

Chapter 6

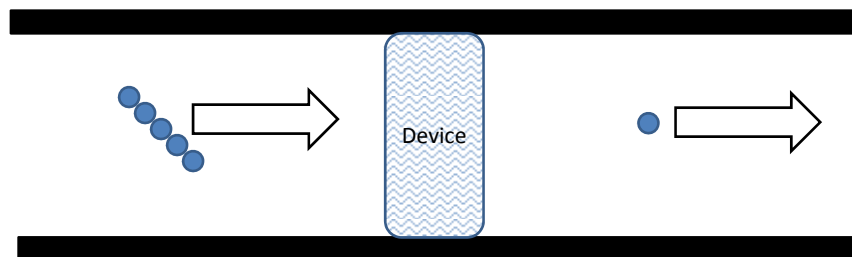
Separation of Particles from Gas

- Separation efficiency
- Separation mechanisms
 - *Gravitational settling*
 - *Electrostatic precipitation*
 - *Centrifugal separation*
 - *Filtration*

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Separation Efficiency

- Consider the simplest case
- 5 particles, same size, going through a filter, 1 made it.



- **Penetration** efficiency = $1/5 = 20\%$
- **Collection** efficiency = $4/5 = 80\%$

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Monodisperse and Polydisperse

- **Monodisperse:** characterized by particles of uniform size in a dispersed phase
- **Polydisperse:** characterized as particles of varied sizes in the dispersed phase of a disperse system



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Monodisperse Particles

- N_i particles, same size d_p , going through a air cleaner,
- N_o made it.



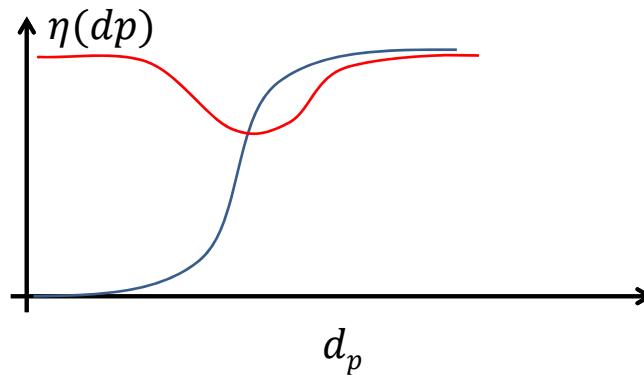
- Penetration efficiency $P(d_p) = \frac{N_o}{N_i}$
- Separation efficiency $\eta(d_p) = 1 - P(d_p) = \left(1 - \frac{N_o}{N_i}\right)_{d_p}$

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Polydisperse

☐ Size dependent

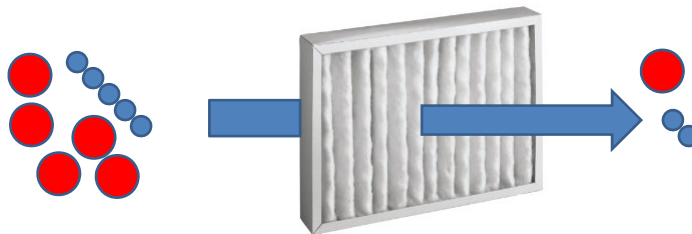
- Efficiency for each size may be different
- Fractional efficiency or grade efficiency



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Total number efficiency (polydisperse)

- In: 10 particles (5 of the 10 um, the others 1 um)
- Out: 1 of them 10-um, 2 of them 1-um



- Total number penetration efficiency = $3/10$
- Total number collection efficiency = $7/10$

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In general

Total number efficiency

$$\eta = \frac{\sum_i \eta(d_p) N_i(d_p)}{\sum_i N_i(d_p)} \quad \eta = \frac{\int_0^{\infty} \eta(d_p) N_i(d_p) d(d_p)}{\int_0^{\infty} N_i(d_p) d(d_p)}$$

Total mass efficiency from fractional efficiency

$$\eta = \frac{\int_0^{\infty} m_{po}(d_p) d(d_p)}{\int_0^{\infty} m_{pi}(d_p) d(d_p)} = \frac{\int_0^{\infty} \eta(d_p) m_{pi}(d_p) d(d_p)}{\int_0^{\infty} m_{pi}(d_p) d(d_p)}$$

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Equipment for particle counting



Scanning mobility particle sizer
(SMPS) x-1000 nm



Aerodynamic Particle Sizer
(APS 3321), 0.35-20 μm



Example (Start with a simple case)

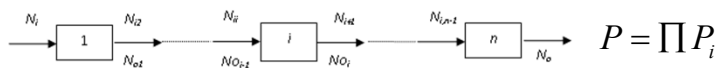
- A particle counter is used to measure the particle number concentration up- and down-stream of a particulate air cleaning device as a function of aerodynamic diameter.
- The up- and down-stream particle number concentrations are

$d_p(\mu\text{m})$	Up- N_i	Down- N_o
1	1000	10
5	100	6
10	5	0
20	1	0

- Calculated the fractional efficiency, and
- the total mass efficiency of this air cleaning device.

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Particle Separation Efficiency of Multiple Devices



$$\eta(d_p) = 1 - P = 1 - \prod P_i = 1 - \prod [1 - \eta_i(d_p)]$$

- **Critical assumption:** particle separation efficiencies of the identical units are the same.
- This is actually not justified in many engineering applications because the particle separation of a unit depends on the incoming particle concentration

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Example

- Two identical filters, filtration efficiency is each 85%, what is the total efficiency if two of them working in serial

