Answer Key

Lesson

THE TWO GOLDEN RULES OF QUANTUM MECHANICS WITH LIGHT POLARIZATION

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Answer Key

Outline

THE TWO GOLDEN RULES OF QUANTUM MECHANICS

ACTIVITY GOAL:
Investigate the superposition and measurement principle using the polarization of light

LEARNING OBJECTIVES

- The quantum nature of light polarization.
- Mutually exclusive states and measurement choice.
- The superposition and measurement principles in quantum mechanics
- Probabilistic nature of quantum mechanics and probability amplitudes.

ACTIVITY OUTLINE

We start by defining **mutually exclusive states** of polarization, and linking the wave and particle pictures through Malus' law.

CONCEPT:
Quantum measurement is a probabilistic process.

We then show how any polarization state can be broken down into components and thought of as a **superposition** of other polarization states.

CONCEPT:
Superposition is relative to the measurement context.

By introducing more polarizers, we observe that measurements can change a quantum state, and explain it using the idea of **wave collapse**.

CONCEPT:
Measuring an unknown quantum state will change it.

PREREQUISITE KNOWLEDGE

Light is made of particles called **photons.**
Light is an **electromagnetic wave** with a **polarization.**
Cartesian co-ordinates and trigonometry.

SUPPLIES REQUIRED

3 polarizers with labelled orientations

Lesson

THE TWO GOLDEN RULES OF QUANTUM MECHANICS

PHOTONS AND THE WAVE-PARTICLE DEBATE

In classical physics, we can neatly separate objects into two categories: particles and waves. In quantum mechanics, that distinction no longer holds. In its simplest form, quantum mechanics is based on the idea that **everything is made of particles that have wave-like properties** – this is the **wave-particle duality.** In order to fully appreciate wave-particle duality, we need to highlight some fundamental differences between the two:

Particles	Waves
Exist at one place (localized)	Exist over a large space (delocalized)
Have well-defined properties like mass and volume	Have well-defined properties like wavelength and frequency
Have kinetic collisions	Show wave interference
Are countable	Are continuous

Light is a perfect example of how quantum mechanics doesn't always allow for things to fit into such neat categories. Light behaves like a wave, in that it has a well-defined frequency, can spread out over a large space, and exhibits interference rather than kinetic collisions with other light.

But, as learned from Planck's studies of blackbody radiation and Einstein's explanation of the photoelectric effect, light is made up of indivisible, **countable** "chunks" called **photons**.

In this lesson, we'll consider what it means for a photon to behave like a wave through its **polarization**. We'll discover the rules of quantum superposition and measurement, and how they connect to the wave-particle nature of a photon.

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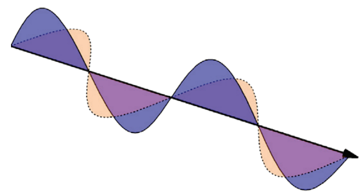
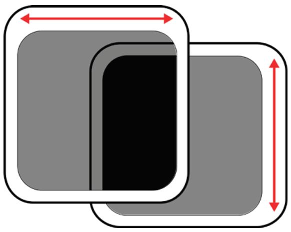
POLARIZATION AND MALUS' LAW



Recall that light is an **electromagnetic wave**, carrying an oscillating electric field perpendicular to the direction it travels in. In the image to the right, the black arrow shows the direction of the light wave. The electric field could be oscillating vertically up-and-down (blue), horizontally side-to-side (orange), or anywhere in between. The direction of the electric field is called the light's **polarization**.



Most objects don't care about the polarization of light that they interact with. **Polarizers** are special sheets of material that absorb or transmit light depending on its polarization. If we hold a polarizer vertically, it will absorb all horizontally polarized light and transmit all vertically polarized light. If we place another polarizer after the first one, we'll find that all the light after the first is now vertically polarized.

For classical polarized light, such from a laser pointer or an LCD computer monitor, we can calculate the intensity of light that makes it through the polarizer using **Malus' Law**. It states that the intensity of light I_{out}, in Watts, that makes it through the polarizer is given by:

$$I_{out} = I_{in}\cos^2\theta,$$

where I_{in} is the intensity of the incoming light and θ is the angle between the polarization of the incoming light and the axis of the polarizer. The polarizer absorbs the rest of the light.

This is consistent with the **wave** picture of light, as the wave component parallel to the polarizer transmits, and the one perpendicular to the polarizer is absorbed. But we know that ultimately, light is made up of indivisible "chunks" – photons. The intensity of light is directly related to the number of photons per second, but what if we send a **single** photon of light to a polarizer?

Consider a single photon at a polarizer. There are only two possible things that can happen:

A. The photon goes through;
B. The photon is absorbed

However, when we have many photons at the same time, we must get Malus' law back. We can resolve this problem if each photon has a certain **probability** of going through the polarizer, and a probability of being absorbed. If the photon's polarization is an angle θ from the polarizer, the probability of being transmitted is:

$$Prob(transmit) = \cos^2\theta$$

1. What is the probability that the photon is absorbed by the polarizer?

   The probability of it being transmitted is $\cos^2\theta$, and the only other possibility is that it is absorbed. Since these probabilities must add up to 1,

$$Prob(absorbed) = 1 - \cos^2\theta = \sin^2\theta$$

2. Suppose horizontally polarized **classical light** is sent to a polarizer at an angle θ from the horizontal axis. What is the ratio of the input to output intensity?

