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Optimal Bandwidth Allocation for Data Transmission in Virtual Data Centers

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***Abstract*—In this paper, we propose to use the transmission time of data blocks as the metric to measure the network utilization, and study the bandwidth allocation for multi-path data transmission in virtual data centers. We start with the case that data transmission requires no minimum bandwidth, and formulate the bandwidth allocation problem as two convex optimization problems. In the first problem, the maximum transmission time of all data blocks is minimized under the given data block sizes and link capacities, which is named as the *maximum transmission time minimization (MTTM)* problem. The second convex optimization problem minimizes the transmission time summation of all data blocks, and the problem is named as the *transmission time summation minimization (TTSM)* problem. These two problems are then generalized to the case that data transmission has minimum bandwidth requirement, and the corresponding problems are named as general MTTM *(G-MTTM)* and general TTSM *(G-TTSM)* problems. By exploring the properties of the G-MTTM problem, we design a fast algorithm for it, named as *TTM-based* algorithm. Extensive simulation results demonstrate that the G-MTTM and G-TTSM problems can be efficiently solved, and to use transmission time of data blocks as the metric for network utilization measurement and optimization is practical.**

***Index Terms*—Virtual data center, transmission time minimization, bandwidth allocation, convex optimization, linear programming.**

I. Introduction

As a powerful cloud computing platform, the data center has recently received significant attention. Nowadays, many large companies, such as Google, Amazon, Microsoft and Facebook, rely on data centers for a wide range of cloud services, like computing, storage, Web search, on-line shopping, social networking, etc. [1] [2].

To efficiently share and multiplex the vast computing power, storage and network resources of a data center among numerous tenants, there is an emerging trend towards data center virtualization [3]–[10]. Similar to server virtualization, enabling tenants to use dedicated cores, memories and disk spaces in the form of virtual machines (VMs), the data center virtualization provides tenants with virtual data centers (VDCs), which consists of virtual machines, virtual switches and virtual routers interconnected by virtual links. In short, besides server virtualization, data center virtualization includes virtualization of the data center network. With data center virtualization, each tenant can run its applications in a relatively isolated environment, which is a promising technique to meet the various requirements of many network applications, including bandwidth guarantees, low performance interference, application-specific protocols and address spaces implementation, etc. [3], [11]–[14].

Compared to the relatively mature server virtualization techniques, data center network virtualization is still in its infancy, and related proposals in the literature include NetLord, NetShare, FairCloud, SecondNet, etc. [5]–[9]. These proposals have pros and cons in terms of deployability, scalability, robustness, etc. Nevertheless, with current data center network virtualization techniques, to deploy a VDC network as per the tenants' demands in terms of network topologies and link capacities is now feasible [10]–[13]. As the tenants are generally charged to use the VDCs, it is important for them to take full advantage of the allocated resources in the VDCs, and we study the VDC network utilization in this paper. In particular, we focus on the bandwidth allocation to efficiently transfer multiple data blocks distributed in the VDC, which is an appropriate abstraction for many applications (e.g., MapReduce, Dryad, CIEL, and Spark) in the data center involving large amount of data transfer [15]–[20].

The network utilization is generally measured by the used bandwidth with respect to link capacities, and a good bandwidth allocation scheme uses as much bandwidth as possible [6]–[10]. However, we argue that though the used bandwidth reflects the network utilization intuitively, it fails to capture the essence of tenants' concern on network utilization. To the tenants, high network utilization generally means low data transmission time, and this is true when we allocate bandwidth for only one data block. When allocating bandwidth for multiple data blocks, to simply maximize the used bandwidth might be problematic. This is because that the sizes of different data blocks must be taken into consideration in the bandwidth allocation, and maximizing the used bandwidth alone might cause the mismatch between the allocated bandwidth and the size of the data block, i.e., small-sized data blocks use more bandwidths than large-sized data blocks, leading to long transmission time of the large-sized data blocks.

Therefore, we propose to use transmission time of the data blocks as the metric to measure the network utilization, and consider two typical objectives of bandwidth allocation in the real world. The first objective is to minimize the maximum transmission time of all data blocks, which is generally the concern of large-scale data collection and distribution. That is, all data block transmissions are either from one source or

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heading to the same destination, the performance of which is largely affected by the slowest data block transmission [16]–[18]. The second objective is to minimize the transmission time summation of all data blocks, under which no or little correlation exists among different data block transmissions, therefore, each individual data block is transmitted as fast as possible with provided network resource. The bandwidth allocations under these two objectives are studied, and we find the corresponding optimal bandwidth allocations under given data block sizes and VDC network.

The rest of the paper proceeds as follows. We start from the case that data transmission requires no minimum bandwidth, and formulate the problem of bandwidth allocation as two optimization problems, denoted as the *maximum transmission time minimization (MTTM)* and the *transmission time summation minimization (TTSM)* problems, respectively, in Section II. The MTTM and TTSM problems are then generalized to the case that data transmission requires minimum bandwidth, leading to the general MTTM *(G-MTTM)* and general TTSM *(G-TTSM)* problems in Section III. We evaluate the solution time of the G-MTTM and G-TTSM problems in Section IV, and Section V concludes the paper.

II. Problem Formulation

In this section, we formulate the problem of bandwidth allocation for transmitting multiple data blocks in a VDC as two convex optimization problems, and transform one of them to a linear optimization problem by exploring its property.

A. Notations

In this subsection, we introduce the notations to be used in our problem formulation.

We assume that there are N data blocks to be transmitted in a given VDC, and the size of data block i, i.e., the i_{th} data block is denoted as S_i. As the multi-path technique is usually adopted in the data center networks for load balancing and fault tolerance [3], we take multi-path data transmission into consideration in our problem formulation, and each data block is transmitted along M paths, the routes of which are determined by the underlying multi-path technique. The j_{th} path used to transmit data block i is denoted as x_{ij}, and we also use x_{ij} to denote the bandwidth allocated to the path for convenience. Suppose that there are K links in the VDC network, and the capacity of link k, i.e., the k_{th} link is denoted as B_k, we use c_{ij}^k as an indicator of whether path x_{ij} contains link k. That is,

$$c_{ij}^k = \begin{cases} 1, & \text{if path } x_{ij} \text{ contains link } k \\ 0, & \text{otherwise} \end{cases}$$

where $1 \le k \le K$.

B. The MTTM and TTSM Problems

We minimize the maximum transmission time of these N data blocks,

$$\max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | 1 \le i \le N\}$$

in Table 1, where constraint (1) means that the bandwidth summation of all paths containing link k is bounded by the link capacity, B_k, and name the optimization problem as the *maximum transmission time minimization (MTTM)* problem.