- Solution

$$P_1 = P_2 = 0.15$$

$$P = 0.0225$$

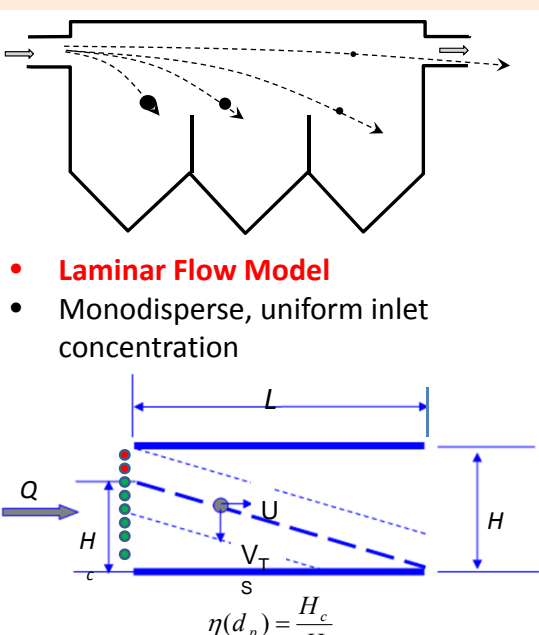
$$\eta = 0.9775 = 97.75\%$$

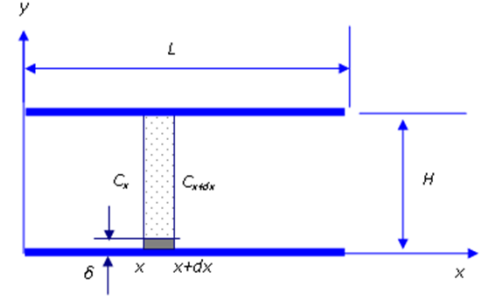
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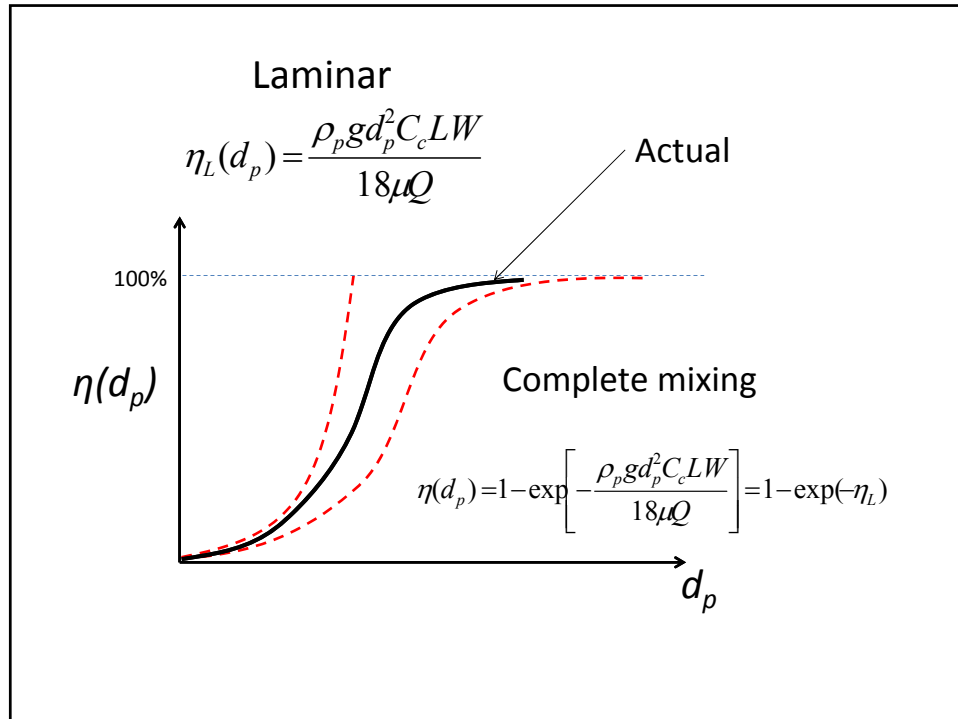
Major Particulate Separation Technologies

- Gravitational settling
- Electrostatic precipitation
- Cyclone
- Filtration

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Gravity Settling Chamber	
 <ul style="list-style-type: none"> Laminar Flow Model Monodisperse, uniform inlet concentration 	$\eta(d_p) = \frac{H_c}{H}$
	$t = \frac{L}{U} = \frac{H_c}{V_{TS}}$
	$H_c = \frac{V_{TS}L}{U}$
	$\eta(d_p) = \frac{V_{TS}L}{UH}$
	$U = \frac{Q}{WH}$
	$V_{TS} = \frac{\rho_p d_p^2 g C_c}{18\mu}$
$\eta(d_p) = \frac{\rho_p g d_p^2 C_c LW}{18\mu Q}$	

Complete Mixing Model	
 <ul style="list-style-type: none"> Particles and air are completely mixed at any cross section perpendicular to the direction of airflow. In dx, the concentration of particles within it is assumed to be uniform and the particles settle down at V_{TS}. During dt the particles at the bottom of the chamber within a distance $\delta = V_{TS} dt$ above the lower collecting plate are considered collected 	$C_x U W H = C V_{TS} W (dx) + C_{x+dx} U W H$
	$\frac{dC}{C} = -\frac{V_{TS}}{H} dt$
	$\frac{dC}{C} = -\frac{V_{TS}}{HU} dx$
	$\int_{C_i}^{C_o} \frac{dC}{C} = -\int_0^L \frac{V_{TS}}{HU} dx$
	$P = \exp\left(-\frac{V_{TS}L}{HU}\right) = \exp\left(-\frac{V_{TS}LHW}{HQ}\right)$
	$\eta = 1 - \exp\left(-\frac{V_{TS}LW}{Q}\right)$



Example

- Consider a gravity settling chamber that is 1-m wide ($W=1$ m) and 1-m high ($H=1$ m).
- Air flow rate is $1 \text{ m}^3/\text{s}$ ($=3600 \text{ m}^3/\text{hr}$)
- Under standard ambient condition and laminar flow assumption,
- Estimate its separation efficiency vs. aerodynamic diameter

$$\eta_L(d_p) = \frac{\rho_p g d_p^2 C_c L W}{18 \mu Q}$$

$$C_c \approx 1$$

$$\begin{aligned} \eta_L(d_p) &= \frac{\rho_p g d_p^2 L W}{18 \mu Q} \\ &= \frac{(1000)(9.81)(1)(1)}{(18)(1.81 \times 10^{-5})(1)} d_p^2 \\ &= 3.01 \times 10^7 d_p^2 \end{aligned}$$

