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# An integrated model for quantitative and qualitative description of spatial direction relations

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Each of the existing models for direction relations has its advantages and disadvantages, but none of them can meet the five criteria used to evaluate a satisfactory model, i.e. correctness, completeness, efficiency, quantification and qualification. Hence, this paper proposes a new model that integrates the advantages of existing ones using two strategies. First, the method for partitioning direction regions is improved so that the new model is correct, complete and efficient; second, the idea for calculating and describing direction relations in the direction-relation matrix model and the Voronoi-based model is integrated into the new model so that direction relations can be represented both qualitatively and quantitatively. Our experiments show that the model can calculate direction relations between arbitrary object pairs in two-dimensional spaces and the results are acceptable to a majority of people.

**Keywords:** spatial direction relations; arbitrary objects; object pairs

## 1. Introduction

Direction relation, along with topological relation (Egenhofer & Franzosa 1991; Li *et al.* 2002; Schneider & Behr 2006), distance relation (Hong 1995; Liu & Chen 2003) and similarity relation (Yan 2010), plays an important role in the communities of geographic information sciences, cartography, spatial cognition and various location-based services (Cicerone & Di Felice 2004). Its functions in spatial database construction (Kim & Um 1999), qualitative spatial reasoning (Frank 1992, 1996; Sharma 1996; Mitra 2002; Wolter & Lee 2010; Mossakowski & Moratz 2012), spatial computation (Ligozat 1998; Renz & Mitra 2004) and spatial retrieval (Papadias *et al.* 1994) have aroused the interest of many researchers. It has also been used in many practical operations (Zimmermann & Freksa, 1996), such as combat (direction relation allows soldiers to identify, locate and predict the location of enemies), driving (direction relation helps drivers avoid contact with other vehicles and environmental obstacles) and aircraft piloting (direction relation assists pilots to avoid terrain, other aircraft and environmental obstacles).

Many models for describing and/or calculating direction relations have been proposed. They mainly include the cone-based model (Haar 1976; Peuquet & Zhan 1987; Abdelmoty & Williams 1994; Frank 1996; Shekhar & Liu 1998), the 2D projection model (Frank 1992; Papadias *et al.* 1994; Safar & Shahabi 1999),

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the direction-relation matrix model (Goyal 2000) and the Voronoi-based model (Yan *et al.* 2006). Although these models have been used in spatial direction description and qualitative spatial reasoning, each of them has its disadvantages (see section 2). Thus, this paper will focus on proposing a new model that can integrate the advantages of the existing models for calculating and describing spatial direction relations.

After the introduction, existing models will be critically discussed (section 2); then a new model will be proposed (section 3); after that, experiments will be shown to demonstrate the acceptability and adaptability of the new model (section 4); finally, some conclusions will be made (section 5).

## 2. Analysis of existing models

Generally, a model for direction relations should meet at least the following five criteria (Goyal 2000; Yan *et al.* 2006).

(1) Correctness: direction relations calculated by the model should be consistent with human recognition, i.e. the results are acceptable to the majority of people.
(2) Completeness: the model can calculate the direction relations between arbitrary types of object pairs (e.g. point-point, point-line, line-polygon etc.).
(3) Quantification: the model can represent direction relations quantitatively (it uses angles or/and percentage values that denote the target object falling in corresponding cardinal direction regions).
(4) Qualification: the model can represent direction relations qualitatively (it uses cardinal directions, e.g. *N*, *NE*, *E*, etc.).
(5) Efficiency in retrieval: this refers to the time used to detect that the objects exist in a specific direction.

To facilitate the discussion it is designated in this paper that

- *A* is the reference object and *B* is the target object;
- Dir(*A*,*B*) is the qualitative description of direction relations from *A* to *B*;
- *D*(*A*,*B*) is the quantitative description of direction relations from *A* to *B*;
- the objects discussed in this paper are in two-dimensional spaces, including points, lines (i.e. linesegments or curves) and polygons (which may be concave or convex); and
- an extrinsic reference frame is employed in this paper for direction relations. Here, an extrinsic reference frame is usually set up on the Earth's surface by means of a rectangular coordinate system with the positive/negative direction of an axis corresponding to a cardinal direction (i.e. north, east, south or west).

