Answer Key

Lesson

QUANTUM PARADOXES WITH INTERFEROMETERS

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Answer Key

Outline

QUANTUM PARADOXES WITH INTERFEROMETERS

ACTIVITY GOAL:
Through a thought experiment, show that quantum behaviours can be used to solve problems in new ways.

LEARNING OBJECTIVES

- Wave and particle behaviours.
- Classical vs. quantum problem solving.
- Measurement and collapse.

ACTIVITY OUTLINE

This short activity presents a thought experiment about quantum fireworks to engage students to consider the consequences of wave and particle behaviour and use them to solve an (admittedly fictitious) problem. The ideas learned here can be extrapolated to understanding the key properties of quantum algorithms.

PREREQUISITE KNOWLEDGE

The *Wave-Particle Duality Revisited* lesson
Mach-Zehnder Interferometers

An alternate framing of this activity in terms of polarization states is possible.

Lesson

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THE FIREWORK SALESPERSON

Let's put ourselves into a thought experiment. You are a salesperson at a new fireworks company that promises the world's absolute best fireworks. They put on the most amazing show in the sky and absolutely amaze any audience lucky enough to see them. The problem is that the fireworks are incredibly volatile, only taking a single photon to set off. They are also prone to errors, often resulting in duds that fail to light up at all. You need to sell these fireworks, but customers need to know in advance whether they will work.

Scenario 1 – Dud	Scenario 2 – Working
	
A photon will pass through a dud firework without interacting with it.	A photon will be absorbed by a live firework, resulting in the firework going off,

Your task is to find a way to guarantee that the firework is functioning. You cannot simply set off the firework, since that would destroy it. **What else can you do to test the firework?**

The Quantum Firework Problem

- You must guarantee to a customer that the firework is functional and not a dud.
- A single photon interacting with the firework will set it off and destroy it, while a dud will not interact with the photon.
- It is acceptable to lose some fireworks in testing.
- It is **not** acceptable to sell a firework that has any probability of being a dud.
- You cannot open the firework in any other way, or else it will go off.
- Using the tools of quantum interference, design an approach to the problem before turning the page.

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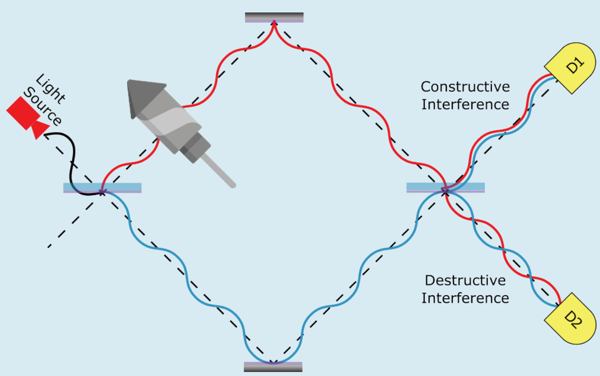
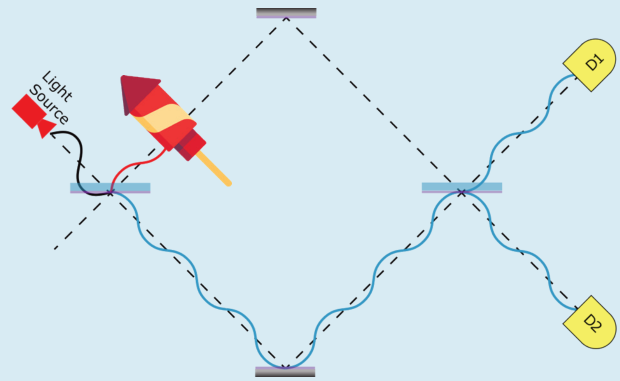
Answer Key

ANSWER

Put the firework in one arm of a Mach-Zehnder interferometer. Consider what happens in each case.

<table>
<tr><th>Scenario 1 – Dud</th><th>Scenario 2 – Working</th></tr>
<tr><td></td><td></td></tr>
<tr><td>A photon will pass through a dud firework without interacting with it.</td><td>A photon will be absorbed by a live firework, resulting in the firework going off,</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>The photon passes through the dud without interacting with it, remaining in a superposition of both paths. When the superposition recombines, there will be constructive interference towards Detector 1 and destructive interference toward Detector 2.</td><td>The photon has a 50% chance of interacting with the firework and setting it off. If it does not, it will definitely take the lower path. At the second beam splitter, there will be no interference, resulting in an equal probability of Detector 1 or 2 going off.</td></tr>
</table>

Event	Probability	Event	Probability
Firework goes off	0%	Firework goes off	50%
Detector 1 detects	100%	Detector 1 detects	25%
Detector 2 detected	0%	Detector 2 detects	25%

We see that Detector 2 only ever goes off if the firework works. This means that, if we ever see a photon at Detector 2, we can guarantee that the firework works without ever setting it off!

1. If Detector 1 goes off, what can we conclude? What can we do to help increase our confidence in the outcome if it is uncertain?

   Detector 1 has a probability of going off in either case. If we see Detector 1 go off, we can repeat the experiment by sending another photon into the interferometer, repeating until either the firework goes off or Detector 2 detects. If we repeat many times, we can statistically infer that the firework is likely a dud and discard it.

2. Discuss how the quantum superposition principle and interference helped you solve the quantum firework problem.

   The photon went through both the top and bottom paths in superposition. In terms of queries, it didn't simply go through the firework path and the empty path; it went through both **in superposition**.

QUANTUM COMPUTING CONNECTIONS

The firework problem is an unrealistic thought experiment, but the core concepts explored help us understand how quantum computers work. While the firework problem is not solvable except by a quantum solution, quantum algorithms solve problems that classical computers can solve. However, the process by which they solve these problems uses properties like superposition and measurement interference, allowing them to solve them much more efficiently in some cases. This is not true to every problem you can imagine; for example, multiplying numbers together is easy for both classical and quantum computers, but find the factors of a large number is much easier for quantum computers of the future than modern computers.

It's important to note that quantum computers are not better at solving **all** problems than classical computers. Quantum computers are exponentially better at finding the factors of numbers, but are no better at all at multiplying two numbers together. Finding problems that can be solved more efficiently by a quantum computer is still a very active field of research!

While photonic interferometers are one path toward building quantum computers, there are many other promising ways that are being researcher by scientists and engineers around the world. Some examples include superconducting circuits, the energy levels of trapped atoms, and the spin state of quantum dots. While most of the benefits of quantum computers are still in the future, significant progress has been made in the last twenty years in making the building blocks of this quantum technology.

Answer Key

Based on the Elitzur-Vaidman bomb tester, see:
A.C. Elitzur and L. Vaidman, "*Quantum Mechanical Interaction-Free Measurements*".
Foundations of Physics **23**, 987 (1993).

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