











Table 13-1. Proposed Occupational Exposure Limits of Nanoaerosols					
Nanoaerosol	Occupational exposure limit	Parameters			
Titanium dioxide	0.1 mg/m <sup>3</sup>	0.1 risk level particles \100 nm			
General dust	3 mg/m <sup>3</sup>				
Photocopier toner	0.6 mg/m <sup>3</sup>	Tolerable risk			
	0.06 mg/m <sup>3</sup>	2009 acceptable risk			
	0.006 mg/m <sup>3</sup>	2018 acceptable risk			
Biopersistent granular materials (e.g. metal oxides)	20,000 particles/cm <sup>3</sup>	Density>6,000 kg/m3			
	40,000 particles/cm <sup>3</sup>	Density<6,000 kg/m3			
Carbon Nanotubes (CNTs)	0.01 f/cm <sup>3</sup>	Exposure risk ratio for asbestos			
Fibrous	0.01 f/cm <sup>3</sup>	3:1; length 75,000 nm			
Multi-walled CNTs	0.0025 mg/m <sup>3</sup>	Nanocyl product only			
		/			





- An important assumption of Stokes' law is that there is no slipping between the gas and the rigid particles.
- However, when the particle is getting smaller and smaller, approaching the mean free path of the gas molecules, this assumption is no longer valid.
- The Knudsen Number

$$Kn = \lambda / \left(\frac{d_p}{2}\right) = 2\lambda / d_p$$

 $\lambda$  = mean free path of the gas







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## Single Fiber Efficiency per unit length of fiber by Interception

$$\eta_{it} = 0.6 \, \left(\frac{1-\alpha}{Y}\right) \frac{R^2}{1+R} \left(1 + \frac{1.996 K n_f}{R}\right)$$

- $\alpha = (1 porosity)$ , is the solidity of filter;
- $R = d_p/d_f$  is the ratio of particle diameter to fiber diameter, and
- *Y* is the Kuwabara hydrodynamic factor defined below, with slip effect taken into consideration

$$Y = -\frac{\ln\alpha}{2} - \frac{3}{4} + \alpha - \frac{\alpha^2}{4}$$

## Example 13.1: Nanoaerosol transport efficiency

A filter is made of fiberglass with a solidity of 5%, and it is 5-cm thick. The average diameter of the fibers is 5  $\mu$ m. When the face speed is 0.15 m/s, calculate and plot the fractional transport efficiency per unit length of fiber as a function of particle aerodynamic diameter in the range of 1- 100 nm under standard conditions, by interception and diffusion, respectively.

## Example 13.1: Solution

*In this problem, the following parameters are considered as constant* 

$$d_f = 5 \ \mu m, \qquad \alpha = 0.05, \qquad Kn_f = \frac{2\lambda}{d_f} = 0.0264$$
$$Y = \frac{\ln \alpha}{2} - \frac{3}{4} + \alpha - \frac{\alpha^2}{4} = 1.033$$
$$\mu = 1.81 \times 10^{-5} \ Pa.s \qquad U_0 = 0.05 \ m/s$$

*The following variables can be calculated in an Excel sheet for different particle diameters* 

$$C_c = 1 + Kn_p \left[ 1.142 + 0.558exp\left(-\frac{0.999}{Kn_p}\right) \right]$$
$$R = \frac{d_p}{d_f} \qquad D_p = \frac{kTC_c}{3\pi\mu d_p} \qquad Pe = \frac{U_0 d_f}{D_p}$$

The single fiber filtration efficiency by inertial interception, impaction and diffusion, per unit length of fiber is calculated using

$$\eta_D = 0.84 P e^{-0.43}$$
  
$$\eta_{it} = 0.6 \, \left(\frac{1-\alpha}{Y}\right) \frac{R^2}{1+R} \left(1 + \frac{1.996 K n_f}{R}\right)$$





• The number concentration of particles lost per unit volume from the bulk air over the distance dx equals to that captured by the fiber with a single fiber efficiency  $\eta_{sf}$  corresponding to an approaching flow rate of  $U_0 d_f \cdot ds_f$ , where  $d_f \cdot ds_f$ defines the cross section area of the fiber with the length of  $ds_f$  and the diameter of  $d_f$ 

$$Q \cdot dC_N(x) = -\eta_{sf}C_N(x)U_0d_f \cdot ds_f$$

• all these particles passed through a single fiber with a cross section area of  $(d_f \cdot ds_f)$  with an approaching speed of  $U_0$  and a single fiber efficiency of  $\eta_{sf}$ 

$$A_{c}U_{\infty} \cdot dC_{N}(x) = -C_{N}(x)\eta_{sf}U_{0}d_{f} \cdot \frac{4\alpha A_{c} \cdot dx}{\pi d_{f}^{2}}$$
$$\frac{dC_{N}(x)}{C_{N}(x)} = -\left(\frac{U_{0}}{U_{\infty}}\frac{\eta_{sf}4\alpha}{\pi d_{f}}\right)dx$$
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$$\begin{aligned} \text{Adhesion Efficiency Analysis} \\ \eta_{ad} &= \frac{\int_{0}^{v_{cr}} f(v_{im}) dv_{im}}{\int_{0}^{\infty} f(v_{im}) dv_{im}} = \frac{\int_{0}^{v_{cr}} v_{im}^{2} \exp\left(-\frac{mv_{im}^{2}}{2KT}\right) dv_{im}}{\int_{0}^{\infty} v_{im}^{2} \exp\left(-\frac{mv_{im}^{2}}{2KT}\right) dv_{im}} \end{aligned}$$