$d_p(\mu\text{m})$	$\eta(d_p)$
10	0.03%
100	3%
150	7%
200	12%
250	19%
350	37%
500	75%
575	100%

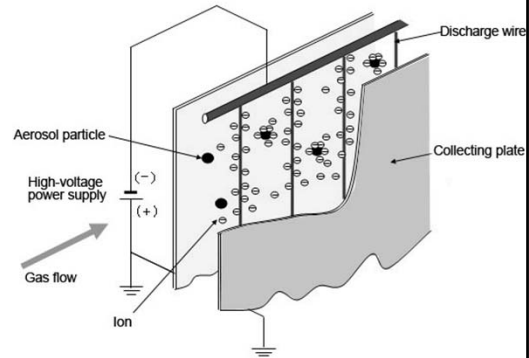
- The stringent control requirements adopted in the late 1960s through early 1970s have resulted in a sharp decline in the use of Gravity settling chamber.

- There are very few gravity settling chambers still in commercial use.

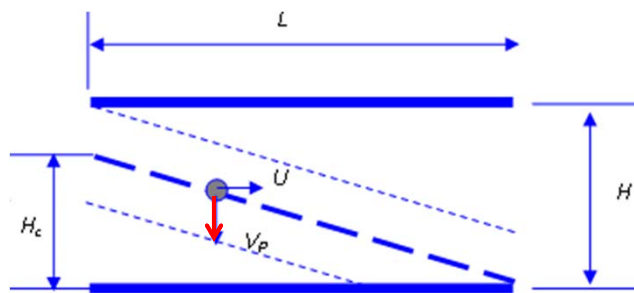
- ☐ But it happens everywhere in air duct



Electrostatic Precipitation



Model analysis by similar approach to gravitational settling



“Settling” in electrostatic force field

$\eta(d_p) = \frac{V_p L}{UH} = \frac{V_p L b}{UHb} = \frac{V_p A}{Q} \quad (\text{Laminar})$
$\eta(d_p) = 1 - \exp\left(-\frac{V_p A}{Q}\right) \quad (\text{Turbulent})$

Terminal precipitation velocity

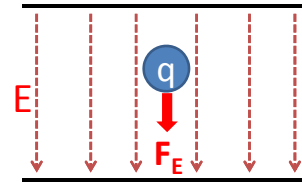
Electrical force = Drag force

$$F_E = F_D$$

$$qE = \frac{3\pi V_p d_p \mu}{C_c}$$

$$V_p = \frac{qEC_c}{3\pi d_p \mu}$$

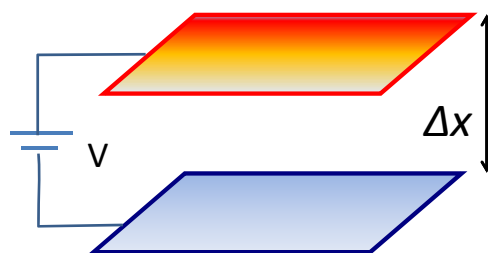
If you know q and E !



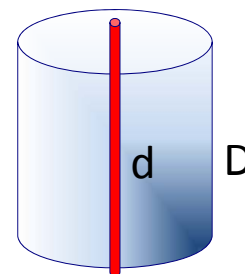
- q is the charge carried by the particles
- E is the electric field strength

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Electrical Field Intensity, E



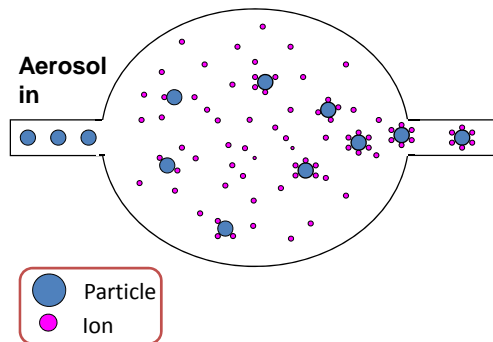
$$E = \frac{V}{\Delta x}$$



$$E(r) = \frac{V}{r \ln(D/d)}$$

Particle Charging

- Particles passing through a space filled with ions will be charged



- **Field charging** – ion attached to particles in an electrical field driven by the electrical force
- **Diffusion charging** – ions attached to particles due to Brownian motion of ions (not the electrical field).

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1. Diffusion Charging

$$n = \frac{d_p kT}{2K_E e^2} \ln \left[1 + \left(\frac{N_0 c_i \pi d_p K_E e^2}{2kT} \right) t \right]$$

Under standard conditions,

- $K_E = 9 \times 10^9 \text{ Nm}^2/\text{C}$,
- $c_i = 240 \text{ m/s}$,
- $k = 1.38 \times 10^{-23} \text{ J/K}$,
- $e = 1.6 \times 10^{-19} \text{ C}$.
- A typical concentration of ions is $N_0 = 5 \times 10^{14} \text{ ion/m}^3$,
- The diameters of most airborne particles are smaller than $10 \mu\text{m}$ and time t is in the order of unit or less

2. Field charging

$$n = \frac{E d_p^2}{4K_E e} \left(\frac{3\varepsilon_r}{\varepsilon_r + 2} \right) \left(\frac{t}{t + \tau} \right)$$

- E = intensity of the electric field with a typical value of 10^6 V/m ,
- $\varepsilon_r = \varepsilon/\varepsilon_0 =$ **relative permittivity** or **dielectric constant** of the particle with respect to a vacuum,
 - $\varepsilon_0 = 8.854 \times 10^{-12} \text{ C/V.m}$, is the permittivity of a vacuum.
 - The permittivity of typical particles can be found in handbooks.

Field Charging

$$n = \frac{Ed_p^2}{4K_E e} \left(\frac{3\varepsilon_r}{\varepsilon_r + 2} \right) \left(\frac{t}{t + \tau} \right)$$

The number of ions that is eventually charged to a particle depends on three factors:

- time t ,
- the concentration of ions in the charging zone N_0 , and
- the electric mobility of these ions B_e , which determines the moving speed of the ions in response to the electric field E .