TABLE 1
Maximum Transmission Time Minimization (MTTM) Problem Formulation for Multi-path Data Transmission in a Virtual Data Center

Minimize:

$$\max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | 1 \le i \le N\}$$

Subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}^k x_{ij} \le B_k, 1 \le k \le K, \quad (1)$$

$$c_{ij}^k = \begin{cases} 1, & \text{if path } x_{ij} \text{ contains link } k \\ 0, & \text{otherwise} \end{cases}$$

For presentational convenience, we define

$$U = \{x_{ij} | 1 \le i \le N,\ 1 \le j \le M\} \quad (2)$$

is a feasible solution of the MTTM problem if $x_{ij} \in U$ satisfies all constraints in Table 1. The optimal solution and optimal value of the MTTM problem are denoted as U^* and t^*, respectively. That is,

$$t^* = \max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | x_{ij} \in U^*,\ 1 \le i \le N\}$$

By altering the objective function in Table 1 to the summation of all data block transmission time,

$$\sum_{i=1}^{N} \frac{S_i}{\sum_{j=1}^{M} x_{ij}}$$

we have the *transmission time summation minimization (TTSM)* problem in Table 2.

To show the convexity of the above two optimization problems, we begin with a convex function $\frac{1}{x}$, where $x > 0$.

As $\frac{1}{x}$ is convex when $x > 0$,

$$\frac{1}{\theta x + (1-\theta) y} \le \frac{\theta}{x} + \frac{1-\theta}{y}$$

where $0 \le \theta \le 1$, and $x, y > 0$. Let $x = \sum_{i=1}^{n} x_i,\ y = \sum_{i=1}^{n} y_i$, we have

$$\frac{1}{\theta \sum_{i=1}^{n} x_i + (1-\theta)\sum_{i=1}^{n} y_i} \le \frac{\theta}{\sum_{i=1}^{n} x_i} + \frac{1-\theta}{\sum_{i=1}^{n} y_i}$$

Furthermore,

$$\theta \sum_{i=1}^{n} x_i + (1-\theta)\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \theta x_i + (1-\theta) y_i$$

TABLE 2
TRANSMISSION TIME SUMMATION MINIMIZATION (TTSM) PROBLEM FORMULATION FOR MULTI-PATH DATA TRANSMISSION IN A VIRTUAL DATA CENTER

Minimize:

$$\sum_{i=1}^{N} \frac{S_i}{\sum_{j=1}^{M} x_{ij}}$$

Subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}^k x_{ij} \leq B_k, 1 \leq k \leq K, \quad (3)$$

$$c_{ij}^k = \begin{cases} 1, & \text{if path } x_{ij} \text{ contains link } k \\ 0, & \text{otherwise} \end{cases}$$

therefore,

$$\frac{1}{\sum_{i=1}^{n} \theta x_i + (1-\theta) y_i} \leq \frac{\theta}{\sum_{i=1}^{n} x_i} + \frac{1-\theta}{\sum_{i=1}^{n} y_i}$$

which proves the convexity of $\frac{1}{\sum_{i=1}^{n} x_i}$ when $\sum_{i=1}^{n} x_i > 0$.

Since $\forall i \in [1, N]$,

$$\frac{S_i}{\sum_{j=1}^{M} x_{ij}}, \text{ where } \sum_{j=1}^{M} x_{ij} > 0$$

is convex, and the maximum (or summation) of multiple convex functions is still, the object functions of the MTTM and TTSM problems are convex. Moreover, the constraints of these two optimization problems are affine, the MTTM and TTSM problems are thus both convex optimization problems.

There are a variety of algorithms for solving a convex optimization problem in the literature [24], and we can adopt any of them for our problems. For presentational convenience, we uniformly name these algorithms as *convex algorithms* in the rest of our paper. Nevertheless, by exploring the properties of the MTTM problem, we found that the MTTM problem can be transformed into a linear optimization problem. As will be demonstrated by the simulation results in Section IV, the linear optimization problem after transformation can be solved much faster than the MTTM problem by convex algorithms.

C. The TTM Problem

To transform the MTTM problem into a linear optimization problem, we construct another optimization problem named as *Transmission Time Minimization (TTM)* problem in Table 3, and prove that the optimal solution of the TTM problem is also optimal for the MTTM problem.

In the TTM problem, the transmission time of all data blocks are equal, as reflected by constraint (5) in Table 3, and the object is to minimize this equal transmission time. We also say U defined by (2) is a feasible solution of the TTM problem if $x_{ij} \in U$ satisfy all constraints in Table 3, and the optimal solution of the TTM problem is denoted as U_1^*. Since the object of the TTM problem is equivalent to maximizing $\sum_{j=1}^{M} x_{1j}$, which is a linear function, and constraint (5) can be replaced by the following linear constraint,

$$\frac{\sum_{j=1}^{M} x_{1j}}{S_1} = \frac{\sum_{j=1}^{M} x_{ij}}{S_i}, \ 1 \leq i \leq N$$

the TTM problem can be readily transformed into a linear optimization problem. Therefore, the MTTM problem can be transformed into a linear optimization problem as well if the MTTM and TTM problems share identical optimal solution, and we have the following lemma.

TABLE 3
TRANSMISSION TIME MINIMIZATION (TTM) PROBLEM FORMULATION FOR MULTI-PATH DATA TRANSMISSION IN A VIRTUAL DATA CENTER

Minimize:

$$\frac{S_1}{\sum_{j=1}^{M} x_{1j}}$$

Subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}^k x_{ij} \leq B_k, 1 \leq k \leq K, \quad (4)$$

$$c_{ij}^k = \begin{cases} 1, & \text{if path } x_{ij} \text{ contains link } k \\ 0, & \text{otherwise} \end{cases}$$

$$\frac{S_1}{\sum_{j=1}^{M} x_{1j}} = \frac{S_i}{\sum_{j=1}^{M} x_{ij}}, \ 1 \leq i \leq N \quad (5)$$

Lemma 1: U_1^* is the optimal solution of the MTTM problem.

Proof: As the TTM problem in Table 3 includes all the constraints of the MTTM problem in Table 1, U_1^* is a feasible solution of the MTTM problem. Therefore,

$$t^* \leq \max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | x_{ij} \in U_1^*\} = t_1^*$$

by constraint (5) in Table 3.