### *Cone-based model*

The cone-based model (Haar 1976; Peuquet & Zhan 1987; Shekhar & Liu 1998) partitions the two-dimensional space around the centroid of the reference object into four direction regions (Figure 1), with one region corresponding to one of the four cardinal directions (i.e. *N*, *E*, *S*, *W*). The direction of the target object with respect to the reference object is determined by the target object's presence in a direction region for the reference object. If the target object coincides with the reference object, the direction between them is called 'same'.

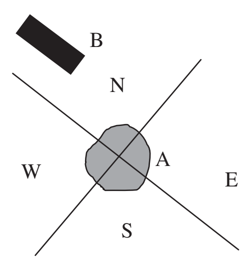



**Figure 1.** Principle of the cone-base model.

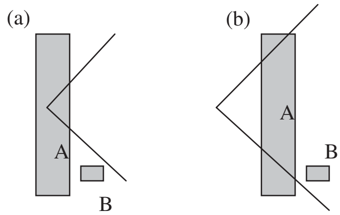



**Figure 2.** If the distance between the two objects is much smaller than their sizes, the cone-based model needs to be adjusted. (a) $B$ is visually to the east of $A$, but it does not fall in the east partition; (b) after the direction region of the east partition is adjusted, $B$ is to the east of $A$.

This model can efficiently detect whether a target object exists in a given direction, and gives a qualitative but not quantitative description of direction relations. If the distance between the two objects is much larger than their size, the model works well; otherwise a special method must be used to adjust the direction regions (e.g. Figure 2). If objects are overlapping, intertwined or horseshoe-shaped, this model uses centroids to judge directions (Peuquet & Zhan 1987) and the results are sometimes misleading (e.g. Figure 3). In addition, if a target object is in multiple directions, such as $\{N, NE, E\}$, this model does not provide a knowledge structure to represent such multiple directions (Goyal 2000).

### D projection model

The 2D projection model (Frank 1992, 1996; Papadias *et al.* 1994; Safar & Shahabi 1999) represents spatial relations between objects using MBRs (minimum bounding rectangles); hence, it is also called the MBR-based model. The MBR of an object is a minimum rectangle that encloses the object and whose four edges are either horizontal or vertical.

Reasoning between projections of MBRs on the $x$- and $y$-axes is performed using 1D interval relations. For example, in Figure 4, the projection of $B$ on the $x$-axis ($\text{proj}_x^B$) is *before* $\text{proj}_x^A$ and $\text{proj}_y^B$ is *before* $\text{proj}_y^A$; therefore, the relation between MBRs of objects $B$ and $A$ is (*before*, *before*). Using this method, one can characterize relations between MBRs of objects uniquely. There are 13 possible relations on an axis (Allen 1983; Nabil *et al.* 1995) in 1D space; therefore, this model distinguishes $13 \times 13 = 169$ relations in 2D space.

The 2D projection model approximates objects by their minimum bounding rectangles; therefore, the spatial relation may not necessarily be the same as the relation between exact representations of the objects, because the model cannot capture the details of objects in direction descriptions (Goyal 2000). So this model is only used for the qualitative description of direction relations.

### Direction-relation matrix model

The direction-relation matrix model (Goyal 2000) partitions space around the MBR of the reference object into nine direction regions (Figure 5), i.e. $N$, $NE$, $E$, $SE$, $S$, $SW$, $W$, $NW$ and

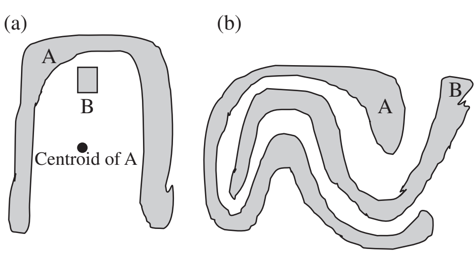



**Figure 3.** Two errors in the cone-based model: (a) the centroid of $B$ is to the north of the centroid of $A$, but $B$ is not visually to the north of $A$; (b) when A and B are intertwined, the answer is misleading.