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Saturation Field Charge

$$n = \frac{Ed_p^2}{4K_E e} \left(\frac{3\varepsilon_r}{\varepsilon_r + 2} \right) \left(\frac{t}{t + \tau} \right)$$

- $t \sim \infty$, $\frac{t}{t + \tau} \sim 1$
- τ is the charging constant and it varies with the field condition. Typically = 0.003 s

$$n_s = \frac{Ed_p^2}{4K_E e} \left(\frac{3\varepsilon_r}{\varepsilon_r + 2} \right)$$

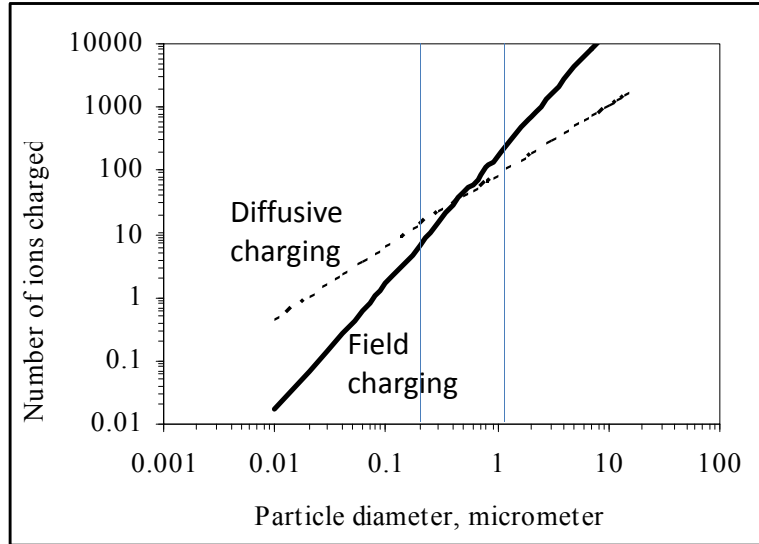
$$\tau = \frac{1}{\pi N_0 K_E e B_e}$$

- $K_E = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$, is a force constant.
- the concentration of ions in the charging zone N_0 , and
- the electric mobility of these ions B_e , which determines the moving speed of the ions in response to the electric field E .

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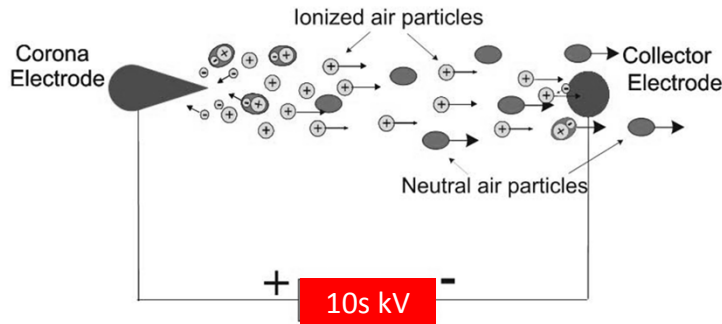
Combined charging

$$n = \frac{d_p kT}{2K_E e^2} \ln \left[1 + \left(\frac{N_0 c_i \pi d_p K_E e^2}{2kT} \right) t \right] + \frac{E d_p^2}{4K_E e} \left(\frac{3\epsilon_r}{\epsilon_r + 2} \right) \left(\frac{t}{t + \tau} \right)$$



How to produce sufficient ions?

- **Corona discharge** is an electrical discharge brought on by the ionization of a fluid surrounding a conductor that is electrically energized



Summary of Equations for Parallel Plate ESP

$$\eta(d_p) = \frac{V_P A}{Q} \quad \text{Laminar}$$

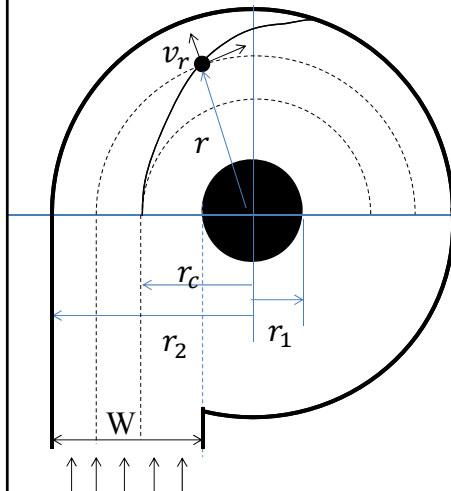
$$\eta(d_p) = 1 - \exp\left[-\frac{V_P A}{Q}\right] \quad \text{Complete mixing}$$

$$V_P = \frac{qEC_c}{3\pi d_p \mu} = \frac{neEC_c}{3\pi d_p \mu}$$

$$n = \frac{d_p kT}{2K_E e^2} \ln \left[1 + \left(\frac{N_0 c_i \pi d_p K_E e^2}{2kT} \right) t \right] + \frac{Ed_p^2}{4K_E e} \left(\frac{3\varepsilon_r}{\varepsilon_r + 2} \right) \left(\frac{t}{t + \tau} \right)$$

