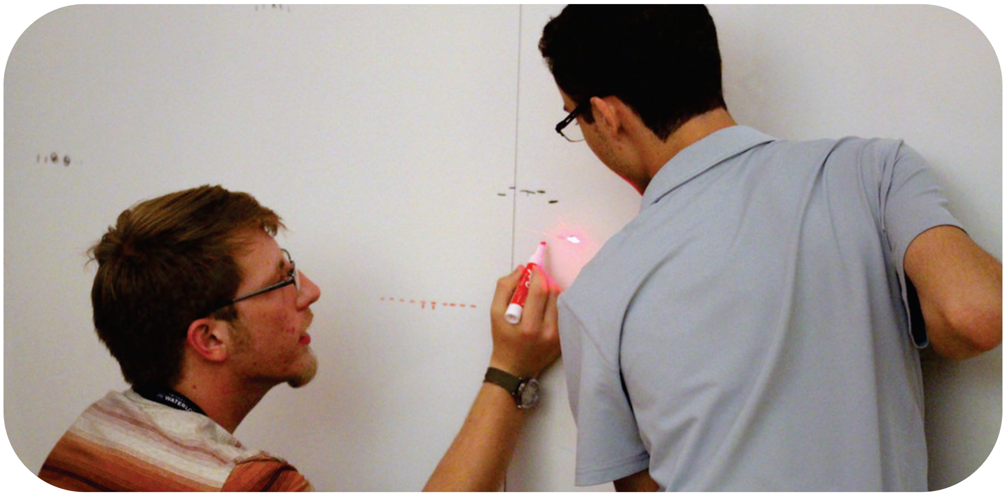
Educator's Guide

INTERFERENCE & MEASUREMENT: MEASURING THE WIDTH OF YOUR HAIR



Institute for Quantum Computing
University of Waterloo
200 University Avenue West
Waterloo, Ontario, Canada N2L 3G1

uwaterloo.ca/iqc

Interference is one of the fundamental behaviours exhibited by waves, including sound, light, and even quantum probabilities. In this hands-on activity, we'll use light-wave interference to infer the width of our hair and connect to how many quantum sensors work.

ACTIVITY AT A GLANCE

Students will use a laser beam to illuminate their hair and measure the diffraction/interference pattern on the wall. By measuring the distance between constructive and destructive interference, they can infer the width of their hair.

CONCEPT: **Superposition and Interference**
The laser light can travel around the hair in multiple ways, which interference with each other either constructively or destructively.

CONCEPT: **Quantum Sensing**
Using interference allows us to measure properties that would otherwise be impossible to measure.

CONCEPT: Uncertainty
The smaller the hair is, the more the laser beam will spread out. This can be connected to a more certain position resulting in a less certain momentum.

LEARNING OBJECTIVES

- Explore the wave behaviour of laser light.
- Connect ideas about precision measurement to wave behaviour and quantum systems.

THE BIGGER PICTURE

Interference is a wave property that occurs when two waves meet that applies to several systems, including those on the quantum-mechanical scale. Understanding interference is key to quantum technologies include quantum computing and quantum sensing.

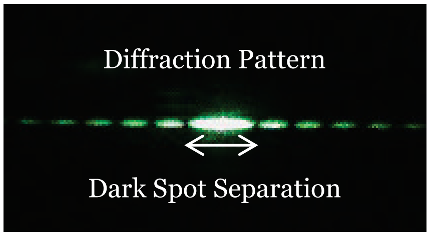
RECOMMENDED AGES

This activity is appropriate for students in **middle school and high school**. It can be run with younger students if they have help with using the laser and performing the calculation. The derivation of the formula used to calculate the width of the hair and connection to the uncertainty principle is most appropriate for upper high-school students.

ACTIVITY TIME

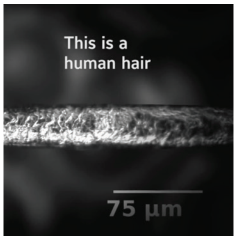
The hands-on activity can be completed in 20-30 minutes.

BACKGROUND



When a laser beam illuminates a very thin object, it scatters around it in different directions. If we look the laser spot far away, we'll see ***interference***, where light can either add up to make a bright spot (*constructive interference*) or cancel out to make a dark spot (*destructive interference*). This effect occurs for all waves, including light, sound, and water waves.