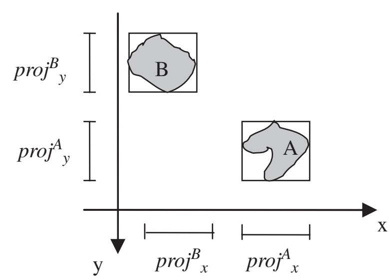



**Figure 4.** Principle of the 2D projection model.

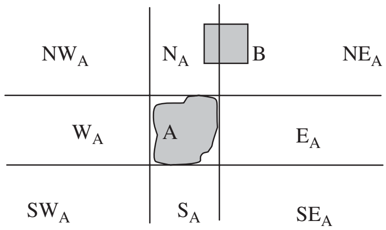



**Figure 5.** Principles of the direction-relation matrix model.

$O$ (same direction). A direction-relation matrix is constructed to record whether a section of the target object falls into a specific region (expression (1)).

$$\mathrm{Dir}(A,B) = \begin{bmatrix} NW_A \cap B & N_A \cap B & NE_A \cap B \\ W_A \cap B & O_A \cap B & E_A \cap B \\ SW_A \cap B & S_A \cap B & SE_A \cap B \end{bmatrix} \tag{1}$$

Expression (1) is too coarse to effectively express quantitative direction relations, for it only uses some 1s and 0s to record directions. To improve the reliability of the model, a detailed direction-relation matrix capturing more details by recording the area ratio of the target object in each region is employed (expression (2)).

$$D(A,B) = \begin{bmatrix} \frac{\mathrm{Area}(NW_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(N_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(NE_A \cap B)}{\mathrm{Area}(B)} \\ \frac{\mathrm{Area}(W_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(O_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(E_A \cap B)}{\mathrm{Area}(B)} \\ \frac{\mathrm{Area}(SW_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(S_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(SE_A \cap B)}{\mathrm{Area}(B)} \end{bmatrix} \tag{2}$$

The direction-relation matrix model can provide both quantitative and qualitative description of direction relations, and provides a knowledge structure for recording multiple directions. Nevertheless, this model can only calculate direction relations between extended objects. In other words, it cannot work if the reference/target object is a point or a line.

### *Voronoi-based model*

The Voronoi-based model (Yan *et al.* 2006) is based on the idea that people describe directions between two objects using multiple directions but not a single one; hence, 'direction group' is used in this model. A direction group consists of multiple directions, and each direction includes two components: the azimuths of the normals of direction Voronoi edges between two objects and the corresponding weights of the azimuths (Figure 6). The former can be calculated by means of Delaunay triangulation of the vertices and the points of intersection of the two objects; the latter can be calculated using the common areas of the two objects or the lengths of their direction Voronoi diagram edges.

The Voronoi-based model can give both quantitative and qualitative direction relations between arbitrary objects. Nevertheless, computation of Voronoi edges makes this model inefficient compared with the cone-based model, the 2D projection model and the direction-relation matrix model.

### *Comparison of the existing models*

A comparison of the existing models is shown in Table 1. It can be concluded that

(1) the cone-based model is of the highest efficiency and can calculate direction relations between arbitrary object pairs, though it cannot always give correct answers;
(2) the 2D projection model presents qualitative direction relations and its efficiency is medium, but it is not good

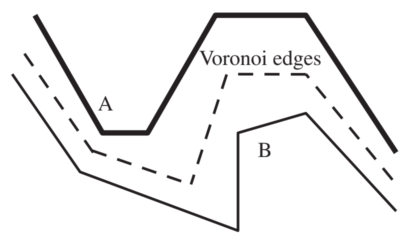



**Figure 6.** Principle of the Voronoi-based model.

**Table 1.** A comparison of the existing models

| Models | Correctness | Completeness | Qualification | Quantification | Efficiency |
|---|---|---|---|---|---|
| Cone-based model | Not always | *Yes* | Yes | No | *High* |
| 2D projection model | Not always | *No* | Yes | No | *Medium* |
| Direction-relation matrix model | Not always | *No* | Yes | Yes | *Medium* |
| Voronoi-based model | Yes | *Yes* | Yes | Yes | *Low* |

at correctness, completeness and quantification;

(3) the direction-relation matrix model is a qualitative and quantitative model, but it is not always correct; and

(4) the Voronoi-based model is of low efficiency, but it is good at the other four aspects.

Obviously, none of the existing models meets all five criteria.

## 3. New integrated model

Two strategies are employed to ensure that the new model can meet the five criteria discussed in section 2.

(1) The methods for partitioning direction regions used in the cone-based model and the direction relation matrix mode are improved and employed so that the new model is correct, complete and efficient.

(2) The idea for calculating and describing direction relations in the direction-relation matrix model and the Voronoi-based model is integrated into the new model so that direction relations can be represented both qualitatively and quantitatively.