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“Centrifugal force”



$$\bullet \eta(d_p) = \frac{r_2 - r_c}{r_2 - r_1} = \frac{r_2 - r_c}{W}$$

$$\bullet \bar{u}_g = \frac{Q}{WH}$$

$$\bullet m_p \frac{dv_r}{dt} = F_C - F_D = 0$$

$$\bullet F_C = m_p \frac{v_\theta^2}{r}$$

$$\bullet F_D = 8\pi\mu d_p v_r$$

$$v_r = \frac{\rho_g d_p^2 v_\theta^2}{18\mu r}$$

Crawford model (1976)*

$$v_r = \frac{\rho_p d_p^2 v_\theta^2}{18\mu r}$$

$$v_\theta = u_\theta = \frac{Q}{H \ln(r_2/r_1)} \frac{1}{r}$$

$$v_r = \frac{\rho_p d_p^2}{18\mu} \left[\frac{Q}{H \ln(r_2/r_1)} \right]^2 \frac{1}{r^3}$$

$$rd\theta = v_\theta dt$$

$$dr = v_r dt \quad \frac{dr}{rd\theta} = \frac{v_r}{v_\theta}$$

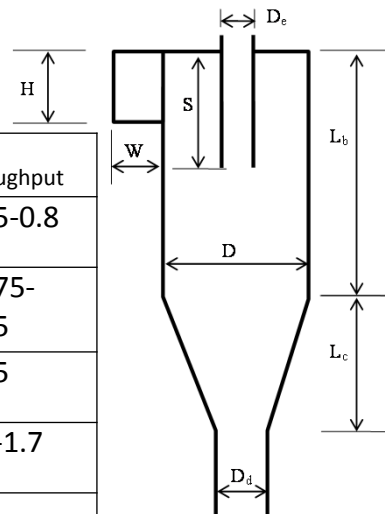
$$r^2 - r_c^2 = \left[\frac{\rho_p d_p^2 Q}{9\mu H \ln(r_2/r_1)} \right] \theta$$

$$\eta(d_p) = \frac{r_2 - \left[r_2^2 - \frac{\rho_p d_p^2 Q \theta_2}{9\mu H \ln(r_2/r_1)} \right]^{1/2}}{r_2 - r_1}$$

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Lapple (1951) model

	High efficiency	Conventional	High throughput
Height of inlet H/D	0.5-0.44	0.5	0.75-0.8
Width of inlet W/D	0.2-.021	0.25	0.375-0.35
Diameter of gas exit De/D	0.4-0.5	0.5	0.75
Length of vortex finder S/D	0.5	0.625-0.6	1.5-1.7
Length of body, L _b /D	1.4	1.75	1.7
Cone length, L _c /D	2.5	2	2.5-2
Diameter of dust outlet	0.375-0.4	0.25-0.4	0.375-0.4



$$\eta(d_p) = \frac{r_2 - r_c}{W} = \frac{\rho_g d_p^2 v_\theta^2}{18\mu r W} t$$

Lapple model

$$v_r = \frac{r_2 - r_c}{t} \quad t = \frac{2\pi r N_e}{\bar{u}_g}$$

$$v_\theta \approx u_\theta \approx \bar{u}_g$$

$$\eta(d_p) = \frac{\rho_g d_p^2 v_\theta^2}{18\mu r W} \frac{2\pi r N_e}{\bar{u}_g} \xrightarrow{N_e = \frac{L_B + 0.5L_C}{H}} \eta(d_p) = \left(\frac{\pi \bar{u}_g N_e \rho_p}{9\mu W} \right) d_p^2$$

- Lapple gave an efficiency formula for traditional reverse flow cyclones in terms of cut size,

$$d_{50} = \left[\frac{4.5\mu W}{\pi N_e \bar{u}_g \rho_p} \right]^{1/2} \quad \eta(d_p) = \frac{1}{1 + (d_{50}/d_p)^2}$$

Example 5.3: Cyclone efficiency

- A conventional cyclone has a body diameter of 20 cm and other geometries are listed in the table as follows. It operates at an inlet volumetric flow rate of 360 m³/hr. Assume standard condition, plot its fractional efficiency curve vs. particle aerodynamic diameter.

Height of inlet H/D	0.5
Width of inlet W/D	0.25
Length of body, L _B /D	1.75
Cone length, L _C /D	2

Solution

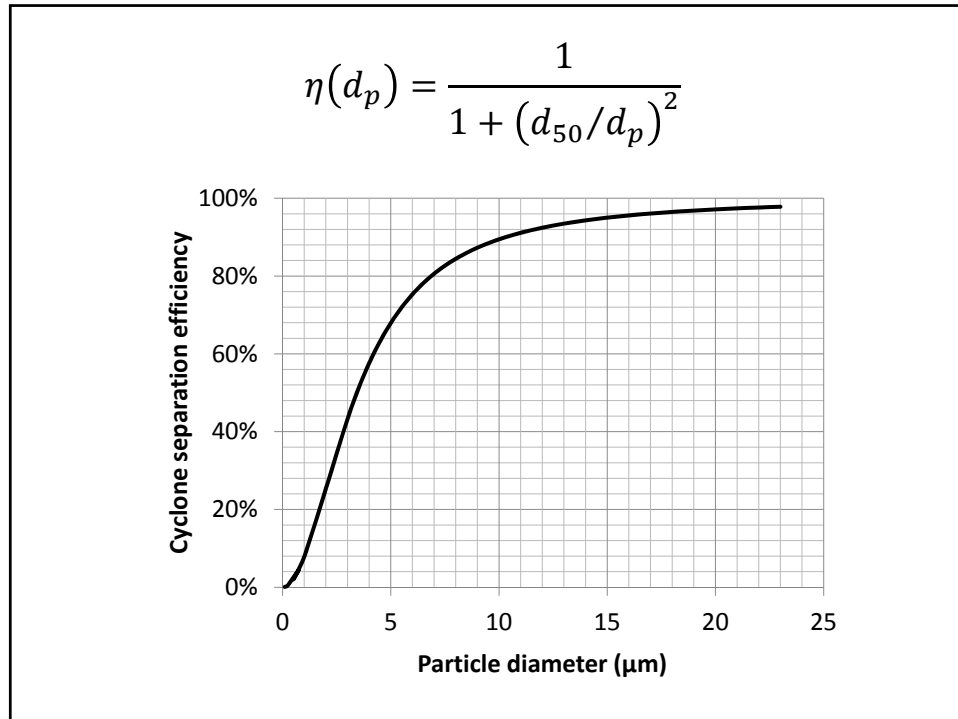
	Dimension
Body diameter D (m)	0.2
Height of inlet H	0.1
Width of inlet W	0.05
Diameter of gas exit D _e	0.1
Length of body, L _B	0.35
Cone length, L _C	0.4

