Quantum Ideas

ENTANGLEMENT

Created by the IQC Scientific Outreach team
Contact: iqc-outreach@uwaterloo.ca

Institute for Quantum Computing
University of Waterloo
200 University Avenue West
Waterloo, Ontario, Canada N2L 3G1

uwaterloo.ca/iqc

Quantum Ideas

ENTANGLEMENT

LESSON GOAL: Understand what quantum entanglement can and cannot do.

LEARNING OBJECTIVES

- Quantum vs. classical correlations.
- Non-local coherence and interference.
- Quantum games and Bell's inequalities.

ACTIVITY OUTLINE

First, we'll explore what it means for two objects to be correlated, and how this concept is completely classical in nature.

CONCEPT:
Correlation is a relationship between two objects or variables and has nothing to do with quantum mechanics.

By recalling the idea of coherence and superposition from earlier lessons, we'll explore how quantum correlations can differ from classical correlations.

CONCEPT:
Coherent superpositions of states involving more than one quantum object can reveal interesting results when measured in different bases.

Finally, we'll overview some of the uses of entanglement in quantum technologies.

CONCEPT:
Entanglement is a key resource in quantum technologies.

PREREQUISITE KNOWLEDGE

Polarization of Light
Two Golden Rules of Quantum Mechanics

Quantum Ideas
ENTANGLEMENT

BACKGROUND

So far, our discussions have been limited to single quantum systems in isolation, such as one photon or one spin. In experiments, it is very difficult (but not impossible!) to truly work with only one single quantum system. Many times, we're interested in how many quantum systems like electrons or photons behave together, and what their relationships to each other are.

If multiple quantum systems interact with each other, their properties may become dependent on each other, to the point where they don't have well-defined properties of their own. When the properties of quantum systems cannot be separated from one another, we call them **entangled**.

> **Entangled quantum state:** a quantum state of two or more objects that cannot be individually defined.

So how do entangled relations differ from regular relations? Entanglement can be broken down as the combination of two effects: correlation and coherence. **Correlation** describes a relationship or dependency between two events or objects, where what happens to one provides information about the other. **Coherence** is a wave property that allows for superpositions to be well-defined.

To start, we'll review the concepts of correlation and coherence, and see what happens when we bring the two together.

CORRELATION

A usual definition of a "random outcome" is one that cannot be predicted in advance. For example, a fair coin flip can be heads or tails, but it is impossible to tell before the flip which it will be; there's a 50% probability of each. Similarly, if we put a pair of shoes in a box, mix them around, and draw one out, the probability of it being a right shoe is 50%, and same for it being a left shoe.

However, let's say that we take one pair of shoes, split them into two boxes, and give one to Alice and one to Bob. Alice has a 50/50 chance of finding a right or left shoe in her box, but if she opens it and finds a right shoe, she knows with 100% certainty that Bob has the left shoe. Alice and Bob's results are individually random, but perfectly *correlated*.

COHERENCE

We've seen that quantum states can exhibit randomness much like the coins or shoes example from above. If we encode a photon in the +45 diagonal state and measure it in the horizontal/vertical (H/V) basis, the outcome will be random (probability of 50% for horizontal, 50% for vertical). This is because diagonally polarized light is a 50/50 superposition of horizontal and vertical, which we write as:

$$\nearrow = \frac{1}{\sqrt{2}}(\rightarrow + \uparrow)$$

The key difference between the photon and the coin is that we can choose which *basis* we measure the photon. If we chose to measure the ±45-degree basis, we would have measured +45° 100% of the time, and -45° 0% of the time.

In a quantum superposition, the sign matters, and hence the $\rightarrow + \uparrow$ and $\rightarrow - \uparrow$ superpositions give different answers in the ±45-degree basis despite both being perfectly random in the H/V basis. They aren't just randomly horizontal or vertical, but rather a *coherent superposition* of two possibilities.

This property of superposition, called *coherence*, is what allows for constructive and destructive interference.