On the other hand, as

$$t^* = \max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | x_{ij} \in U^*\}$$

and decreasing x_{ij} will not violate constraint (4) in Table 3, we construct a feasible of the TTM problem, say, U, by decreasing $x_{ij} \in U^*$ such that

$$\frac{S_i}{\sum_{j=1}^{M} x_{ij}} = t^*, \text{ where } x_{ij} \in U, \ 1 \leq i \leq N$$

Since t_1^* is the minimum transmission time of data block 1,

$$t_1^* \leq \frac{S_1}{\sum_{x_{1j} \in U} x_{1j}}$$

Hence, $t_1^* = t^*$, and the lemma holds. ∎

By Lemma 1, we can obtain the optimal solution of the MTTM problem by solving the TTM problem. Furthermore, U_1^* uses the least bandwidths to minimize the maximum transmission time, as stated by the following corollary.

Corollary 1:

$$\sum_{x_{ij} \in U_1^*} x_{ij} \leq \sum_{x_{ij} \in U^*} x_{ij}$$

Proof: We prove the corollary by contradiction, and assume that

$$\sum_{x_{ij} \in U_1^*} x_{ij} > \sum_{x_{ij} \in U^*} x_{ij}$$

By our assumption, there must exist some $i' \in [1, N]$, such that

$$\sum_{x_{i'j} \in U_1^*} x_{i'j} > \sum_{x_{i'j} \in U^*} x_{i'j}$$

Therefore,

$$t_1^* = \frac{S_{i'}}{\sum_{x_{i'j} \in U_n^*} x_{i'j}} < \frac{S_{i'}}{\sum_{x_{i'j} \in U^*} x_{i'j}} \leq t^*$$

However, U_1^* is the optimal solution of the MTTM problem, and $t_1^* = t^*$ by Lemma 1. Hence, the assumption is not true, and the lemma holds. ■

In this section, we discuss the bandwidth allocation problem for data transmission without minimum bandwidth requirement. Next, we extend our discussion to the case that data transmission requires minimum communication bandwidth.

III. Generalization of the MTTM and TTSM Problems

In this section, we generalize our proposed MTTM and TTSM problems to data transmission with minimum bandwidth requirement, which is common for data center applications [11]–[13], and denote these two problems after generalization as the general MTTM (*G-MTTM*) and general TTSM (*G-TTSM*) problems, respectively.

A. The G-MTTM and G-TTSM Problems

The minimum required bandwidth of data block i is denoted as b_i, and the G-MTTM and G-TTSM problems are formulated in Table 4 and 5, respectively, where constraint (7) and (9) on minimum bandwidth requirements are added. As the added constraints are affine, the G-MTTM and G-TTSM problems are still convex optimization problems, and can be solved by convex algorithms.

B. The TTM-n Problem

As discussed in the previous section, the MTTM problem can be transformed into a linear optimization problem. Likewise, we can decompose the G-MTTM problem into several linear optimization problems by exploring its property, which leads to faster solution of the problem.

To show the decomposition of the G-MTTM problem, we construct an optimization problem, denoted as *Transmission Time Minimization-n (TTM-n)* problem in Table 6, where the first $n-1$ data blocks are allocated their minimum required bandwidths, and the rest data blocks share equal transmission time, reflected by constraint (11) and (12), respectively. The object of the TTM-n problem is to minimize the transmission time of data block n. Similar to the TTM problem, as the object of the TTM-n problem is equivalent to maximizing $\sum_{j=1}^{M} x_{nj}$, which is linear, and constraint (12) can be replaced by the following linear constraint,

$$\frac{\sum_{j=1}^{M} x_{nj}}{S_n} = \frac{\sum_{j=1}^{M} x_{ij}}{S_i}, \ n \leq i \leq N$$

the TTM-n problem can be readily transformed into a linear optimization problem.

TABLE 4
General Maximum Transmission Time Minimization (G-MTTM) Problem Formulation for Multi-path Data Transmission in a Virtual Data Center

Minimize:

$$\max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | 1 \leq i \leq N\}$$

Subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}^k x_{ij} \leq B_k, 1 \leq k \leq K, \tag{6}$$

$$c_{ij}^k = \begin{cases} 1, \text{ if path } x_{ij} \text{ contains link } k \\ 0, \text{ otherwise} \end{cases}$$

$$\sum_{j=1}^{M} x_{ij} \geq b_i, 1 \leq i \leq N \tag{7}$$

TABLE 5
General Transmission Time Summation Minimization (G-TTSM) Problem Formulation for Multi-path Data Transmission in a Virtual Data Center

Minimize:

$$\sum_{i=1}^{N} \frac{S_i}{\sum_{j=1}^{M} x_{ij}}$$

Subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}^k x_{ij} \leq B_k, 1 \leq k \leq K, \tag{8}$$

$$c_{ij}^k = \begin{cases} 1, \text{ if path } x_{ij} \text{ contains link } k \\ 0, \text{ otherwise} \end{cases}$$

$$\sum_{j=1}^{M} x_{ij} \geq b_i, 1 \leq i \leq N \tag{9}$$

TABLE 6
TRANSMISSION TIME MINIMIZATION-n (TTM-n) PROBLEM FORMULATION FOR MULTI-PATH DATA TRANSMISSION IN A VIRTUAL DATA CENTER

Minimize:

$$\frac{S_n}{\sum_{j=1}^{M} x_{nj}}$$

Subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}^{k} x_{ij} \leq B_k, 1 \leq k \leq K, \tag{10}$$

$$c_{ij}^{k} = \begin{cases} 1, & \text{if path } x_{ij} \text{ contains link } k \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{j=1}^{M} x_{ij} = b_i, 1 \leq i < n \tag{11}$$

$$\frac{S_n}{\sum_{j=1}^{M} x_{nj}} = \frac{S_i}{\sum_{j=1}^{M} x_{ij}},\ n \leq i \leq N \tag{12}$$

Next, we show the correlation between the optimal solutions of the G-MTTM and TTM-n problems, based on which we design an optimal algorithm for the G-MTTM problem, named as the *TTM-based* algorithm. As will be demonstrated by our simulation results in Section IV, the TTM-based algorithm can solve the G-MTTM problem faster than the convex algorithms.

For presentational convenience, we say that U defined by (2) is a feasible solution of the G-MTTM (or TTM-n) problem if $x_{ij} \in U$ satisfies all constraints in Table 4 (or Table 6). The optimal solution and value of the G-MTTM (or TTM-n) problem are denoted as U_g^* and t_g^* (or U_n^* and t_n^*), respectively. That is,

$$t_g^* = \max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | x_{ij} \in U_g^*,\ 1 \leq i \leq N\}$$

and

$$t_n^* = \frac{S_n}{\sum_{x_{nj} \in U_n^*} x_{nj}}$$

In the next subsection, we will propose the TTM-based algorithm and prove its optimality.