This effect is most noticeable when the thin object is comparable to the wavelength of light, which is less than a millionth of a meter. By measuring the pattern, the wavelength of the laser light, and the distance to the screen, we can accurately measure the width of the object.



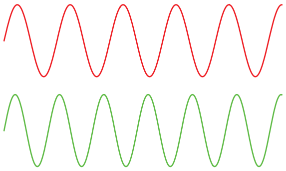
Hairs are much less than a millimeter thick, so we can't use tools like rulers to measure them. In this activity, we'll use interference to measure the width of your hair!

COLOUR AND WAVELENGTH

Light is an *electromagnetic wave*. That means that it carries an electric field that gets stronger and weaker and stronger and weaker, over and over again. The number of times per second where a complete cycle occurs is called the *frequency*, and the distance between two maximums is called its *wavelength*.

Just as we hear a sound wave of a higher frequency as a higher tone, we see light of different frequencies as different *colours*. The colour of the light corresponds to its wavelength.

What's your laser wavelength?



Red Laser Wavelength = 650 nanometres = 0.0000650 centimetres = 0.0000256 inches

Green Laser Wavelength = 532 nanometres = 0.0000532 centimetres = 0.0000209 inches

A USEFUL EQUATION

The separation of the dark spots depends on the width of the object, the wavelength of the light, and the distance between the object and the wall. If we re-arrange the equation to find the width of the object, we find that:

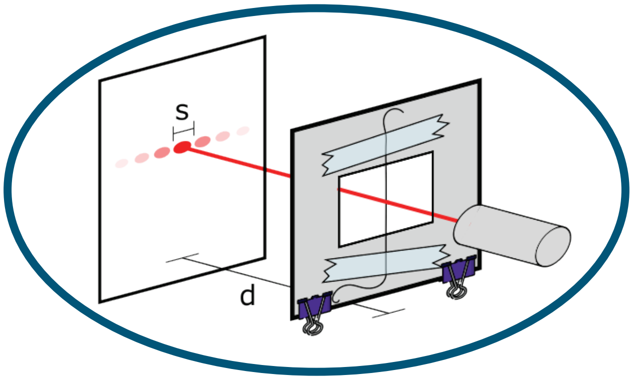
$$Hair\ Width = 2 \times \frac{Wavelength\ of\ laser\ \times\ Distance\ From\ Hair\ to\ Wall}{Separation\ of\ Dark\ Spots}$$

Canadian bills have diffraction signatures to fight counterfeiting.
These can be found in the circle in the middle of the maple leaf.
Shine a laser pointer through them to see what patterns they make!

MATERIALS

- Laser pointer
- Measuring tape
- Calculator
- Scissors
- Tape
- A strand of hair
- Clips

PROCEDURE



1. **Mount** the hair so that it stands up vertically.
2. **Create** a long, clear path from the mount to a wall.
3. **Measure** the distance "**d**" from the hair to the wall.
4. **Shine** the laser onto the wall through the hair.
5. **Observe** the pattern made on the wall by the hair.
6. **Measure** the separation "**s**" between the dark spots.
7. **Calculate** the width "**w**" of your hair as:

$$w = \frac{2d\lambda}{s}$$

Note that all numbers must be entered in the same units (cm or inch).

What's the width of your hair?

Trace the pattern here!

Laser Wavelength (**λ**) = ________________

Distance to wall (**d**) = ________________

Dark spot separation (**s**) = ________________

Hair width (**w**) = ________________

____

1. Most human hairs are between one-hundred and one-thousandth of a centimeter wide. Why might there be a difference between different people? Compare with other groups to determine the range.
2. What else could you measure using this method?

Diffraction patterns like these can be used to measure the size of small objects and identify the structure of complex crystals. We can also see interference patterns with other quantum particles, like electrons, atoms, and even large molecules! This shows that matter, on the quantum level, can behave like a wave.