ENTANGLEMENT: COHERENCE + CORRELATION

Let's reimagine the correlated shoe scenario but using quantum polarization states instead. We send Alice and Bob each a photon, prepared such that they either both have horizontal or both have vertical. We'll color Alice's photon red and Bob's blue (so ↑→, for example, would mean Alice has a vertical photon and Bob a horizontal one) and write the situation as:

$$\rightarrow\rightarrow \; OR \; \uparrow\uparrow$$

Alice and Bob's measurements are both completely random on their end; they have no idea whether to expect a horizontally or vertically polarized photo. However, if Alice measures a horizontal photon, she knows with certainty that Bob also has a horizontal photon. If Alice measures a vertical photon, she'll know that Bob has a vertical photon as well. This situation has exactly the same classical correlations we saw with the shoe example.

However, in quantum mechanics, Alice and Bob can choose to measure in different bases, such as the ±45-degree basis. Just as we can write ↗ and ↘ in the ±45-degree basis, we can write → and ↑ in the ±45-degree basis as:

$$\rightarrow = \frac{1}{\sqrt{2}}(\nearrow + \searrow), \qquad \uparrow = \frac{1}{\sqrt{2}}(\nearrow - \searrow)$$

Substituting these into both possible situations and expanding, we find that:

$$\rightarrow\rightarrow = \frac{1}{2}(\nearrow + \searrow)(\nearrow + \searrow) = \frac{\nearrow\nearrow + \nearrow\searrow + \searrow\nearrow + \searrow\searrow}{2}$$

$$\uparrow\uparrow = \frac{1}{2}(\nearrow - \searrow)(\nearrow - \searrow) = \frac{\nearrow\nearrow - \nearrow\searrow - \searrow\nearrow + \searrow\searrow}{2}$$

So in either case, by measuring in the ±45-degree basis, we ruin the correlations we had before. This makes some intuitive sense, as measuring an H/V basis state in the ±45-degree basis does give a random answer. From this angle, two people measuring a state with H/V correlations should get completely random answers without any correlations.

However, we can make one small change and see an incredible difference. Let's say that, instead of the correlated state described above, Alice and Bob share the state:

$$\frac{1}{\sqrt{2}}(\rightarrow\rightarrow + \uparrow\uparrow)$$

This has the same properties as the original situation when measured in the H/V basis; Alice and Bob will always agree on the outcome. However, instead of being →→ or ↑↑, we have a *coherent superposition* of →→ and ↑↑.

If we do the same expansion, we find that:

$$\frac{\rightarrow\rightarrow + \uparrow\uparrow}{\sqrt{2}} = \frac{(\nearrow + \searrow)(\nearrow + \searrow) + (\nearrow - \searrow)(\nearrow - \searrow)}{2\sqrt{2}}$$

$$= \frac{\nearrow\nearrow + \nearrow\searrow + \searrow\nearrow + \searrow\searrow + \nearrow\nearrow - \nearrow\searrow - \searrow\nearrow + \searrow\searrow}{2\sqrt{2}}$$

$$= \frac{\nearrow\nearrow + \searrow\searrow}{\sqrt{2}}$$

When we have a coherent superposition of the two possibilities, we see that all of the cases where the polarizations are opposite (↘↗ and ↗↘) *destructively* interfere, while the cases where the polarizations are the same (↗↗ and ↘↘) *constructively* interfere. This coherent correlation results in photons that maintain their correlation even if the measurement basis is changed.

So why do we call them entangled? Recall that Alice and Bob both always get a completely random outcome regardless of which basis they measure in. They only see an interesting pattern when they compare their results against each other. Alice's quantum state is not defined without also considering Bob's quantum state, and vice versa. The two photons are said to be *entangled*, in that they share a property (polarization) even if separated by extreme distances.

1. Are Alice and Bob's measurement results always correlated? What if Alice measures in the H/V basis and Bob in the ±45-degree basis?

2. The states $\frac{1}{\sqrt{2}}(\rightarrow\rightarrow + \uparrow\uparrow)$ and $\frac{1}{\sqrt{2}}(\rightarrow\rightarrow - \uparrow\uparrow)$ both show the same correlations in the H/V basis. Is there a difference if we measure them in the ±45° basis?

FASTER THAN LIGHT?

Most would agree that there's nothing "spooky" about correlation, but quantum correlations have proven more difficult to merge with classical intuition.

Let's dive into a specific situation where Alice and Bob are separated by a vast distance. Perhaps many light-years, such that they can't send each other messages quickly, even if they have a speed-of-light communication channel. Before measurement, both of their photons are completely randomly polarized. However, once Alice measures hers to be, for example, horizontally polarized, Bob's will instantaneously "collapse" to the same state, despite the vast distance between them.