C. The TTM-based Algorithm

Before proposing the TTM-based algorithm, we first derive the following lemmas and theorems, which are used to prove the optimality of the TTM-based algorithm. Without loss of generality, we assume

$$\frac{S_{i-1}}{b_{i-1}} \leq \frac{S_i}{b_i}, \text{ for } 1 \leq i \leq N$$

where $\frac{S_0}{b_0}$ is set as 0, and $\frac{S_i}{b_i}$ is regarded as ∞ when $b_i = 0$.

Lemma 2: $t_g^* \leq \frac{S_N}{b_N}$.

Proof: By constraint (7) in Table 4,

$$\sum_{j=1}^{M} x_{ij} \geq b_i, \text{ where } x_{ij} \in U_g^*,\ 1 \leq i \leq N$$

by which we have

$$\frac{S_i}{\sum_{j=1}^{M} x_{ij}} \leq \frac{S_i}{b_i}, \text{ where } x_{ij} \in U_g^*,\ 1 \leq i \leq N$$

therefore,

$$t_g^* \leq \max\{\frac{S_i}{b_i} | 1 \leq i \leq N\}$$

In addition,

$$\frac{S_{i-1}}{b_{i-1}} \leq \frac{S_i}{b_i}, \text{ for } 1 \leq i \leq N$$

thus,

$$\max\{\frac{S_i}{b_i} | 1 \leq i \leq N\} = \frac{S_N}{b_N}$$

■

Lemma 2 gives the upper bound of t_g^*, the next lemma tells the ordering relation between t_g^* and t_n^*.

Lemma 3: $t_n^* \leq t_g^*$, for $1 \leq n \leq N$.

Proof: By the definition of t_g^*,

$$t_g^* \geq \frac{S_i}{\sum_{j=1}^{M} x_{ij}}, \text{ where } x_{ij} \in U_g^*,\ 1 \leq i \leq N$$

Since decreasing $x_{ij} \in U_g^*$ will not violate constraint (6) in Table 4 or constraint (10) in Table 6, and

$$\sum_{j=1}^{M} x_{ij} \geq b_i, \text{ where } x_{ij} \in U_g^*,\ 1 \leq i \leq N$$

by constraint (7) in Table 4, we construct a feasible solution of the TTM-n problem, say, U, by decreasing $x_{ij} \in U_g^*$, such that

$$\sum_{j=1}^{M} x_{ij} = b_i, \text{ where } x_{ij} \in U,\ 1 \leq i < n$$

and

$$t_g^* = \frac{S_i}{\sum_{j=1}^{M} x_{ij}}, \text{ where } x_{ij} \in U,\ n \leq i \leq N$$

As t_n^* is the minimum transmission time of data block S_n in the TTM-n problem, and

$$\frac{S_n}{\sum_{x_{nj} \in U} x_{nj}} = t_g^*$$

the lemma holds. ■

By the proof of Lemma 3, a feasible solution of the TTM-n problem can be constructed from the optimal solution of the G-MTTM problem. In other words, if the TTM-n problem is infeasible, neither is the G-MTTM problem, as stated by the following corollary.

Corollary 2: The G-MTTM problem is infeasible if $\exists n \in [1, N]$, such that the TTM-n problem is infeasible.

Also, by Lemma 3, we have the following theorem on U_n^* and the feasible solution of the G-MTTM problem.

Theorem 1: U_n^* is a feasible solution of the G-MTTM problem if and only if $t_g^* \leq \frac{S_n}{b_n}$.

Proof: Sufficiency. We prove the sufficiency by contradiction, and assume that U_n^* is infeasible for the G-MTTM problem when $t_g^* \leq \frac{S_n}{b_n}$.

Comparing the constraints in Table 4 and 6, constraint (7) in Table 4 must be violated in U_n^*. In other words, $\exists i' \in [n, N]$, such that

$$\sum_{x_{i'j} \in U_n^*} x_{i'j} < b_{i'}$$

Hence,

$$t_n^* = \frac{S_n}{\sum_{x_{nj} \in U_n^*} x_{nj}} = \frac{S_{i'}}{\sum_{x_{i'j} \in U_n^*} x_{i'j}} > \frac{S_{i'}}{b_{i'}}$$

by constraint (12) in Table 6. In addition,

$$\frac{S_n}{b_n} \leq \frac{S_{i'}}{b_{i'}}, \text{ when } n \leq i'$$

thus, $t_n^* > \frac{S_n}{b_n}$. Since $t_g^* \leq \frac{S_n}{b_n}$, we have $t_g^* < t_n^*$, which contradicts Lemma 3. Therefore, the assumption is not true, and the sufficiency holds.

Necessity. We prove the necessity by contradiction, and assume that $t_g^* > \frac{S_n}{b_n}$ when U_n^* is feasible for the G-MTTM problem.

As t_g^* is the minimum transmission time of all data blocks,

$$\max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | x_{ij} \in U_n^*,\ 1 \leq i \leq N\} \geq t_g^*$$

By constraint (11) and (12) in Table 6,

$$\sum_{j=1}^{M} x_{ij} = b_i, \text{ where } x_{ij} \in U_n^*,\ 1 \leq i < n$$

and

$$t_n^* = \frac{S_i}{\sum_{j=1}^{M} x_{ij}}, \text{ where } x_{ij} \in U_n^*,\ n \leq i \leq N$$

therefore,

$$\max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | x_{ij} \in U_n^*,\ 1 \leq i \leq N\} = \max\{t_n^*, \frac{S_i}{b_i} | 1 \leq i < n\}$$

According to our assumption,

$$t_g^* > \frac{S_n}{b_n} \geq \frac{S_i}{b_i}, \text{ for } 1 \leq i < n$$

we have

$$\max\{t_n^*, \frac{S_i}{b_i} | 1 \leq i < n\} = t_n^* > \frac{S_n}{b_n}$$

Hence,

$$t_n^* = \frac{S_n}{\sum_{x_{nj} \in U_n^*} x_{nj}} > \frac{S_n}{b_n}$$

by which we have

$$\sum_{x_{nj} \in U_n^*} x_{nj} < b_n$$

and U_n^* violates constraint (7) in Table 4. Therefore, the assumption is not true, and the necessity holds. ■

Theorem 1 tells the condition of U_n^* being feasible for the G-MTTM problem, and we further give the condition of U_n^* being optimal for the G-MTTM problem in the next theorem.

Theorem 2: If $\frac{S_{n-1}}{b_{n-1}} < t_g^* \leq \frac{S_n}{b_n}$, U_n^* is the optimal solution of the G-MTTM problem.