### *Partition of direction regions*

The principle of proximity in Gestalt psychology (Wertheimer 1923) tells us that objects (or parts of objects) at close distance have a tendency to be perceived as a group (Palmer 1992; Rock 1996). This principle implies that different parts of objects take different roles in a human being's direction judgments. If the principle of proximity is taken into consideration, together with the 'selection of perception' in direction judgments, it can be concluded that direction judgment actually depends on the adjacency of parts/sides of the two objects. Nevertheless, the cone-based model selects the centroid of the reference object as the starting point to partition the direction regions, and views all parts of the reference object as the same in direction judgments, which obviously violates the principle of proximity in Gestalt psychology. To fill this theoretical gap, the direction regions in the new model are partitioned using a new method that integrates the ones used in the cone-based model and the direction-relation matrix model.

The direction regions of the reference objects with different geometric characteristics, i.e. polygonal, linear and point, are partitioned using different methods.

- If the reference object is a polygon, direction regions can be partitioned as follows.

First, calculate the MBR of $A$, and extend the four edges of the MBR to construct nine rectangular regions, i.e. $N_M, NE_M, E_M, SE_M$, $S_M, SW_M, W_M, NW_M$ and $\text{Same}_M$ (Figure 7(a)).

Second, calculate the centroid of the reference object and partition the space into eight direction regions using the cone-based model (Figure 7(b)).

Third, move the intersection of the two edges of each cardinal direction region (including $N_C$, $E_C$, $S_C$ and $W_C$) to the corresponding mid-point of the MBR (Figure 7(c)).

Last, let $C_A = \overline{\text{Same}_A \cup N_A \cup W_A \cup S_A \cup E_A}$; then the nine direction regions of $A$ (i.e. $N_A, E_A, S_A, W_A$, $\text{Same}_A, NE_A, SE_A, SW_A$ and $NW_A$) can be obtained using the following formulae.

(1) $N_A = N_M \cup N_C$;
(2) $E_A = E_M \cup E_C$;
(3) $S_A = S_M \cup S_C$;
(4) $W_A = W_M \cup W_C$;
(5) $\text{Same}_A = \text{Same}_M$;
(6) $NE_A = NE_A \cap C_A$;
(7) $SE_A = SE_A \cap C_A$
(8) $SW_A = SW_A \cap C_A$; and
(9) $NW_A = NW_A \cap C_A$.

- If the reference object is a point, the direction region partition is the same as that of the cone-based model (Figure 8(a)).
- If the reference object is a linear object (Figure 8(b)), calculate the MBR of the curve, then partition its direction regions the same as those for polygonal objects. Especially, if the reference object is a straight line segment (Figure 8(c) and (d)), the MBR becomes a line segment. The partition method for the direction region is similar to that for polygonal objects, too.

***Qualitative representation of direction relations***

After the partition of the direction regions surrounding $A$, it is obvious that $B$ must be in one or more of the nine regions. Thus, a matrix with nine elements may be constructed with formula (3) to record the direction relations between $A$ and $B$.

$$\text{Dir}(A,B) = \begin{bmatrix} NW_A \cap B & N_A \cap B & NE_A \cap B \\ W_A \cap B & \text{Same}_A \cap B & E_A \cap B \\ SW_A \cap B & S_A \cap B & SE_A \cap B \end{bmatrix} \tag{3}$$

where, for each of the nine elements, if the intersection is ?, its value is 0; else, its value is 1.

