Week 4 Tutorial: Function Limits

Section 5: Mechanical Engineering (Stream 4)

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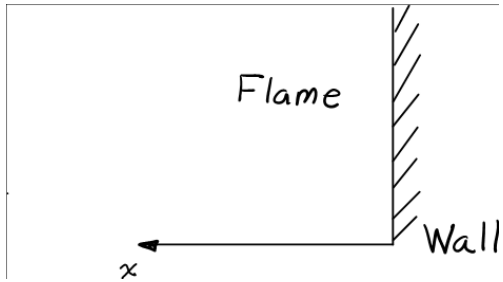

To estimate heat transfer in a furnace, the temperature is measured at points off the furnace wall. If x is distance off the wall, the temperature, T, in the regions $0[m] \leq x \leq 0.09[m]$ and $0.11[m] \leq x \leq 0.25[m]$, can be represented by:

$$T(x) = A + \frac{Bx^2}{(x - C)^2}$$

where A,B, and C are constants. Given the following measurements, estimate the three constants to ensure accurate limits.

$x[m]$	0.001	0.005	0.010	0.095	0.105	0.75
$T[K]$	400	400	400	1250	1250	425

Photo courtesy of University of Waterloo Fire Research Laboratory

Question 2: Left / Right Limits

Problem 18, Exercise 2.2, Page 119
Evaluate, if possible, the limit of

$$f(x) = \frac{\sqrt{1+x} - 1}{x^2}$$

at $x = 0$? As part of your solution calculate $f(x)$ at ± 0.01 and ± 0.001.

Question 3: Discontinuity / Heaviside Functions

Problem 4, Exercise 2.5, Page 142
Express the function

$$f(x) = \begin{cases} 0 & x < -1 \\ x - 2 & -1 < x < 3 \\ x - 4 & 3 < x < 5 \\ 0 & 5 < x \end{cases}$$

in terms of Heaviside functions. Sketch a graph of the function.

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Assignment 3

Due: Week, Oct. 6 - 10, 2008

1. Textbook, Page 115, Exercises 2.1, # 15, 27, 52.*+

2. Textbook, Page 119, Exercises 2.2, # 19, 21.

3. Textbook, Page 127, Exercises 2.3, # 19, 33, 47.

4. Textbook, Page 135, Exercises 2.4, # 3.

5.* (a) Sketch the graph of $f(x)= 2 -(x + 2)h(x + 4) + 2xh(x) + (1- x)h(x-2)$, where $h(x)$ is the Heaviside function.

   (b) Express $g(x)$ in terms of the Heaviside function, if

$$g(x)=\begin{cases} \sqrt{-x} & , \quad x<0 \\ 2x+2 & , \quad 0<x<1 \\ 2-x & , \quad 1<x<3 \\ 0 & , \quad x>3 \end{cases}$$

6.* Consider a population of a species for which each individual has a constant probability, *b*, of producing a new individual (birth) and a constant probability, *d*, of going extinct (death). The extinction probability as a function of time, *t*, can be approximated by ($b \neq d$):

$$P(t) = \frac{d(e^{(b-d)t} - 1)}{be^{(b-d)t} - d}$$

   Compute $\lim_{t\to\infty} P(t)$ if (i) $b > d$ and (ii) $b < d$.

Recommended Problems (NOT to be handed in)

Textbook, Page 115, Exercises 2.1, # 2-40 (even)
Textbook, Page 119, Exercises 2.2, # 2-20 (even)
Textbook, Page 127, Exercises 2.3, # 2-20 (even), 28, 32, 34
Textbook, Page 135, Exercises 2.4, # 2, 8, 16, 20, 30, 32, 34, 38
Textbook, Page 142, Exercises 2.5, # 2, 4, 6, 20

* - indicates that this problem was selected to be graded
+ - indicates that the correct answer to this problem is provided in the course textbook

EXERCISES 2.1

In Exercises 1–41 find the indicated limit, if it exists.