Proof: By Theorem 1, U_n^* is feasible for the G-MTTM problem, which means that

$$\max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | x_{ij} \in U_n^*,\ 1 \leq i \leq N\} \geq t_g^*$$

With constraint (11) and (12) in Table 6, we have

$$\max\{\frac{S_i}{\sum_{j=1}^{M} x_{ij}} | x_{ij} \in U_n^*,\ 1 \leq i \leq N\} = \max\{t_n^*, \frac{S_i}{b_i} | 1 \leq i < n\}$$

In addition,

$$t_g^* > \frac{S_{n-1}}{b_{n-1}} \geq \frac{S_i}{b_i}, \text{ for } 1 \leq i < n$$

Hence,

$$\max\{t_n^*, \frac{S_i}{b_i} | 1 \leq i < n\} = t_n^* \geq t_g^*$$

Applying Lemma 3, $t_n^* = t_g^*$, and U_n^* is the optimal solution of the G-MTTM problem. ■

Note that if U_n^* is the optimal solution of the G-MTTM problem, it uses the least bandwidths, which is proved in the following corollary.

Corollary 3: If U_n^* is the optimal solution of the G-MTTM problem,

$$\sum_{x_{ij} \in U_n^*} x_{ij} \leq \sum_{x_{ij} \in U_g^*} x_{ij}$$

Proof: We prove the corollary by contradiction, and assume that

$$\sum_{x_{ij} \in U_n^*} x_{ij} > \sum_{x_{ij} \in U_g^*} x_{ij}$$

By our assumption, there must exist some $i' \in [1, N]$, such that

$$\sum_{x_{i'j} \in U_n^*} x_{i'j} > \sum_{x_{i'j} \in U_g^*} x_{i'j}$$

As

$$\sum_{j=1}^{M} x_{ij} = b_i, \text{ where } x_{ij} \in U_n^*,\ 1 \leq i < n$$

by constraint (11) in Table 6 and

$$\sum_{j=1}^{M} x_{ij} \geq b_i, \text{ where } x_{ij} \in U_g^*,\ 1 \leq i \leq N$$

by constraint (7) in Table 4, we have $n \le i' \le N$. Therefore,

$$t_n^* = \frac{S_{i'}}{\sum_{x_{i'j} \in U_n^*} x_{i'j}} < \frac{S_{i'}}{\sum_{x_{i'j} \in U_g^*} x_{i'j}} \le t_g^*$$

However, $t_n^* = t_g^*$ if U_n^* is the optimal solution of the G-MTTM problem, therefore, the assumption is not true, and the corollary holds. ■

The next lemma states the condition of U_n^* being infeasible for the G-MTTM problem.

Lemma 4: U_n^* is infeasible for the G-MTTM problem if and only if $t_n^* > \frac{S_n}{b_n}$.

Proof: Sufficiency. $t_n^* > \frac{S_n}{b_n}$ means

$$t_n^* = \frac{S_n}{\sum_{x_{nj} \in U_n^*} x_{nj}} > \frac{S_n}{b_n}$$

Therefore,

$$\sum_{x_{nj} \in U_n^*} x_{nj} < b_n$$

violating constraint (7) in Table 4, and U_n^* is infeasible for the G-MTTM problem.

Necessity. If U_n^* is infeasible for the G-MTTM problem, constraint (7) in Table 4 must be violated for some $i' \in [n, N]$. That is, $\exists i' \in [n, N]$, such that

$$\sum_{x_{i'j} \in U_n^*} x_{i'j} < b_i$$

As

$$\frac{S_n}{b_n} \le \frac{S_{i'}}{b_{i'}}, \text{ when } n \le i'$$

we have that

$$t_n^* = \frac{S_n}{\sum_{x_{nj} \in U_n^*} x_{nj}} = \frac{S_{i'}}{\sum_{x_{i'j} \in U_n^*} x_{i'j}} > \frac{S_{i'}}{b_{i'}} \ge \frac{S_n}{b_n}$$

by constraint (12) in Table 6. ■

Theorem 1 and Lemma 4 in conjunction prove the following lemma.

Lemma 5: $t_g^* > \frac{S_n}{b_n}$ if and only if $t_n^* > \frac{S_n}{b_n}$.

With the above lemmas and theorems, we propose the TTM-based algorithm for the G-MTTM problem in Table 7, and prove its optimality in Lemma 6.

Lemma 6: The TTM-based algorithm in Table 7 is optimal for the G-MTTM problem.

Proof: As

$$\frac{S_{i-1}}{b_{i-1}} \le \frac{S_i}{b_i}, \text{ for } 1 \le i \le N$$

and $t_n^* \le t_g^*$ by Lemma 2, $t_n^* > \frac{S_{n'-1}}{b_{n'-1}}$ means that

$$t_g^* > \frac{S_n}{b_n}, \text{ for } 1 \le n \le n' - 1$$

By Theorem 1, U_n^* is infeasible for the G-MTTM problem for $1 \le n \le n' - 1$, and we cannot find the optimal solution of the G-MTTM problem by solving the TTM-n problem if $1 \le n \le n' - 1$.

TABLE 7
HIGH-LEVEL DESCRIPTION OF THE TTM-BASED ALGORITHM FOR THE G-MTTM PROBLEM

Let $n = 1$ initially.
while $n \le N$
  solve the TTM-n problem;
  if the TTM-n problem is infeasible
    the G-MTTM problem is infeasible;
    break;
  else if $t_n^* \le \frac{S_n}{b_n}$
    U_n^* is the optimal solution of the G-MTTM problem;
    break;
  else if $\exists n' \in [n, N]$, such that $\frac{S_{n'-1}}{b_{n'-1}} < t_n^* \le \frac{S_{n'}}{b_{n'}}$
    $n = n'$;
  else
    the G-MTTM problem is infeasible;
    break;
  end if;
end while;
END

Hence, we can set $n = n'$, and if $t_{n'}^* \le \frac{S_{n'}}{b_{n'}}$, $t_g^* \le \frac{S_{n'}}{b_{n'}}$ by Lemma 5, meaning that

$$\frac{S_{n'-1}}{b_{n'-1}} < t_g^* \le \frac{S_{n'}}{b_{n'}}$$

Therefore, $U_{n'}^*$ is the optimal solution of the G-MTTM problem by Theorem 2.

On the other hand, if

$$\forall i \in [1, N], \; t_n^* > \frac{S_i}{b_i}$$

we have

$$t_g^* \ge t_n^* > \frac{S_N}{b_N}$$

according to Lemma 5, which contradicts Lemma 2, and the G-MTTM problem is infeasible. ■

As will be seen in the next section, the TTM-based algorithm can solve the G-MTTM problem faster than the convex algorithms. Next, we analyze the time complexity of the TTM-based algorithm.

D. Time Complexity Analysis

In this subsection, we analyze the time complexity of the propose TTM-based algorithm.