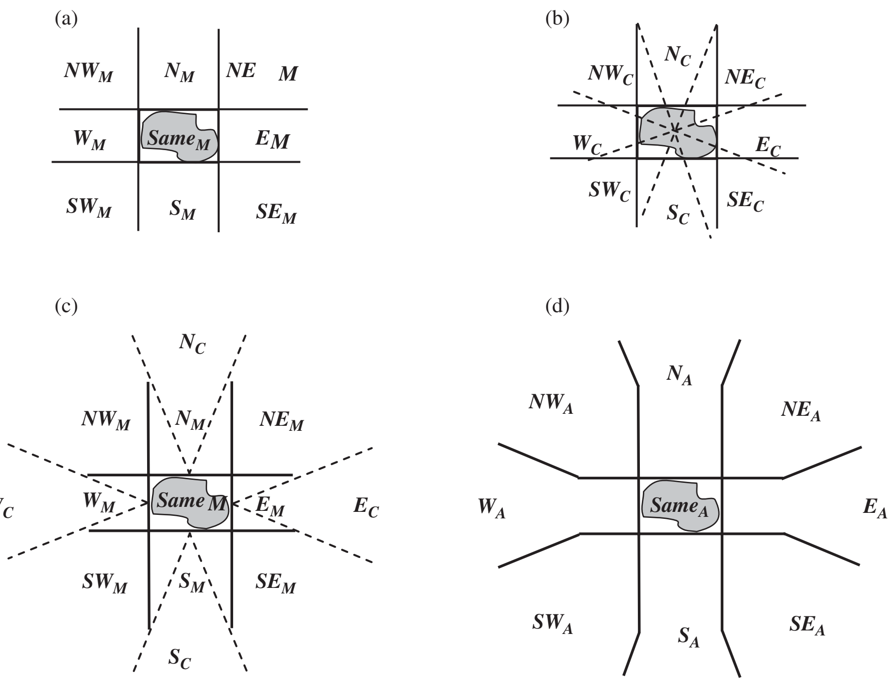



**Figure 7.** Partition of the nine direction regions. (a) Calculate the MBR of the reference object; (b) partition the eight direction regions using the cone-based model; (c) move the eight direction regions to the edges of the MBR; and (d) obtain the nine new direction regions.

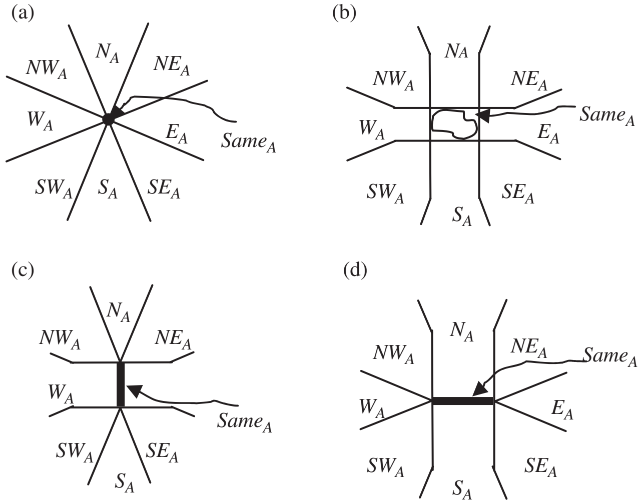



**Figure 8.** Partition of direction regions if the reference object is a point or a linear object. (a) a point; (b) a curve; (c) a straight line segment; and (d) a horizontal line segment.



Formula (3) tells whether $B$ appears in each of the nine direction regions or not. In other words, it presents a qualitative description of direction relations. Taking Figure 9 as an example, a qualitative expression can be obtained as follows:

$$\mathrm{Dir}(A,B) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

This qualitative description may also be expressed as:

$$\mathrm{Dir}(A,B) = \{E, SE, S\}$$

In other words, $B$ appears in the east, south-east and south direction regions of $A$.

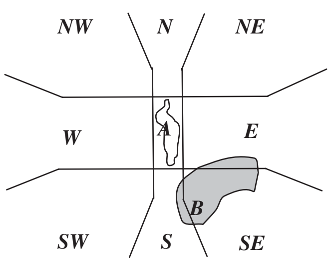



**Figure 9.** Computation of direction relations using the new model.

### *Quantitative representation of direction relations*

If $B$ is a polygon, formula (3) may be transformed into formula (4) in light of the idea in the direction-relation matrix model to quantitatively describe direction relations (Goyal 2000).

$$D(A,B) = \begin{bmatrix} \frac{\mathrm{Area}(NW_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(N_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(NE_A \cap B)}{\mathrm{Area}(B)} \\ \frac{\mathrm{Area}(W_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(\mathrm{Same}_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(E_A \cap B)}{\mathrm{Area}(B)} \\ \frac{\mathrm{Area}(SW_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(S_A \cap B)}{\mathrm{Area}(B)} & \frac{\mathrm{Area}(SE_A \cap B)}{\mathrm{Area}(B)} \end{bmatrix} \tag{4}$$

where each of the nine elements is the percentage of the area of $B$ falling in the corresponding direction regions.

Taking Figure 9 as an example, its quantitative expression can be obtained as follows:

$$D(A,B) = \begin{bmatrix} 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.13 \\ 0.00 & 0.12 & 0.75 \end{bmatrix}$$

This matrix can be explained more clearly: 13% of $B$ is to the east of $A$; 12% of $B$ is to the south of $A$; and 75% of $B$ is to the southeast of $A$.