$$\cdot \int [x^{2} \exp(-ax^{2})] dx &= \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{ax})}{4a^{1.5}} - \frac{x \times \exp(-ax^{2})}{2a} \end{aligned}$$

$$\cdot \operatorname{erf} \text{ is the error function, and erf}(0) = 0; \operatorname{erf}(\infty) = 1. \end{aligned}$$

$$\cdot x \cdot \exp(-ax^{2}) \to 0 \text{ when } x \to \infty \end{aligned}$$

$$\int_{0}^{v_{cr}} v_{im}^{2} \exp\left(-\frac{mv_{im}^{2}}{2KT}\right) dv_{im} = \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\frac{m}{2KT}}v_{cr}\right)}{4\left(\frac{m}{2KT}\right)^{1.5}} - \frac{v_{cr}\exp\left(-\frac{m}{2KT}v_{cr}^{2}\right)}{\frac{m}{KT}} \end{aligned}$$

$$\int_{0}^{\infty} v_{im}^{2} \exp\left(-\frac{mv_{im}^{2}}{2KT}\right) dv_{im} = \frac{\sqrt{\pi}}{4\left(\frac{m}{2KT}\right)^{1.5}}$$

$$\eta_{ad} = \operatorname{erf}\left(\sqrt{\frac{m}{2KT}}v_{cr}\right) - \sqrt{\frac{2m}{\pi KT}}v_{cr}\cdot\exp\left(-\frac{m}{2KT}v_{cr}^{2}\right) \end{aligned}$$



$$erf(z) \approx 1 - \frac{1}{(1 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4)^4}$$
$$a_1 = 0.278393,$$
$$a_2 = 0.230389,$$
$$a_3 = 0.00972 \text{ and}$$
$$a_4 = 0.078108.$$
The maximum error is 5 × 10<sup>-4</sup> (Fortran 77 manual).



- Several models of adhesion energy  $(E_{ad})$  were developed before and they were summarized by Givehchi and Tan (2014).
- Two of them are introduced here,
  - 1. JKR model (Johnson, Kendall and Roberts, 1971)
  - 2. DMT model (Derjaguin-Muller-Toporov, 1974)



Table 13-2.Material Properties for Thermal Rebound Calculation					
Material	Hamaker constant	Density (kg/m <sup>3</sup> )	Mechanical constant $(K_i \times 10^{11} m^2/N)$ GivenchiWang & Kasper		
			(2014)	(1991)	
Polystyrene	0.79	1005	10.130	8.86	
Glass (Dry)	0.85	2180	0.443	-	
NaCl	0.7	2165	0.746	2.35	
WOx (Tungsten)	1.216	19250	0.071	-	
Steel	2.12	7840	0.137	0.139	
Nickle			-	0.137	
Copper	3.3	8890	0.218	0.216	
Fused guartz	0.65		_	0.432	

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With the mechanical constant of  $K_p = 0.75 \times 10^{-11} \frac{m^2}{N} \qquad K_f = 0.443 \times 10^{-11} \frac{m^2}{N}$ The composite Young's modulus of bodies  $Y^* = \frac{4}{3\pi} \left( \frac{1}{K_p + K_f} \right) = \frac{4}{3\pi} \left( \frac{1}{0.75 \times 10^{-11} + 0.443 \times 10^{-11}} \right)$   $= 3.51 \times 10^{11} (Pa)$ The characteristic radius of two bodies  $R^* = \frac{1}{2} \left( \frac{1}{d_p} + \frac{1}{d_f} \right)^{-1}$ The impact contact area is determined by  $a = \left[ \frac{R^*}{Y^*} (2\Delta\gamma\pi R^*) \right]^{1/3}$ Then the adhesion energy is  $E_{ad} = \Delta\gamma\pi a^2$ Then the critical speed and impact speed are calculated using  $v_{cr} = \sqrt{\frac{12E_{ad}}{\pi\rho_p d_p^3 e^2}}; \ \bar{v}_{im} = \sqrt{\frac{48kT}{\pi^2\rho_p d_p^3}}$ 47

Then the adhesion efficiency is determined by  

$$z = \frac{2}{\sqrt{\pi}} \frac{v_{cr}}{\bar{v}_{im}}$$

$$\eta_{ad} = \operatorname{erf}(z) - \frac{2z}{\sqrt{\pi}} \exp[-z^2]$$
In calculation using spread sheet, the error function is  
approximated with  

$$\operatorname{erf}(z) \approx 1 - \frac{1}{(1 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4)^4}$$
where  $a_1 = 0.278393$ ,  $a_2 = 0.230389$ ,  $a_3 = 0.000972$  and  $a_4 = 0.078108$ .  
The results are shown in Figure 13.7.

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![](_page_27_Figure_1.jpeg)

![](_page_27_Figure_2.jpeg)

![](_page_28_Figure_1.jpeg)

![](_page_28_Figure_2.jpeg)

![](_page_29_Figure_1.jpeg)

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![](_page_30_Figure_1.jpeg)