While entanglement does intertwine these distant particles, it does ***not*** allow information to travel faster than the speed of light. When Alice measures her particle as horizontal, she instantaneously knows what polarization state Bob has (also horizontal). However, her measurement is still completely random; she knows what Bob will measure if he chooses the same basis but has no control over it (she was equally likely to, in this case, measure vertical polarization).

Only when Alice and Bob compare results or send each other information are they able to see or use the quantum correlations, and that process is limited to the speed of light.

APPLICATIONS OF ENTANGLEMENT

While entanglement does not allow information to travel faster than the speed of light, faking quantum entanglement in today's most rigorous experiments would require information to travel faster than the speed of light. We can put this to the test by measuring the correlations of two distant photons and seeing if they obey **Bell's Inequality**, a Heisenberg-like condition which must be obeyed by classical correlations but can be violated by entangled states. Verifying that entangled pairs of photons can violate Bell's inequality not only teaches us about the quantum nature of our world, but also verifies that no classical influences or eavesdroppers could have interacted with our photons. This allows us to build even more secure versions of quantum key distribution, called **device-independent QKD**.

Entanglement is also an essential part of quantum computing. Indeed, quantum computing using only quantum systems in separate superposition provides no advantage over classical computing. But more interestingly, we can use entangled states to implement **quantum error correction**, where a cluster of entangled quantum bits (qubits) act together as one super-qubit which is resistant to errors. Quantum error correction is an essential step towards engineering large-scale quantum computation devices.

QUANTUM CONCEPTS

1. Measuring one half of an entangled state instantaneously tells you what the state of the other half is, even if it is many light-years away. Explain why this does **not** allow for faster-than-light communication.

QUANTUM LEAP: CHALLENGE QUESTIONS

1. Bell's inequalities rule out local hidden variables as explanations for entanglement. In experimental tests of entanglement based on Bell's Inequalities, researchers are often concerned about loopholes that may describe their experiment without requiring entanglement. Two of the most famous loopholes are the **locality loophole** and the **detection efficiency loophole**. Research one of these loopholes and explain how Bell's Inequality could be violated in-principle without needing entanglement by exploiting the loophole.

GLOSSARY

- **Bell's Inequalities**, most famously the **Bell-CHSH Inequality**, are limits on the types of correlations that can be realized by conspiring with hidden variables that obey classical mechanics. They can be violated by using quantum-correlated (entangled) particles.
- **Coherence** is a property of waves that allows for interference. **Coherent superpositions** is a term that emphasizes that the superposition state can give rise to interference if measured in the right way.
- **Correlation** is a classical relationship between two variables, such that information about one provides either partial or complete information about the other.
- **Entangled quantum states** are quantum states of multiple objects that must be considered together and cannot be expressed as separate from each other.
- **Non-local games** are scenarios where two players must provide information to referees without communicating to each other, and win if they satisfy certain conditions. In some games, like the **CHSH game**, players can gain an advantage by sharing entangled quantum states.
- **Separable quantum states** are quantum states of multiple objects that can be expressed independently of each other.

FURTHER READING AND REFERENCES

- D. Kaiser, “How the Hippies Saved Physics.” W. W. Norton Company, 2011.
- J. L. Miller, “Three groups close the loopholes in tests of Bell’s theorem.” Physics Today **69**, 1, 14 (2016).

VIDEOS

- Veritasium, “Quantum Entanglement & Spooky Action at a Distance.” 2015 Jan 12.

Created by John Donohue
Published by the **IQC Scientific Outreach team**
Contact: iqc-outreach@uwaterloo.ca

Institute for Quantum Computing
University of Waterloo
200 University Ave. W.
Waterloo, ON, Canada, N2L3G1

About IQC
The Institute for Quantum Computing (IQC) is a world-leading research centre in quantum information science and technology at the University of Waterloo. IQC’s mission is to develop and advance quantum information science and technology through interdisciplinary collaboration at the highest international level. Enabled by IQC’s unique infrastructure, the world’s top experimentalists and theorists are making powerful new advances in fields spanning quantum computing, communications, sensors and materials. IQC’s award-winning outreach opportunities foster scientific curiosity and discovery among students, teachers and the community.

uwaterloo.ca/institute-for-quantum-computing