- If $B$ is a linear object, formula (3) may be transformed into formula (5):

$$D(A,B) = \begin{bmatrix} \frac{\text{Length}(NW_A \cap B)}{\text{Length}(B)} & \frac{\text{Length}(N_A \cap B)}{\text{Length}(B)} & \frac{\text{Length}(NE_A \cap B)}{\text{Length}(B)} \\ \frac{\text{Length}(W_A \cap B)}{\text{Length}(B)} & \frac{\text{Length}(\text{Same}_A \cap B)}{\text{Length}(B)} & \frac{\text{Length}(E_A \cap B)}{\text{Length}(B)} \\ \frac{\text{Length}(SW_A \cap B)}{\text{Length}(B)} & \frac{\text{Length}(S_A \cap B)}{\text{Length}(B)} & \frac{\text{Length}(SE_A \cap B)}{\text{Length}(B)} \end{bmatrix} \quad (5)$$

where each element is the percentage of the length of $B$ falling in the corresponding direction regions.

- If $B$ is a point, formula (3) may be transformed into formula (6):

$$D(A,B) = \begin{bmatrix} P_{NW} & P_N & P_{NE} \\ P_W & P_{\text{same}} & P_E \\ P_{SW} & P_S & P_{SE} \end{bmatrix} \quad (6)$$

where one, but only one, of the nine elements in formula (6) is 100%, and the other eight elements are 0.

Formulae (4), (5) and (6) may also be expressed as:

$$D(A,B) = \{<NW, P_{NW}>, <N, P_N>, <NE, P_{NE}>, <W, P_W>, <\text{Same}, P_{smae}>, <E, P_E>, <SW, P_{SW}>, <S, P_S>, <SE, P_{SE}>\} \quad (7)$$

where $P_{NW}, P_N, \ldots P_{SE}$ are the percentages of the areas of the target object falling in the corresponding direction regions.

If the percentage value corresponding to a direction region is 0, the combination of this direction and its percentage value can be deleted from the expression, which makes formula (7) simpler. For example in Figure 9:

$$D(A,B) = \{< E, 13\% >, < S, 12\% >, < SE, 75\% > \}$$

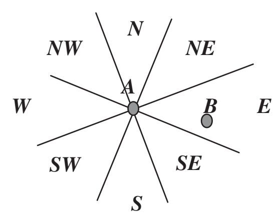
## 4. Experiments and discussions

Judgment of direction relations is rooted in human spatial cognition (Goyal 2000; Bolton & Bass 2009); *hence, the correctness, completeness, quantification and qualification of the new model should be tested by human beings. For this purpose, psychological experiments are designed. Nine pairs of objects that cover all types of object pairs (from* Figure 10 to 18) are used as samples, and 50 undergraduates majoring in geography in Lanzhou Jiaotong University are selected as subjects. The quantitative and qualitative descriptions of direction relations obtained by the new model are attached to each pair of objects. The subjects are required to answer whether they 'agree' or 'do not agree' with each answer.

$P$ in each figure is the percentage of subjects that agree with the direction relations calculated by the new model. The mean $P$ of each experiment is also listed in Table 2. A number of insights can be gained from the experiments.

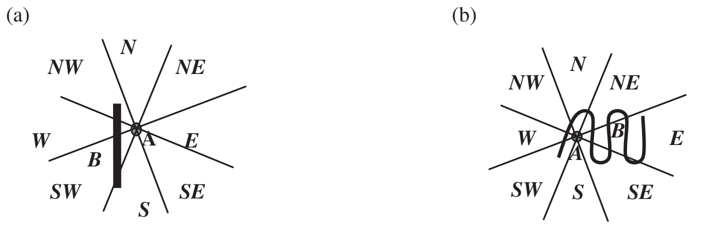
First, the mean value of the percentage of the subjects that agree with the calculation results by means of the new model is 85.6%. *This demonstrates the model produces results that are supported by a large majority of the human test subjects* (i.e. correctness of the model).