As the TTM-based algorithm solves at most N TTM-n problems, and the TTM-n problem can be linearized, i.e., transformed into a linear optimization problem, by replacing its nonlinear object function and constraint with linear ones, we start with the discussion on time complexity of solving a linear optimization problem.

The standard form of the linear optimization problem is to maximize $\mathbf{c}^T\mathbf{x}$ ($\mathbf{c}, \mathbf{x} \in \mathbf{R}^n$) over all vectors $\mathbf{x}$ such that $\mathbf{A}\mathbf{x} = \mathbf{b}$ and $\mathbf{x} \ge \mathbf{0}$. In [22], Khachiyan proved that a linear optimization problem can be solved in polynomial time relative to the length of the binary encoding of the input, denoted as L. In other words, L is the number of bits encoding $\mathbf{A}$, $\mathbf{b}$ and $\mathbf{c}$.

To transform the TTM-n problem to the standard form of linear optimization problem, we add nonnegative slack variables v_k to constraint (12) in Table 6 to convert the inequality to equality, i.e.,

$$\sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}^{k} x_{ij} + v_k = B_k, 1 \leq k \leq K$$

Therefore, in the obtained standard form of the linearized TTM-n problem, **A** is an $(MN+1)\times(K+N)$ matrix, **b** and **c** are $(K+N)$-dimentional and M-dimensional vectors, respectively. Suppose each element in **A**, **b** and **c** is encoded by fixed length of binary, L should be proportional to $(MN+1)\times(K+N)+K+N+M$. According to Khachiyan's result, the time complexity of solving the TTM-n problem is thus polynomial relative to $MN\times(K+N)$. Since the TTM-based algorithm in Table 7 solves at most N TTM-n problems, the time complexity of the TTM-based algorithm is polynomial relative to $MN^2\times(K+N)$. Note that when $n=1$, the TTM-n problem in Table 6 and TTM problem in Table 3 are identical. As the MTTM and TTM problems share the same optimal solution, the time complexity of solving the MTTM problem is polynomial relative to $MN\times(K+N)$ by our analysis.

It is worth pointing out that the above time complexity analysis is mostly of theoretical interest, the ellipsoid algorithm Khachiyan used to prove his result is not useful in practice. Instead, many algorithms can solve the linear optimization problem very efficiently despite their worse or unclear time complexities. Hence, we rely on the algorithms embedded in commercial solvers to solve the TTM-n problem, as will be seen in the next section.

In this section, we extend our discussion to data transmission with minimum bandwidth requirement, and generalize the MTTM and TTSM problems to G-MTTM and G-TTSM problems, respectively. By analyzing the properties of the G-MTTM problem, we propose the TTM-based algorithm for it. In the next section, we evaluate the efficiency of the TTM-based algorithm by extensive simulations.

E. Simulation Configurations

It is difficult, if not impossible, to capture all the features of VDCs deployed in a real data center, which are closely related to many factors, such as the tenants' demands, virtualization technologies, underlying physical data center architecture, etc. [3], [5]–[10]. Therefore, we adopt certain extent of simplification to randomly generate the VDCs used in our simulation, and regard the simulation results after simplification as preliminary and useful references to predict the merit of our work in real world. Next, we introduce the simulation configurations in detail.

We generate 100 VDCs in our simulation, every 10 of them have equal number of VMs, which increases from 20 to 200 at an incremental step of 20 [6]. In a VDC with N VMs, we generate N data block transmissions, the sources and destinations of which are randomly chosen among all VMs in the VDC. Note that for the purpose of matching the data transfer pattern in large-scale data collection and distribution, in the G-MTTM problem, all these N data block transmissions share the same source (or destination) VM in the VDC. The upper bound of the data block size is normalized, and the size of a data block is a uniformly distributed random number in interval $[0,1]$.

To reflect the diversity of VDC network topologies, we randomly interconnect the virtual switches and VMs in the generated VDCs. Nevertheless, we assume that the VDC network is of rich interconnectivity, which resembles the feature of many physical data center networks, and use the data center architecture in [21] as the reference to determine the numbers of virtual switches and virtual links with respect to the number of VMs in the VDC network. In [21], $k^3/4$ servers are interconnected by a 3-tier fat tree network consisting of $5k^2/4$ k-port commodity switches, and the number of physical links is $3k^3/4$. We adopt the same ratio among the numbers of VMs, virtual switches and links in generating VDCs in our simulation. Furthermore, we assume homogenous link capacities in the VDC, which are set as 1, and the data transmission paths are the top M shortest paths in terms of hop distance between the source and destination VMs. To observe the impact of path multiplicity on simulation results, we increase M from 1 to 4.

IV. Performance Evaluation

In this section, we compare the efficiencies of our proposed TTM-based algorithm and convex algorithms in solving G-MTTM problem, and evaluate the solution time of the G-TTSM problem in a variety of VDCs with different network sizes, i.e., the number of VMs, and network topologies.

In our simulation, the required minimum bandwidth of a data block is a uniformly distributed random number in interval $[0,b]$, where b is the upper bound of the required minimum bandwidth with respect to the link capacity, i.e., 1. Therefore, in a VDC with N VMs, the expectation of required minimum bandwidth summation of all data blocks is $b\cdot N/2$. To estimate the difficulty of the VDC network to satisfy the minimum bandwidth requirements in the G-MTTM and G-TTSM problems, we calculate the node degree, i.e., the number of connected links, of the common source (or destination) VM, denoted as d, in the G-MTTM problem, and the average length of all data transmission paths, denoted as h, in the G-TTSM problem. Since the link capacity is 1, and there are $3N$ links in the VDC by our simulation configuration, we define

$$\alpha = \begin{cases} \frac{b\cdot N/2}{d} & \text{if maximum transmission time is minimized} \\ \frac{h\cdot b\cdot N/2}{3N} & \text{if transmission time summation is minimized} \end{cases}$$

When α approaches 1, we consider the minimum bandwidth requirements hard to meet for the VDC network, which is verified by our simulation, and the corresponding optimization problems are often infeasible. To ensure that the optimization problems involved in our simulation are generally feasible, α is set as 0, 0.25 and 0.5 to represent zero, light and moderate bandwidth requirements, respectively.