Facilitator Information

ROOM SETUP

- **Light Level**: A dark room is ideal to ensure that students can clearly make out the constructive and destructive interference. While total blackout is not necessary, if the room is too bright, many students will only see the laser point.
- **Long Pathways**: To see the interference pattern clearly, the laser beam should have at least 2m (6ft) to diffract (spread out) after the hair. Rooms with adjustable seating so students can have long clear straight paths are ideal.
    - Due to the use of lasers, long hallways are not recommended as there is a risk of hitting a passerby in the eye. Additionally, if the pattern spreads too much, it will be too dim to see.
- **Groups**: Groups of at least two (preferably three) are needed so that a student can hold the laser while another measures or traces the pattern.

MATERIALS NEEDED

You will need to acquire the following materials:

- **Worksheet**: One per student or group.
- **Laser Pointer**: A red or green laser pointer is recommended. Laser light is essential; flashlights will not work for this activity. Ensure that students are briefed on laser safety and know to only turn on the laser when its path is controlled and to keep the beam low to avoid accidentally shining it in someone's eyes.
    - The wavelength of the laser is likely 650 nm for red and 532 nm for green, but could vary. If you have a spectrometer or diffraction grating, you can include calibrating the laser pointer as part of the activity.
- **A Strand of Hair**: A student from each group will need to donate a strand of hair to measure.
- **Measuring Tape/Ruler**: A measuring tape will be needed to measure the distance from the hair to the wall. A ruler is recommended for measuring the width of the pattern. Ensure that these are in the right units (e.g., don't give them an tape that measures in inches if you want an answer in centimeters).
- **Craft Supplies**: Students will need to find a way to mount the hair for measurement. Cardboard, cue cards, tape, scissors, and clips are recommended.
    - Clever students may simply tape the hair to the laser pointer.

COMMON ISSUES

- **The number I got doesn't make sense**
    - Units are very important for this calculation. If you are working with a group who is uncomfortable with units, we recommend keeping everything in centimeters.
    - Most hairs will have a width between 10-100 micrometers, or 0.001-0.01 centimeters.
- **I don't see the pattern**
    - Ensure the hair is being illuminated; if you look closely at the hair, you should see a bit of laser light being scattered from it.
    - Darken the room or move to a dark corner if possible.
    - Ensure the beam has enough space to spread out. If you measure within 1 meter (3 feet) of the hair, the pattern won't be visible.
    - Ensure the beam isn't too spread out. The pattern gets dimmer and dimmer as it spreads, since the energy is spread over a larger area. 3 meters (10 feet) of spread is plenty.
- **Am I measuring the right thing?**
    - The distance that is important is from the hair to the wall, not necessarily from the laser to the wall.

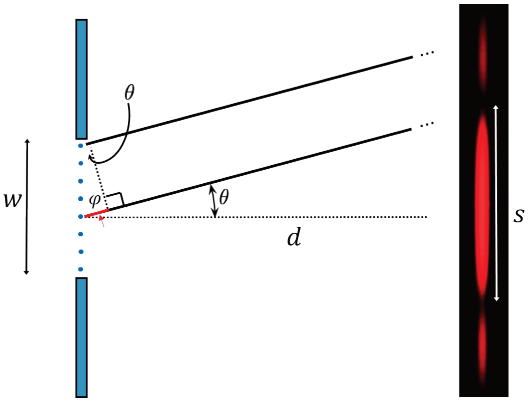
DERIVATION OF THE FORMULA [OPTIONAL]

For students in middle-school or early high-school, we recommend providing the formula without justification. For students who are learning about interference and comfortable with trigonometry, it can be derived straightforwardly.

The easiest way to find the formula is through *Babinet's Principle*. This principle states that the interference pattern we see from a thin blocking object is the same as we see from a thin opening or slit, allowing us to calculate the parameters of the pattern by solving the problem for the more intuitive case of a single-slit object.

The intuition for Babinet's Principle comes from the linearity of wave optics: if we have no object, we simply see a laser point on the wall. If we add the hair to a single slit of the same width, they will completely block the light. The same must hold for their interference patterns: if we add the two patterns together, they must cancel each other out perfectly. This means that they will have different phases, but the same intensity distribution and spacing between constructive and destructive interference peaks.

The intuition for the single-slit interference is below:



Diffraction interference through a single slit (or blocking object) or width w can be understood by deconstructing the plane wave into a series of point sources spanning the width if the slit.