Second, the new model can calculate the direction relations between arbitrary object pairs no matter what size differences and topological relations the two objects have (i.e. the completeness of the model), *including*




$D(A, B) = \{<E, 100\%>\}$ ; $P$=96%

**Figure 10.** A point-point pair.




$D(A, B) = \{< S, 5\% >, < SW, 55\% >, < W, 25\% >, < NW, 15\% >\}$ ; $P$=76%

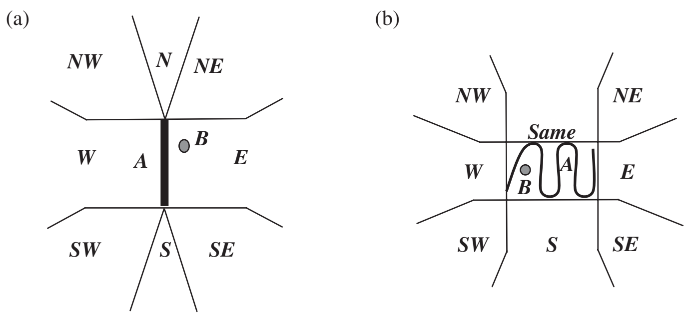
$D(A, B) = \{< N, 7\% >, < NE, 12\% >, < E, 39\% >, < SE, 18\% >, < SW, 8\% >, < W, 8\% >, < NW, 8\% >\}$ ; $P$=72%

**Figure 11.** Point-line pairs: (a) point-line segment; and (b) point-curve.


$D(A, B) = \{< NW, 5\% >, < W, 35\% >, < SW, 60\% >\}$ ; $P$ = 84%

$D(A, B) = \{< N, 6\% >, < NE, 9\% >, < E, 12\% >, < SE, 26\% >, < S, 3\% >, < SW, 29\% < W, 12\% >, < NW, 8\% >\}$ ; $P$=72%

**Figure 12.** Point-polygon pairs: (a) separated point-polygon; and (b) intertwined point-polygon.



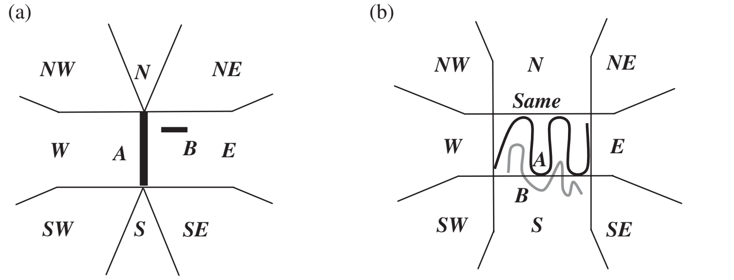

$D(A, B) = \{< E, 100\% >\}$ ; $P$=96%

$D(A, B) = \{< Same, 100\% >\}$ ; $P$=92%

**Figure 13.** Point-line pairs: (a) line segment-point; and (b) curve-point.

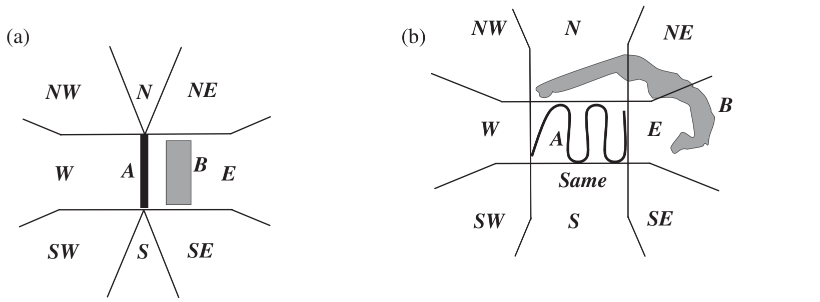
*point-point, point-line, point-polygon, line-point, line-line, line-polygon, polygon-point, polygon-line and polygon-polygon pairs shown in the experiments (from* Figure 10 to 18).

Third, the new model can describe direction relations quantitatively, and it provides a knowledge structure (i.e. the matrix in formula (4) to formula (6)) to record direction relations.



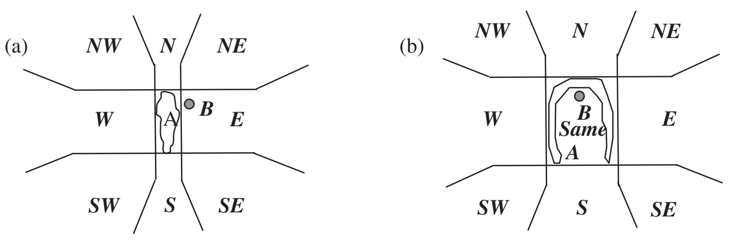

$D(A, B) = \{< E, 100\% >\}$ ; $P$=96%    $D(A, B) = \{< Same, 48 >, < S, 52 >\}$ ; $P$=86%

