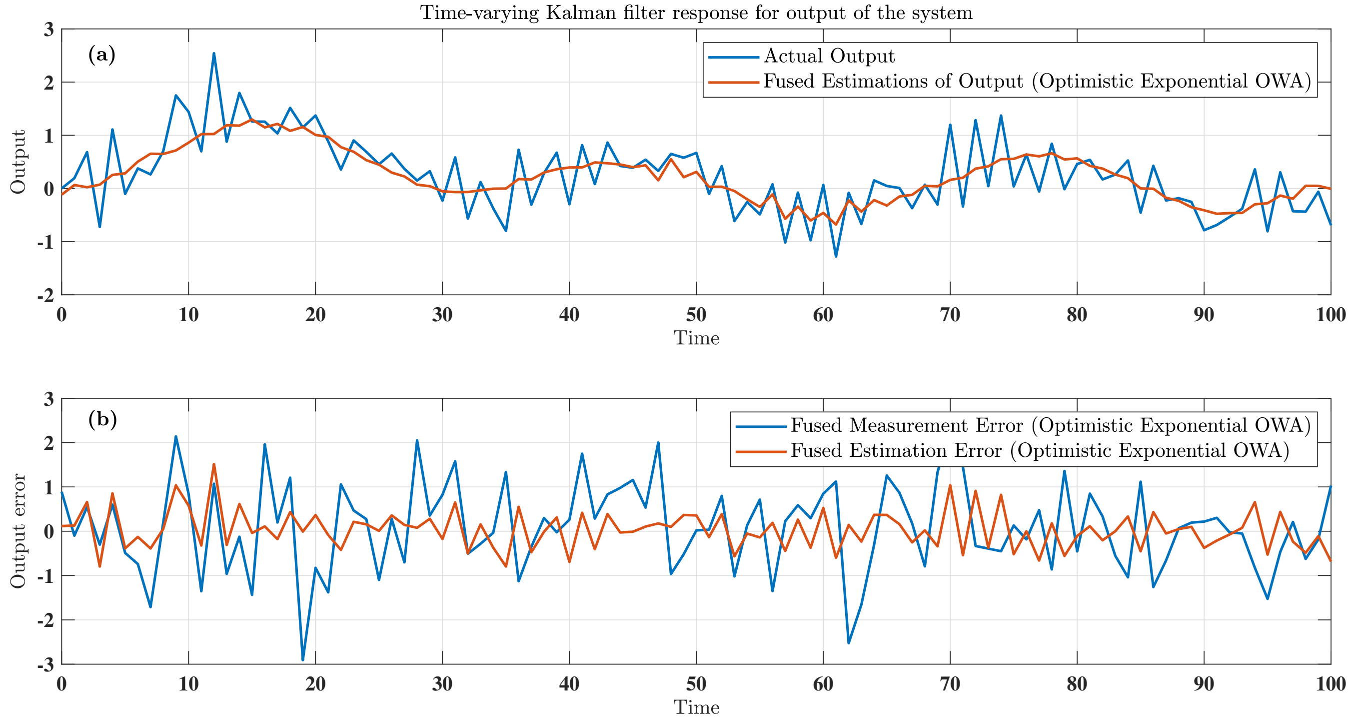

Fig. 10. (a) Actual output and fused estimations of output (b) Fused measurement error and fused estimation error

VI. Conclusion and Future Work

In this paper, a novel approach was proposed to estimate the states and outputs of a linear time-varying system with a high level of accuracy. In the proposed method, a distributed class of time-varying Kalman filter incorporated with a new sensor fusion method has been employed to estimate the states and outputs of a linear time-varying system. In the considered system, both the dynamics and the measurements are assumed to be contaminated by external noises. The filtering and estimation design is based on a wireless sensor network topology, which provides the opportunity to use sensor fusion techniques. OWA sensor fusion method was employed to fuse the estimation of the sensors. The OWA weights were determined to efficiently minimize the estimation error for each observation. That provided a high level of accuracy for the estimation of the states and outputs of the system under study. Besides, two different exponential OWA operators, optimistic and pessimistic methods, were used to achieve a pre-specified level of Orness. Finally, the effectiveness of the method was shown in a numerical example. To continue this work, one can extend our method for nonlinear systems with other state estimators.

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