1. $\lim_{x\to 7} \dfrac{x^2-5}{x+2}$

2. $\lim_{x\to -2} \dfrac{x^3+8}{x+5}$

3. $\lim_{x\to -5} \dfrac{x^2+3x+2}{x^2+25}$

4. $\lim_{x\to 0} \dfrac{x^2+3x}{3x^2-2x}$

5. $\lim_{x\to 3^+} \dfrac{2x-3}{x^2-5}$

6. $\lim_{x\to 2^-} \dfrac{2x-4}{3x+2}$

7. $\lim_{x\to 0^-} \dfrac{x^4+5x^3}{3x^4-x^3}$

8. $\lim_{x\to 2^+} \dfrac{x^2+2x+4}{x-3}$

9. $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$

10. $\lim_{x\to 3^+} \dfrac{x^2-9}{x-3}$

11. $\lim_{x\to 5^-} \dfrac{x^2-25}{x-5}$

12. $\lim_{x\to 3} \dfrac{x^2-2x-3}{3-x}$

13. $\lim_{x\to 2} \dfrac{x^2-4x+4}{x-2}$

14. $\lim_{x\to 2} \dfrac{x^3-6x^2+12x-8}{x^2-4x+4}$

15. $\lim_{x\to 1} \dfrac{x^3-6x^2+11x-6}{x^2-3x+2}$

16. $\lim_{x\to 2} \dfrac{x^3-6x^2+11x-6}{x^2-3x+2}$

19. $\lim_{x\to 0} \dfrac{x^3-6x^2-}{x^2-3}$

21. $\lim_{x\to 1} \sqrt{\dfrac{2-x}{2+x}}$

23. $\lim_{x\to 0} \dfrac{\tan x}{\sin x}$

* 25. $\lim_{x\to 0} \dfrac{\sin 4x}{\sin 2x}$

* 27. $\lim_{x\to 0^+} \dfrac{\sin 2x}{\tan x}$

* 29. $\lim_{x\to 0} \dfrac{\sqrt{1-x}-}{x}$

* 31. $\lim_{x\to 5^-} \dfrac{|x^2-25|}{x^2-25}$

In Exercises 51 and 52 use the squeeze theorem to discuss the limits.

51. $\lim_{x\to 0} x \sin\left(\dfrac{1}{x}\right)$

52. $\lim_{x\to 0} x^4 \cos\left(\dfrac{3}{x}\right)$

EXERCISES 2.2

In Exercises 1–24 evaluate the limit, if it exists.

1. $\lim_{x\to 2^+} \frac{1}{x-2}$

2. $\lim_{x\to 2^-} \frac{1}{x-2}$

3. $\lim_{x\to 2} \frac{1}{x-2}$

4. $\lim_{x\to 2^+} \frac{1}{(x-2)^2}$

5. $\lim_{x\to 2^-} \frac{1}{(x-2)^2}$

6. $\lim_{x\to 2} \frac{1}{(x-2)^2}$

7. $\lim_{x\to 1} \frac{5x}{(x-1)^3}$

8. $\lim_{x\to 1/2} \frac{6x^2+7x-5}{2x-1}$

9. $\lim_{x\to 1} \frac{2x+3}{x^2-2x+1}$

10. $\lim_{x\to 2} \frac{x-2}{x^2-4x+4}$

* 11. $\lim_{x\to 0} \csc x$

12. $\lim_{x\to \pi/4} \sec(x-\pi/4)$

* 13. $\lim_{x\to 3\pi/4} \sec(x-\pi/4)$

14. $\lim_{x\to 0^+} \cot x$

* 15. $\lim_{x\to \pi/2^+} \tan x$

* 16. $\lim_{x\to \pi/2^-} \tan x$

* 17. $\lim_{x\to 1} \frac{x^2-2x+1}{x^3-3x^2+3x-1}$

* 18. $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x^2}$

* 19. $\lim_{x\to 0} \frac{2x}{1-\sqrt{x^2+1}}$

* 20. $\lim_{x\to 4} \frac{|4-x|}{x^2-8x+16}$

* 21. $\lim_{x\to 0^+} \ln(4x)$

* 23. $\lim_{x\to 0} e^{1/x}$

In Exercises 25–28 a
limit, if it exists.

* 25. $\lim_{x\to a^+} \frac{x-a}{x^2-2ax}$

* 27. $\lim_{x\to 0^-} \frac{\sqrt{a+x}-}{x^2}$

* 29. It is not clear wh
that $\lim_{x\to 0^+} x^2 = 0$
term is more dominar
(a) Calculate
1, . . . , 1C
for the lin
(b) Plot grapl
tion in pa

* 30. Repeat Exercise
values of x at which

EXERCISES 2.3

In Exercises 1–38 evaluate the limit, if it exists.