**Figure 14.** Line-line pairs: (a) straight line-horizontal line; and (b) intertwined curves.




$D(A, B) = \{< E, 100\% >\}$ ; $P$=100%    $D(A, B) = \{< N, 35\% >, < NE, 31 >, < E, 34\% >$ ; $P$=88%

**Figure 15.** Line-polygon pairs: (a) line segment-rectangle; and (b) intertwined curve-polygon.

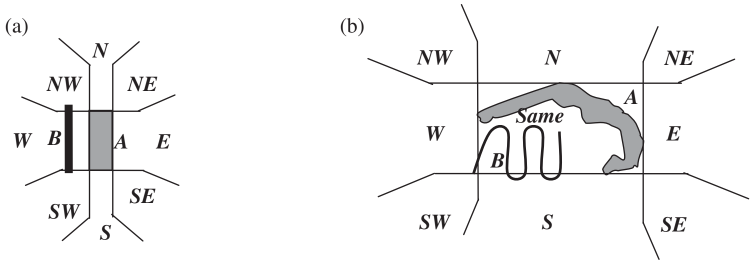



$D(A, B) = \{< E, 100\% >\}$ ; $P$=84%    $D(A, B) = \{< Same, 100\% >\}$ ; $P$=82%

**Figure 16.** Polygon-point pairs: (a) separated polygon-point; and (b) intertwined polygon-point.

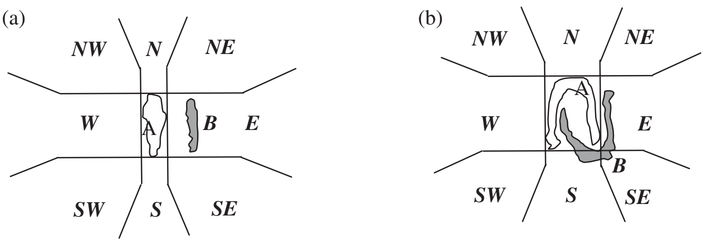
Fourth, the new model can also qualitatively describe direction relations using formula (7).

Finally, previous discussion in this paper has revealed that the new model is more precise in description of direction relations than the cone-based model, because it considers the detail of target objects. This inevitably decreases the efficiency of the new model if it is compared with the cone-based model, because it spends more time in partitioning direction regions and calculating the intersections of the target objects and the direction regions than the cone-based model.




$D(A, B) = \{< SW, 3\% >, < W, 92\% >, < NW, 5\% >\}$ ; $P$=90%    $D(A, B) = \{< Same, 88\% >, < W, 4\% >, < S, 8\% >\}$ ; $P$=74%

**Figure 17.** Polygon-line pairs: (a) rectangle-line segment; and (b) intertwined polygon-curve.




$D(A, B) = \{< E, 100\% >\}$; $P$=100%    $D(A, B) = \{< E, 45\% >, < SE, 6\% >, < S, 14\% >, < Same, 35\% >\}$; $P$=82%

**Figure 18.** Polygon-polygon pairs: (a1) separated polygons; and (b2) intertwined polygons.

**Table 2.** Nine experiments

| Experiments | Reference object | Target object | *P* |
|---|---|---|---|
| Fig.10 | Point | Point | 96% |
| Fig.11 | Point | Line | (76% + 72%)/2 = 74% |
| Fig.12 | Point | Polygon | (84% + 72%)/2 = 78% |
| Fig.13 | Line | Point | (96% + 92%)/2 = 94% |
| Fig.14 | Line | Line | (96% + 86%)/2 = 91% |
| Fig.15 | Line | Polygon | (100% + 88%)/2 = 94% |
| Fig.16 | Polygon | Point | (84% + 82%)/2 = 83% |
| Fig.17 | Polygon | Line | (90% + 74%)/2 = 82% |
| Fig.18 | Polygon | Polygon | (100% + 82%)/2 = 91% |

∗*P* is the percentage of subjects that agree with the direction relations

## 5. Conclusion

This paper proposed a new model for describing direction relations. The new model integrates and improves the direction region partitioning methods used in the cone-based model and the direction relation matrix model so that it can calculate direction relations between arbitrary object pairs in two-dimensional spaces and the results obtained by the new model may be accepted by a majority of people. The results obtained by the new model can be represented qualitatively and quantitatively. A disadvantage of the new model is that it is less efficient than the cone-based model.

Our future study will concentrate on finding a method that may efficiently record the direction relations obtained by the new model in spatial databases and use them in qualitative and quantitative spatial reasoning.

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