In this image, we decompose it into 8 points, and by symmetry, the interference between the 1st and 5th point source will be the same as between the 2nd / 6th, the 3rd / 7th, and the 4th / 8th.

The phase difference φ between the waves from the 1st point (at $x = 0$) and the 5th point (at $x = \frac{w}{2}$) depends on the angle, as $\varphi = \frac{w}{2}\sin\theta$. We see destructive interference when the waves are $\varphi=\pi$ out of phase, meaning that their optical path length difference φ is $\varphi = \frac{w}{2}\sin\theta_n = \frac{n\lambda}{2}$, for any odd value of n.

In the activity, we let the pattern propagate over a distance d and measure the separation s between the $n = 1$ and $n = -1$ points of destructive interference. The separation s is twice the distance of the centre of the pattern to the first point of destructive interference. We can make a similar triangles that both have an angle θ_1 and equate:

$$\frac{\lambda}{w} = \frac{s/2}{d}$$

Re-arranging to find the unknown width w, we find:

$$w = 2\frac{\lambda d}{s}$$

or, in words:

$$Hair\ Width = 2 \times \frac{Wavelength\ of\ laser\ \times\ Distance\ From\ Hair\ to\ Wall}{Separation\ of\ Dark\ Spots}$$

UNCERTAINTY CONNECTION [OPTIONAL]

You can connect the results of this experiment to the uncertainty principle.

The position-momentum version of the uncertainty principle states that the more certain we are about the position of an object (such as a photon), the less certain we are about the momentum. While a laser beam of a single frequency has a well-defined momentum, recall that momentum is a vector quantity. The uncertainty relation between the two (ignoring constants) can be expressed in terms of the position uncertainty Δx and the uncertainty in the x-component of the momentum Δp_x as:

$$\Delta x \propto \frac{1}{\Delta p_x}$$

For laser light, "position" can be understood as where we find the photon and "momentum" as the direction the light would travel afterwards. For a thin blocking object like a hair, our position certainty about the light that interacts with the object is proportional to its width w.

Our momentum certainty is proportional to how quickly it spreads out after the object, which we can infer from the spot size on the wall. If the beam has a well-defined momentum, it will spread out slowly and the spot size s will be small. If the momentum uncertainty is large, the beam will spread rapidly after the object, and the spot size s will be large.

As we see from our equation for the width of the hair, the width of the hair w is inversely proportional to the size of the spot s. This is an example of how uncertainty is a feature of quantum mechanics that arises from its wave-like nature.

FURTHER QUANTUM CONNECTIONS

- **What would happen if you tried this experiment one photon at a time?**
  - If you tried this experiment one photon at a time, we would not see the interference pattern immediately. We would see the photon detected at a random position on the faraway wall instead. However, if we repeated the experiment many times, we would see that the probability distribution exactly matches the interference we see with the laser beam, with brighter areas of constructive interference corresponding to higher probability and dimmer areas of destructive interference corresponding to lower probability.
- **Do we only see this for light?**
  - Objects we traditionally think about as particles can also exhibit wave-like interference. For example, electron diffraction tubes show that beams of electrons also show regions of constructive and destructive interference when accelerated. Tools like electron beam microscopes use the wave nature of electrons to measure objects smaller than one nanometer.

Connections to QIS Key Concepts

	Concept	Fundamental	Activity Connection
9	Quantum Sensing	Quantum sensing uses quantum states to detect and measure physical properties with the highest precision allowed by quantum mechanics.	Properties that are too small to measure directly can be inferred using wave effects.

Educator's guide published by the **IQC Scientific Outreach team.**
Contact: iqc-outreach@uwaterloo.ca

University of Waterloo
200 University Ave. W.
Waterloo, ON, Canada, N2L3G1

About IQC

The Institute for Quantum Computing (IQC) is a world-leading research centre in quantum information science and technology at the University of Waterloo. IQC's mission is to develop and advance quantum information science and technology through interdisciplinary collaboration at the highest international level. Enabled by IQC's unique infrastructure, the world's top experimentalists and theorists are making powerful new advances in fields spanning quantum computing, communications, sensors and materials. IQC's award-winning outreach opportunities foster scientific curiosity and discovery among students, teachers and the community.

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