$$N_e = \frac{L_B + 0.5L_C}{H} = \frac{0.35 + 0.5 \times 0.4}{0.1} = 5.5$$

$$A = HW = 0.005 \text{ (m}^2\text{)}$$

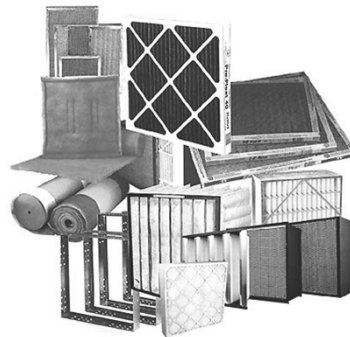
$$\bar{u}_g = \frac{Q}{A} = \frac{360 \frac{\text{m}^3}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}}}{0.005 \text{ m}^2} = 20 \text{ m/s}$$

$$d_{50} = \left[\frac{9\mu W}{2\pi N_e \bar{u}_g (\rho_p - \rho_g)} \right]^{1/2} = \left[\frac{9 \times 1.81 \times 10^{-5} \times 0.05}{2\pi \times 5.5 \times 20 (1000 - 1.21)} \right]^{1/2} = 3.43 \times 10^{-6} \text{ m}$$



Filtration

- Granular filter
- Fibrous filter
- Filter Porosity
- Filter Solidity
- Pressure drop
- Collection efficiencies



Filtration fundamentals

- Two steps:
 1. Particle transport to a collector, quantified by collision efficiency
 2. Attach to a collector, quantified by adhesion efficiency
- In the past, research has been focused on transport efficiency by assuming “always adhere”

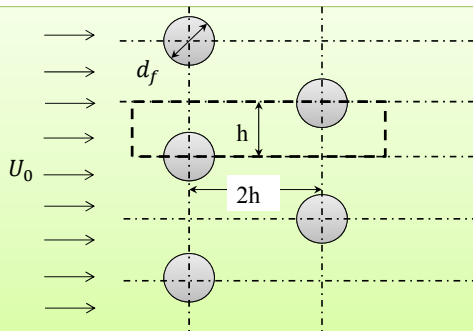
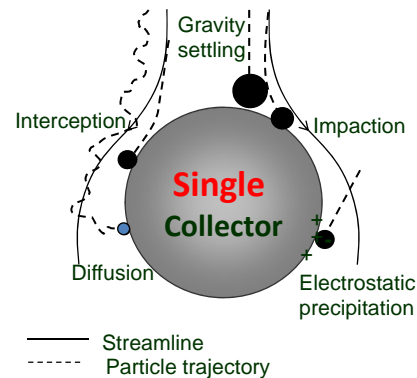
$$\eta = \eta_{ts} \cdot \eta_{ad} \quad \eta_{ad} \equiv 1$$

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Collector surface

Five Transport Mechanisms of A Single Collector

- Adhesion efficiency = 1
- Ignored gravitational Settling
- Electrostatic precipitation not considered



Staggered-array models

- For efficiency per unit length for each mechanism

Single Fiber Efficiency per unit length of fiber by Interception

$$\eta_{it} = \frac{1+R}{2Y} \left[2 \ln(1+R) - (1-\alpha) + \left(1 - \frac{\alpha}{2}\right)(1+R)^{-2} - \frac{\alpha}{2}(1+R)^2 \right]$$

- $\alpha = (1 - \text{porosity})$, is the solidity of filter;
- $R = d_p/d_f$, is the ratio of particle diameter to fiber diameter, and
- Y is the Kuwabara hydrodynamic factor defined below, with slip effect taken into consideration

$$Y = -\frac{\ln \alpha}{2} - \frac{3}{4} + \alpha - \frac{\alpha^2}{4}$$

Single Fiber Efficiency Per Unit Length of Fiber by Inertial Impaction

$$\eta_{ip} = \frac{J}{4Y^2} Stk \quad \leftarrow \quad Stk = \frac{\rho_p d_p^2 U_0 C_c}{18 \mu d_f}$$

$$Y = -\frac{\ln \alpha}{2} - \frac{3}{4} + \alpha - \frac{\alpha^2}{4}$$

$$J = (29.6 - 28\alpha^{0.62})R^2 - 27.5R^{2.8} \quad \text{for } R < 0.4$$

$$J = 2, \text{ for } R \geq 0.4$$

$$R = d_p/d_f$$

Single Fiber Efficiency Per Unit Length of Fiber by Diffusion

$$\eta_D = \frac{3.65(Pe_m)^{-\frac{2}{3}} + 0.624(Pe_m)^{-1}}{2Y}$$

$$D_p = \frac{kTC_c}{3\pi\mu d_p} \quad D_p: \text{Particle diffusion coefficient}$$

$$Pe = \frac{U_0 d_f}{D_p} \quad Pe: \text{Peclet number}$$

$$Pe_m = \frac{Pe}{2Y}$$

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Example Filtration efficiency

A furnace filter is made of fiberglass with a solidity of 3%, it is 2-cm thick. The average diameter of the fiber is 5 μm . Face velocity is 5 cm /s. Estimate the following efficiencies as a function of particle aerodynamic diameter under standard conditions

- a) Single fiber efficiency by interception only*
- b) Single fiber efficiency by impaction only*
- c) Single fiber efficiency by diffusion only*

Solution:

In this equation, the following parameters are considered as constant

$$d_f = 5 \mu\text{m}, \alpha = 0.03, Kn_f = \frac{2\lambda}{d_f} = 0.0264$$

$$Y = \frac{\ln \alpha}{2} - \frac{3}{4} + \alpha - \frac{\alpha^2}{4} = 1.033054, \mu = 1.81 \times 10^{-5} \text{ Pa}\cdot\text{s}, U_0 = 0.05 \text{ m/s}$$

The following variables can be calculated in an Excel sheet for different particle

$$\text{diameters } R = \frac{d_p}{d_f}, Stk_f = \frac{\rho_p d_p^2 C_c U_0}{18 \mu d_f}, Stk_m = \frac{Stk}{2Y}$$

$$I = \begin{cases} (29.6 - 28\alpha^{0.62})R^2 - 27.5R^{2.8} & \text{for } R < 0.4 \\ 2 & \text{for } R \geq 0.4 \end{cases}$$