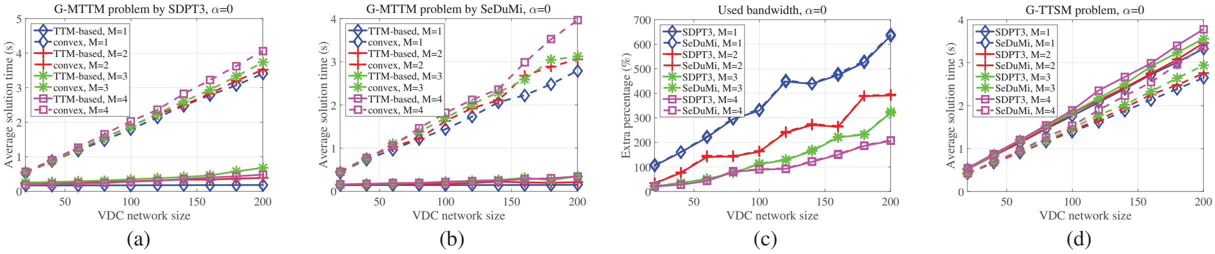

Fig. 1. Simulation results under zero minimum bandwidth requirement. (a) Average solution time of the G-MTTM problem by SDPT3 solver. (b) Average solution time of the G-MTTM problem by SeDuMi solver. (c) Extra percentage of bandwidth used by the convex algorithms. (d) Average solution time of the G-TTSM problem.

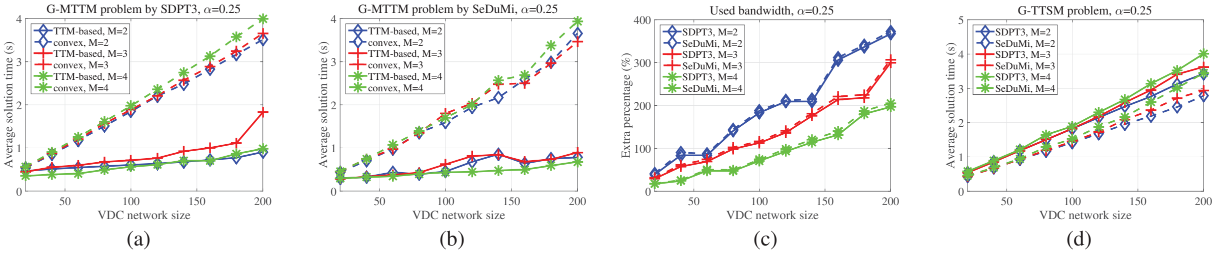

Fig. 2. Simulation results under light minimum bandwidth requirement. (a) Average solution time of the G-MTTM problem by SDPT3 solver. (b) Average solution time of the G-MTTM problem by SeDuMi solver. (c) Extra percentage of bandwidth used by the convex algorithms. (d) Average solution time of the G-TTSM problem.

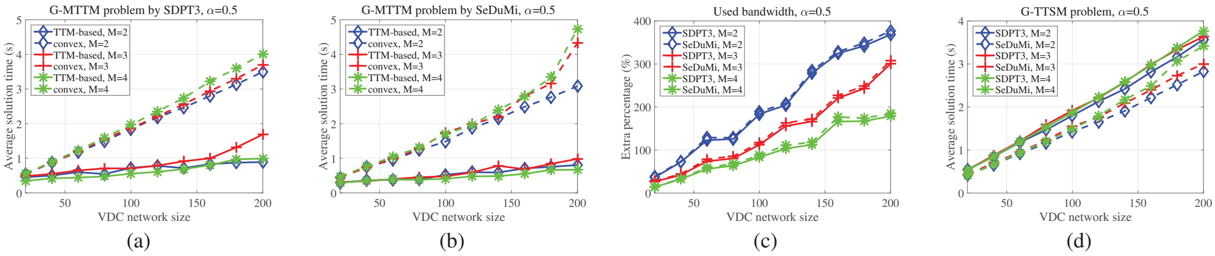

Fig. 3. Simulation results under moderate minimum bandwidth requirement. (a) Average solution time of the G-MTTM problem by SDPT3 solver. (b) Average solution time of the G-MTTM problem by SeDuMi solver. (c) Extra percentage of bandwidth used by the convex algorithms. (d) Average solution time of the G-TTSM problem.

Given the value of α, we randomly generate 100 sets of required minimum bandwidths for all data blocks in a VDC, and solve the corresponding G-MTTM and G-TTSM problems. Note that when $\alpha = 0$, the G-MTTM and G-TTSM problems become the MTTM and TTSM problems, respectively.

As mentioned in the previous section, there are a variety of convex algorithms in the literature, and no one is better than the others on all problems. In our simulation, we use the convex algorithms embedded in two popular solvers to solve the G-MTTM and G-TTTM problems. These two solvers are included in CVX, a package for specifying and solving convex optimization problems are used in our simulation [23] [24]. For the purpose of fair comparison, we solve the linearized TTM-n problem in the TTM-based algorithm by the same solvers.

By the above simulation configurations, we need to solve 1,000 pairs of G-MTTM and G-TTSM problems under given VDC network size, path multiplicity and the value of α. We record the average solution time of the G-MTTM and G-TTSM problems by different algorithms to evaluate the efficiencies of these algorithms. Besides, the optimal solution by the TTM-based algorithm uses the minimum bandwidth to minimize the maximum transmission time by Corollary 3, to compare the bandwidths used by the TTM-based algorithm and convex algorithms, we calculate extra percentage of bandwidth used by the convex algorithms with respect to that of the TTM-based algorithm for each G-MTTM problem, the percentage is then averaged for all these 1,000 G-MTTM problems.

A. Analysis on Simulation Results

In this subsection, we analyze the simulation results, which are illustrated in Fig. 1, 2 and 3. Note that as our simulation results show that the G-MTTM and G-TTSM problems are seldom feasible when $M = 1$ under light and moderate minimum bandwidth requirements, which is due to the lack of flexibility to allocate bandwidth among multiple data transmission paths, we plot the curves starting from $M = 2$ in Fig. 2 and 3.

From these figures, we can see that the average solution time of the G-MTTM and G-TTSM problems by the convex algorithms increases almost linearly with respect to the network size, and never exceeds 5 seconds in our simulation, which shows that the convex algorithms can solve the G-MTTM and G-TTSM problems efficiently. In addition, the TTM-based algorithm solves the G-MTTM problem much faster than the convex algorithm algorithms in our simulation, which is in accordance with our expectation.

We also observe that as the VDC size grows, the convex algorithms use more and more bandwidth than the TTM-based algorithm to minimize the maximum transmission time. This is because that the shared link with the heaviest competition for bandwidth by multiple data transmission paths is the bottleneck for maximum transmission time reduction, and more data transmissions are issued in larger VDCs in our simulation, which intensifies the competition. Therefore, the convex algorithms exploring bandwidth margins on the links left by the TTM-based algorithm cannot lead to better solution, which demonstrates the economy of the TTM-based algorithm in utilizing the network resources. As M increases, the bandwidth used by the convex algorithms to minimize the maximum transmission time is closer to that by the TTM-based algorithm. The reason is that higher path multiplicity provides greater bandwidth allocation flexibility to relieve the competition for the bandwidth on shared links by different data transmission paths, leading to less bandwidth margins on the links in the optimal solution by the TTM-based algorithm.

