Answer Key

Lesson

QUANTUM PARADOXES WITH POLARIZATION

Created by the IQC Scientific Outreach team
Contact: iqc-outreach@uwaterloo.ca

Institute for Quantum Computing
University of Waterloo
200 University Avenue West
Waterloo, Ontario, Canada N2L 3G1

uwaterloo.ca/iqc

Answer Key

QUANTUM PARADOXES WITH POLARIZATION

Outline

ACTIVITY GOAL:
Through a thought experiment, show that quantum behaviours can be used to solve problems in new ways.

LEARNING OBJECTIVES

- Wave and particle behaviours.
- Classical vs. quantum problem solving.
- Measurement and collapse.

ACTIVITY OUTLINE

This short activity presents a thought experiment about quantum fireworks to engage students to consider the consequences of wave and particle behaviour and use them to solve an (admittedly fictitious) problem. The ideas learned here can be extrapolated to understanding the key properties of quantum algorithms.

PREREQUISITE KNOWLEDGE

The *Two Golden Rules* lesson or knowledge about polarization states

An alternate framing of this activity in terms of interferometers is possible.

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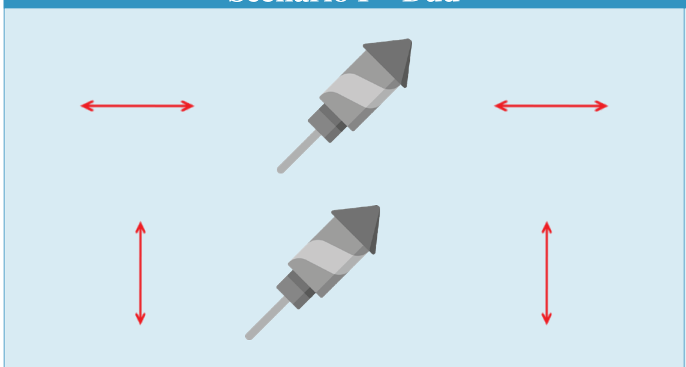
THE FIREWORK SALESPERSON

Let's put ourselves into a thought experiment. You are a salesperson at a new fireworks company that promises the world's absolute best fireworks. They put on the most amazing show in the sky and absolutely amaze any audience lucky enough to see them. The problem is that the fireworks are incredibly volatile, only taking a single photon to set off. They are also prone to errors, often resulting in duds that fail to light up at all. You need to sell these fireworks, but customers need to know in advance whether they will work.

One thing that may help is that the fireworks are sensitive to polarization; only a **vertically polarized photon** will set off the firework.

Scenario 1 – Dud	Scenario 2 – Working
	
A photon of either polarization will pass through a dud firework without interacting with it.	A vertically polarized photon will be absorbed by a live firework, resulting in the firework going off. A horizontal photon will pass through unaffected.

Your task is to find a way to guarantee that the firework is functioning. You cannot simply set off the firework, since that would destroy it. **What else can you do to test the firework?**

The Quantum Firework Problem

- You must guarantee to a customer that the firework is functional and not a dud.
- A single photon interacting with the firework will set it off and destroy it, while a dud will not interact with the photon.
- It is acceptable to lose some fireworks in testing.
- It is **not** acceptable to sell a firework that has any probability of being a dud.
- You cannot open the firework in any other way, or else it will go off.
- Using the tools of polarization measurement, design an approach to the problem before turning the

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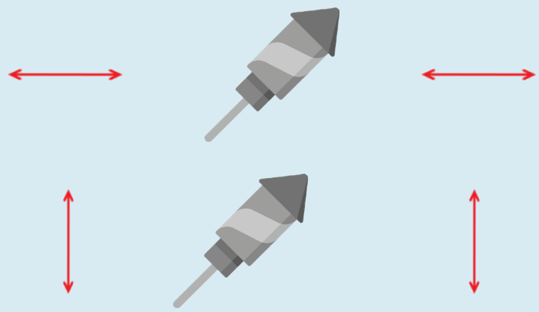
ANSWER

Test the firework with a ***diagonally polarized photon*** as input and a measurement in the 45° basis afterwards. Consider what happens in each case.

Scenario 1 – Dud	Scenario 2 – Working
	
A photon of either polarization will pass through a dud firework without interacting with it.	A vertically polarized photon will be absorbed by a live firework, resulting in the firework going off. A horizontal photon will pass through unaffected.
	
If probed with a diagonal photon, the dud will not affect either the horizontal or vertical component. Therefore, the photon will still be diagonally polarized afterwards.	The photon has an equal horizontal and vertical component. When it interacts with the firework, it has a 50% chance of being found as vertical and setting off the firework. If it does not set it off, it is definitely **horizontal**. A measurement in the 45° basis will produce a random outcome.

Event	Probability	Event	Probability
Firework goes off	0%	Firework goes off	50%
Measure +45°	100%	Measure +45°	25%
Measure -45°	0%	Measure -45°	25%

We see that the photon is only ever found as -45° if the firework is active.

Therefore, if we ever measure -45°, we can confirm the firework works without setting it off!

1. If a +45° photon is measured, what can we conclude? What can we do to help increase our confidence in the outcome if it is uncertain?

   We have some probability of measure the photon as +45° in either case. If we measure a +45° polarized photon, we can repeat the experiment by sending another photon into the interferometer, repeating until either the firework goes off or a -45° photon is measured. If we repeat many times, we can statistically infer that the firework is likely a dud and discard it.

2. Discuss how the quantum superposition principle and measurement bases helped you solve the quantum firework problem.

   The firework effectively measure the photon in the horizontal-vertical basis. The +45° polarized photon can be considered a superposition of horizontal and vertical components, and the measurement of the photon after the firework is also in a different basis. By asking the firework a question in superposition and measuring for the right superposition afterwards, we get information out that we otherwise wouldn't be able to discover.

QUANTUM COMPUTING CONNECTIONS

The firework problem is an unrealistic thought experiment, but the core concepts explored help us understand how quantum computers work. While the firework problem is not solvable except by a quantum solution, quantum algorithms solve problems that classical computers can solve. However, the process by which they solve these problems uses properties like superposition and measurement interference, allowing them to solve them much more efficiently in some cases. This is not true to every problem you can imagine; for example, multiplying numbers together is easy for both classical and quantum computers, but find the factors of a large number is much easier for quantum computers of the future than modern computers.

It's important to note that quantum computers are not better at solving **all** problems than classical computers. Quantum computers are exponentially better at finding the factors of numbers, but are no better at all at multiplying two numbers together. Finding problems that can be solved more efficiently by a quantum computer is still a very active field of research!

While photonic devices are one path toward building quantum computers, there are many other promising ways that are being researcher by scientists and engineers around the world. Some examples include superconducting circuits, the energy levels of trapped atoms, and the spin state of quantum dots. While most of the benefits of quantum computers are still in the future, significant progress has been made in the last twenty years in making the building blocks of this quantum technology.

Answer Key

Based on the Elitzur-Vaidman bomb tester, see:
A.C. Elitzur and L. Vaidman, "*Quantum Mechanical Interaction-Free Measurements*". Foundations of Physics **23**, 987 (1993).

Created by John Donohue
Published by the **IQC Scientific Outreach team**
Contact: iqc-outreach@uwaterloo.ca

Institute for Quantum Computing
University of Waterloo
200 University Ave. W.
Waterloo, ON, Canada, N2L3G1

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