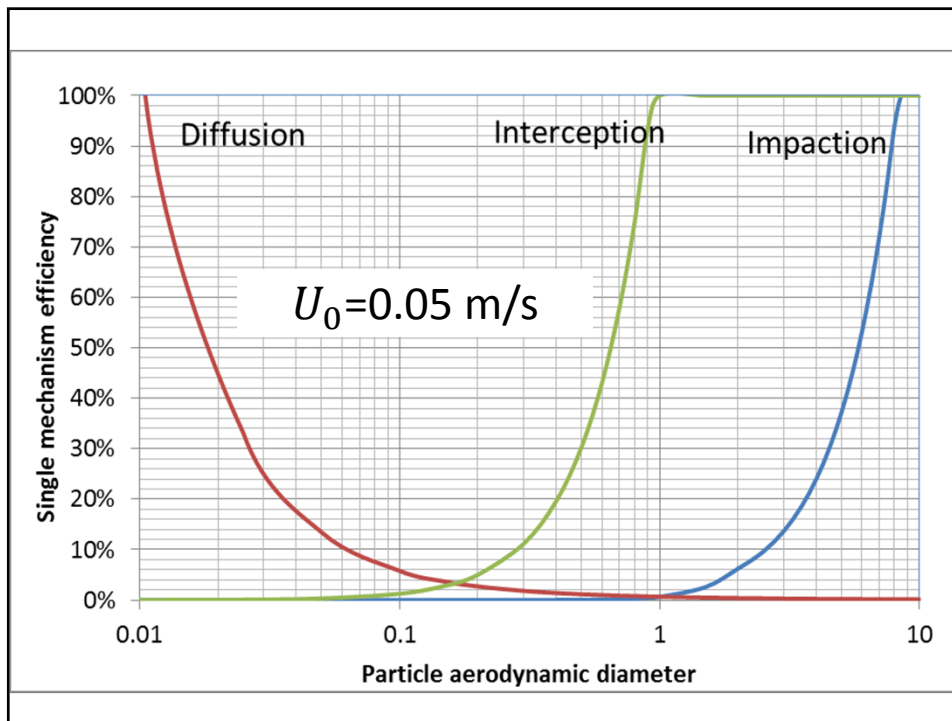
$$D_p = \frac{kTC_c}{3\pi\mu d_p}, Pe = \frac{U_0 d_f}{D_p}, Pe_m = \frac{Pe}{2Y}$$

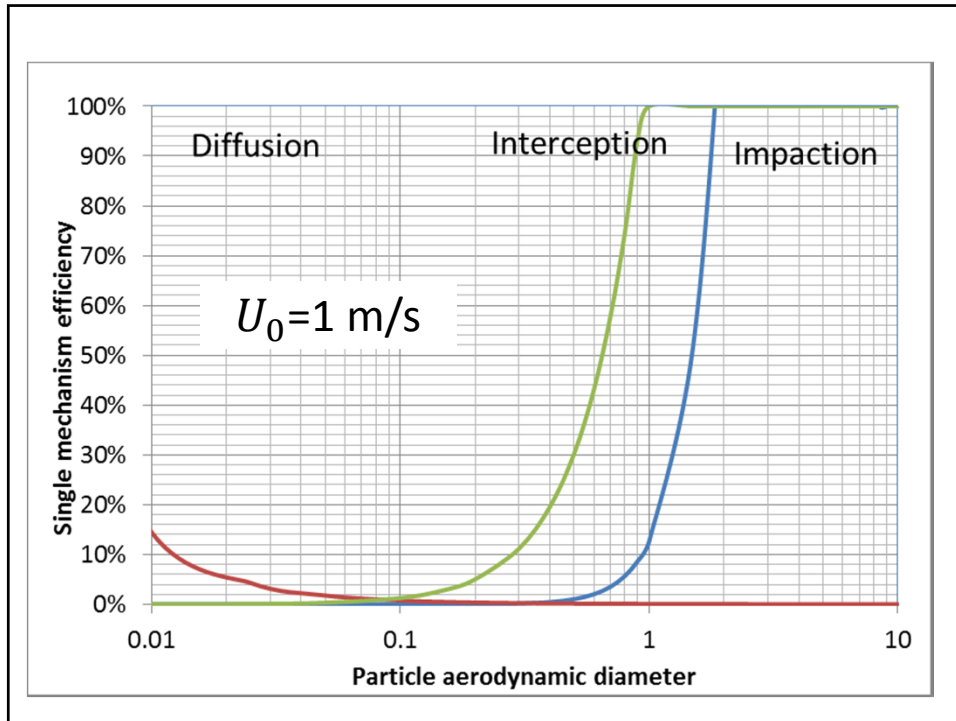
The the single fiber filtration efficiency by inertial interception, impaction and diffusion, per unit length of fiber is calculated using

$$\eta_{it} = \frac{1+R}{2Y} \left[2 \ln(1+R) - (1-\alpha) + \left(1 - \frac{\alpha}{2}\right)(1+R)^{-2} - \frac{\alpha}{2}(1+R)^2 \right]$$

$$\eta_{ip} = \frac{I}{2Y} Stk_m$$

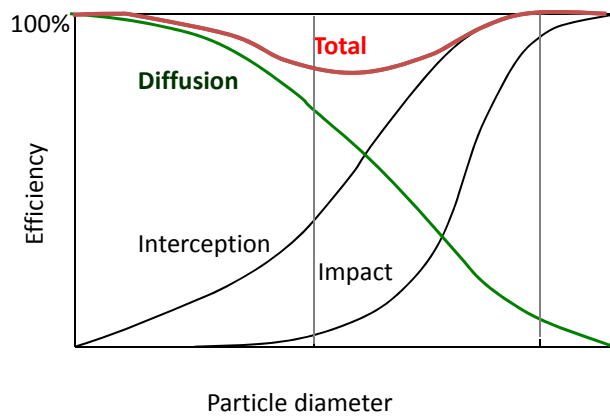
$$\eta_D = \frac{3.65(Pe_m)^{\frac{2}{3}} + 0.624(Pe_m)^{-1}}{2Y}$$