1. $\lim_{x\to\infty} \frac{x+1}{2x-1}$

2. $\lim_{x\to\infty} \frac{1-x}{3+2x}$

3. $\lim_{x\to\infty} \frac{x^2+1}{2x^3+5}$

4. $\lim_{x\to\infty} \frac{1-4x^3}{3+2x-x^2}$

5. $\lim_{x\to-\infty} \frac{2+x-x^2}{3+4x^2}$

6. $\lim_{x\to-\infty} \frac{x^3-2x^2}{3x^3+4x^2}$

7. $\lim_{x\to-\infty} \frac{x^3-2x^2+x+1}{x^4+3x}$

8. $\lim_{x\to-\infty} \frac{x^3-2x^2+x+1}{x^2-x+1}$

9. $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{2x+1}$

* 10. $\lim_{x\to\infty} \frac{3x-1}{\sqrt{5+4x^2}}$

11. $\lim_{x\to-\infty} \frac{\sqrt{1-2x^2}}{x+2}$

* 12. $\lim_{x\to-\infty} \frac{\sqrt{1-2x}}{x+2}$

13. $\lim_{x\to\infty} \sqrt{\frac{2+x}{x-2}}$

* 14. $\lim_{x\to\infty} \frac{\sqrt{3+x}}{\sqrt{x}}$

15. $\lim_{x\to\infty} (x^2-x^3)$

* 16. $\lim_{x\to\infty} \left(x+\frac{1}{x}\right)$

17. $\lim_{x\to\infty} \frac{x}{\sqrt{x+5}}$

* 18. $\lim_{x\to-\infty} \frac{x^2}{\sqrt{3-x}}$

19. $\lim_{x\to-\infty} \frac{x}{\sqrt[3]{4+x^3}}$

* 20. $\lim_{x\to\infty} \frac{3x}{\sqrt[3]{2+4x^3}}$

39. $\lim_{x\to\infty} \frac{x^2+ax-2}{ax^2+5}$

* 40. $\lim_{x\to\infty} \frac{x}{\sqrt{ax^2+3x+2}}$

41. $\lim_{x\to\infty} (\sqrt{x^2+ax}-x)$

* 42. $\lim_{x\to-\infty} \frac{\sqrt{ax^2+7}}{x-3a}$

In Exercises 43–48 identify all horizontal, vertical, and oblique asymptotes for the graph of the function. Determine whether the graph approaches horizontal and oblique asymptotes from above or below.

* 21. $\lim_{x\to\infty} \frac{1}{2x}\cos x$

* 23. $\lim_{x\to\infty} \frac{\sin 4x}{x^2}$

* 25. $\lim_{x\to-\infty} \tan x$

* 27. $\lim_{x\to\infty} (\sqrt{x^2+1}-x$

* 29. $\lim_{x\to\infty} (\sqrt{2x^2+1}-$

* 31. $\lim_{x\to\infty} \frac{\sqrt{3x^2+2}}{x+4}$

* 33. $\lim_{x\to-\infty} \frac{\sqrt{3x^2+2}}{x+4}$

* 35. $\lim_{x\to\infty} (\sqrt{x^2+4}-$

* 36. $\lim_{x\to\infty} (\sqrt[3]{1+x}-$

* 37. $\lim_{x\to\infty} (\sqrt{x^2+x}-$

* 38. $\lim_{x\to-\infty} (\sqrt{x^2+x}-$

* 47. $f(x) = \frac{1-4x^2}{3+2x-x^2}$

* 49. What is the value of $\lim_{x\to}$ li
b, c, d, and e are constants

** 50. What conditions on the
that

$\lim_{x\to\infty} (\sqrt{ax^2+}$

EXERCISES 2.4

In Exercises 1–30 plot a graph of the function indicating any discontinuities. Classify each discontinuity as a removable discontinuity, a jump discontinuity, or an infinite discontinuity, if possible.

1. $f(x) = \dfrac{1}{x+2}$

2. $f(x) = \dfrac{16 - x^2}{x+4}$

3. $f(x) = |x^2 - 5|$

4. $f(x) = \dfrac{12}{x^2+2}$