In summary, our simulation results demonstrate that the G-MTTM and G-TTSM problems can be efficiently solved by the convex algorithms, and our proposed TTM-based algorithm outperforms the convex algorithms in terms of both solution time and used bandwidth in solving the G-MTTM problem.

V. Conclusion

In this paper, we propose to use transmission time of data blocks as the metric to measure the network utilization, and study the bandwidth allocation for multi-path data transmission in VDCs. We start with the case that data transmission requires no minimum bandwidth, and formulate the bandwidth allocation problem as the MTTM and TTSM problems, which are generalized to the G-MTTM and G-TTSM problems, respectively, by introducing minimum bandwidth requirement. We show the convexity of these optimization problems, and design the TTM-based algorithm for the G-TTSM problem by exploring the properties of its optimal solution. Extensive simulation results show that the G-MTTM and G-TTSM problems can be efficiently solved by convex algorithms, and our proposed TTM-based algorithm outperforms the convex algorithms in solving the G-MTTM problem, demonstrating that to measure and optimize the network utilization by transmission time of data blocks is practical.

References

[1] Amazon Elastic Compute Cloud (Amazon EC2). http://aws.amazon.com/ec2/.

[2] D. Carr, "How Google Works," July 2006.

[3] G. Ghoda, N. Meruliya, D. H. Parekh, T. Gajjar, D. Dave and R. Sridaran, "A survey on data center network virtualization," *IEEE International Conference on Computing for Sustainable Global Development,* 2016.

[4] J. Guo, F. Liu, J. C. S. Lui and J. Hai, "Fair Network Bandwidth Allocation in IaaS Datacenters via a Cooperative Game Approach," *IEEE/ACM Transactions on Networking,* vol. 24, no. 2, pp. 873-886, 2016.

[5] J. Mudigonda, P. Yalagandula, B. Stiekes and Y. Pouffary, "NetLord: A Scalable Multi-Tenant Network Architecture for Virtualized Datacenters," *Proc. ACM SIGCOMM,* August 2011.

[6] C. Guo, G. Lu, H. Wang, S. Yang, C. Kong, P. Sun, W. Wu and Y. Zhang, "SecondNet: A Data Center Network Virtualization Architecture with Bandwidth Guarantees," *Proc. ACM CoNEXT,* December 2010.

[7] A. Yassine, A. A. N. Shirehjini and S. Shirmohammadi, "Bandwidth On-demand for Multimedia Big Data Transfer across Geo-Distributed Cloud Data Centers," *IEEE Transactions on Cloud Computing,* 2016.

[8] T. Lam, S. Radhakrishnan, A. Vahdat and G. Varghese, "NetShare: Virtualizing Data Center Networks across Services," *Technical Report CS2010-0957,* May 2010.

[9] L. Popa, G. Kumar, M. Chowdhury, A. Krishnamurthy, S. Ratnasamy and I. Stoica. 2012. "FairCloud: sharing the network in cloud computing," *Proceedings of the ACM SIGCOMM,* pp. 187-198, 2012.

[10] J. Duan, and Y. Yang, "A Load Balancing and Multi-tenancy Oriented Data Center Virtualization Framework," *IEEE Transactions on Parallel and Distributed Systems,* vol. 28, no. 8, pp. 1-1, 2017.

[11] J. Lee, Y. Turner, M. Lee, L. Popa, S. Banerjee, J. Kang and P. Sharma, "Application-driven bandwidth guarantees in datacenter," *SIGCOMM Comput. Commun. Rev.*, vol. 44, no. 4, pp. 467-478, August 2014.

[12] L. Chen, Y. Feng and B. Li, " Efficient Performance-Centric Bandwidth Allocation with Fairness Tradeoff," *IEEE Transactions on Parallel and Distributed Systems*, no. 99, pp. 1-1, 2018.

[13] T. Ma, J. Wu, J. Lan and Y. Hu, "Optimal rate and aggregate bandwidth allocation under service performance requirements in data center backbone networks," *IEEE International Conference on Computer and Communications*, 2017.

[14] T. M. Nam, N. H. Thanh, H. T. Hieu, N. T. Manh, N. V. Huynh and T. Hoang, "Joint Network Embedding and Server Consolidation for Energy–Efficient Dynamic Data Center Virtualization," *Computer Networks*, vol. 125, pp. 76-89, 2017.

[15] M. Chowdhury, M. Zaharia, J. Ma, M. I. Jordan and I. Stoica, "Managing data transfers in computer clusters with orchestra," *SIGCOMM Comput. Commun. Rev.*, vol. 41, no. 4, pp. 98-109, August 2011.

[16] J. Dean and S. Ghemawat, "MapReduce: Simplified data processing on large clusters," *In OSDI,* pp. 137-150, 2004.

[17] M. Isard, M. Budiu, Y. Yu, A. Birrell and D. Fetterly, "Dryad: Distributed data-parallel programs from sequential building blocks." *In EuroSys,* pp. 59-72, 2007.

[18] D. G. Murray, M. Schwarzkopf, C. Smowton, S. Smith, A. Madhavapeddy and S. Hand, "Ciel: A Universal Execution Engine for Distributed Data-Flow Computing," *In NSDI,* 2011.

[19] M. Zaharia, M. Chowdhury, M. J. Franklin, S. Shenker and I. Stoica, "Spark: Cluster Computing with Working Sets," *In HotCloud,* 2010.

[20] Y. Wu, Z. Zhang, C. Wu, C. Guo, Z. Li and F. C. M. Lau, " Orchestrating Bulk Data Transfers across Geo-Distributed Datacenters," *IEEE Transactions on Cloud Computing*, vol. 5, pp. 112-125, 2017.

[21] M. Al-Fares, A. Loukissas and A. Vahdat, "A scalable, commodity data center network architecture," *Proc. of ACM SIGCOMM '08*, pp. 63-74, Aug. 2008.

[22] N. Megiddo, "On the complexity of linear programming," *Advances in economic theory,* vol. 12, pp. 225-268, Cambridge Univ. Press, Cambridge, 1989.

[23] CVX Research, Inc. CVX: Matlab software for disciplined convex programming, version 2.0. http://cvxr.com/cvx, April 2011.

[24] M. Grant and S. Boyd. Graph implementations for nonsmooth convex programs, Recent Advances in Learning and Control (a tribute to M. Vidyasagar), V. Blondel, S. Boyd, and H. Kimura, editors, pages 95-110, Lecture Notes in Control and Information Sciences, Springer, 2008. http://stanford.edu/~boyd/graph_dcp.html.