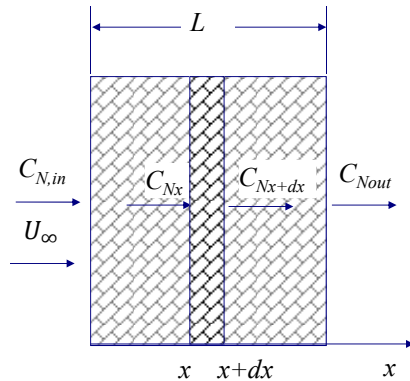


Total Single Fiber Efficiency Per Unit Length of Fiber

$$\eta_{sf} = 1 - (1 - \eta_{it})(1 - \eta_{ip})(1 - \eta_D)$$



Correlation between total single fiber efficiency and overall filtration efficiency



- Within an elemental thickness of dx , the **solidity**, α , from its definition is

$$\alpha = \frac{(\pi d_f^2/4) \cdot ds_f}{A_c \cdot dx}$$

- A_c is the bulk cross section area of the filter that is normal to the face velocity. It can be determined by the air flow rate

$$A_c = \frac{Q}{U_\infty}$$

- U_∞ is the bulk face speed, or the air speed approaching the filter. It is less than that approaching the fiber within the filter, U_0 , because of the existence of solid fibers

$$U_0 = \frac{U_\infty}{1 - \alpha} = \frac{Q}{(1 - \alpha)A_c}$$

- The number concentration of particles lost per unit volume from the bulk air over the distance dx equals to that captured by the fiber with a single fiber efficiency η_{sf} corresponding to an approaching flow rate of $U_0 d_f \cdot ds_f$, where $d_f \cdot ds_f$ defines the cross section area of the fiber with the length of ds_f and the diameter of d_f

$$Q \cdot dC_N(x) = -\eta_{sf} C_N(x) U_0 d_f \cdot ds_f$$

- all these particles passed through a single fiber with a cross section area of $(d_f \cdot ds_f)$ with an approaching speed of U_0 and a single fiber efficiency of η_{sf}

$$A_c U_\infty \cdot dC_N(x) = -C_N(x) \eta_{sf} U_0 d_f \cdot \frac{4\alpha A_c \cdot dx}{\pi d_f^2}$$

$$\frac{dC_N(x)}{C_N(x)} = - \left(\frac{U_0 \eta_{sf} 4\alpha}{U_\infty \pi d_f} \right) dx$$

Overall Filtration Efficiency of a Filter

$$\frac{dC_N(x)}{C_N(x)} = - \left[\frac{\eta_{sf} 4\alpha}{(1-\alpha)\pi d_f} \right] dx$$

$$\int_{C_{Ni}}^{C_{No}} \frac{dC_N(x)}{C_N(x)} = \int_0^L - \left[\frac{\eta_{sf} 4\alpha}{(1-\alpha)\pi d_f} \right] dx$$

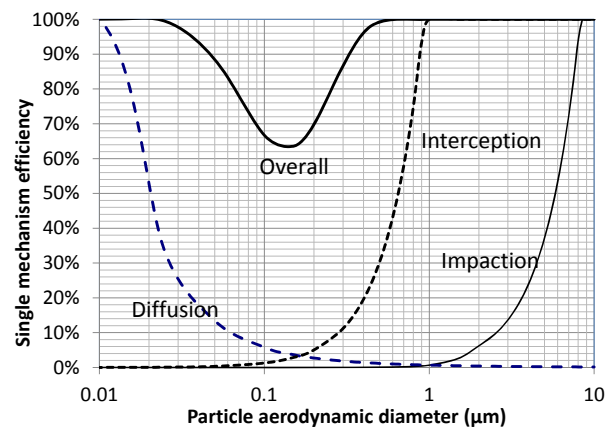
$$P(d_p) = \frac{C_{No}}{C_{Ni}} = \exp \left[\frac{-\eta_{sf} 4\alpha L}{(1-\alpha)\pi d_f} \right]$$

$$\eta(d_p) = 1 - \exp \left[\frac{-\eta_{sf} 4\alpha L}{(1-\alpha)\pi d_f} \right]$$

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Example 5.5: Filtration total efficiency

A filter is made of fiberglass with a solidity of 3%, and it is 2-mm thick. The average diameter of the fiber is 5 μm . When the face velocity is 0.05 m/s, estimate its overall fractional filtration efficiency as a function of particle aerodynamic diameter under standard conditions (consider only interception, impaction, and diffusion).

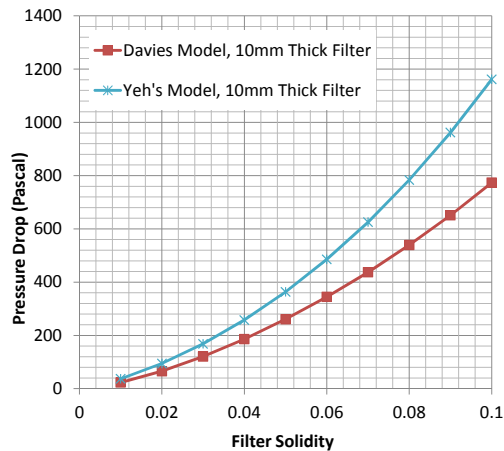


Fiber Filter Pressure Drop

$$\Delta P = C_{\Delta P} \left(4\mu U_0 L / d_f^2 \right)$$

$$C_{\Delta P} = 16\alpha^{1.5} (1 + 56\alpha^3)$$

$$\eta(d_p) = 1 - \exp \left[\frac{-\eta_{sf} 4\alpha L}{(1 - \alpha)\pi d_f} \right]$$



- Both efficiency and pressure drop increase with solidity
- **Particle accumulation** increase the filter solidity
 - it increases efficiency slightly, but increases pressure drop greatly

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