θ	$0°$	$45°$	$-45°$	$-30°$	$60°$	$90°$
I_{out}/I_{in}	1	½	½	¾	¼	0

3. Suppose horizontally polarized **photons** are sent to a polarizer at an angle θ from horizontal. What is the probability of the photon transmitting or being absorbed?

θ	$0°$	$45°$	$-45°$	$-30°$	$60°$	$90°$
Probability transmitted	1	½	½	¾	¼	0
Probability absorbed	0	½	½	¼	¾	1

Malus' law is a consequence of the superposition principle and the probabilistic nature of quantum mechanics. By the end of this activity, we'll be able to answer the key question: **just what does a superposition mean**?

To answer this, we'll need to describe light polarization using **exclusive states**.

Answer Key

MUTUALLY EXCLUSIVE STATES

The **state** of an object defines its properties at a specific time. For example, if we flip a coin, we might find it lands in the state "heads".

Two states are **mutually exclusive** if being found in one state means you definitely aren't in another. In other words, an object can't be in both states at the same time. For example, if we find the coin in the state "heads", we know with certainty it is not in the state "tails". "Heads" and "Tails" are therefore mutually exclusive states.

1. Brainstorm other states that are mutually exclusive to each other.

   There are many possibilities, but some common answers could be a light switch being on or off, a coin being worth 5 cents, 10 cents, etc, or, if Schrödinger is in your class, a cat being alive or dead.

2. Take a vertical polarizer and look through it. Take a second polarizer rotated at 90° and put it front of the first one. How much light coming out of the vertical polarizer makes it through the horizontal one?

   None. Some blue light may be visible due to technical limitations of the polarizers.

3. Are horizontal and vertical polarizations mutually exclusive? Why or why not?

   They are! If a wave oscillates vertically, it has no horizontal component whatsoever, and all the light is blocked. In other words, if the light is vertical, it is definitely not horizontal.

4. Are vertical and diagonal (45°) polarizations mutually exclusive? Can you test this experimentally?

   No they are not. When we hold two polarizers 45° apart, some of the light goes through, confirming that diagonal polarization has some vertical component.

5. Is there any polarization state, other than horizontal, that is mutually exclusive to the vertical state?

   No. All other angles let at least some light through.

6. Can you find a state that is mutually exclusive to the 45° polarization state? Test your prediction using the polarizers.

   Yes, the -45° state. We can verify this experimentally by looking through both a +45° and a -45° polarizer together and noting that no light gets through.

7. Can you think of other collections of states that would be mutually exclusive to each other?

   Any two perpendicular polarizations are mutually exclusive.

   If you've discussed circular polarization in class, such as in the context of 3D movies, left- and right-handed circular polarization are also mutually exclusive. Full discussion of circular polarization is beyond the scope of this activity.

MEASUREMENT BASIS

As we just saw, two perpendicular polarizations form a pair of mutually exclusive states. Let's re-examine the effect of the polarizer with this in mind.

If a horizontally polarized photon hits a horizontal polarizer, it will definitely pass through. If a vertical photon hits the same polarizer, it will definitely be absorbed. By looking for photons after the polarizer, we can tell if they were horizontally or vertically polarized.

Essentially, the horizontal polarizer is asking the photons a question: **are you horizontally or vertically polarized**?

In general, a polarizer performs a **measurement** that distinguishes between two mutually exclusive states!

1. What measurement does a vertical polarizer perform?

   The vertical polarizer distinguishes between horizontal and vertical polarizations. The effect is opposite to the horizontal polarizer, but the question it asks is the same!

2. What measurement does a 45° polarizer perform?

   It distinguishes between 45° polarization and it's mutually exclusive partner, -45°.

3. What measurement does a -30° polarizer perform?

   It distinguishes between -30° polarization and the perpendicular polarization, +60°.

A pair of mutually exclusive states are referred to as a **measurement basis**.

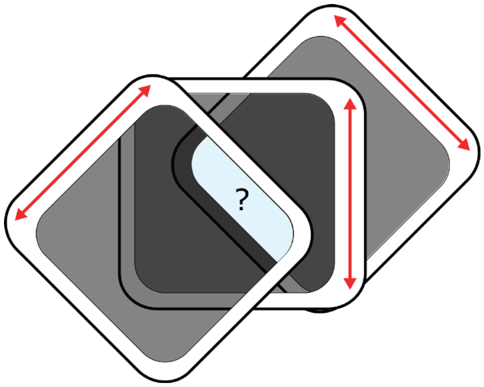
For example, the horizontal and vertical polarizers both perform a measurement in the horizontal/vertical basis, often written in short-form as the HV basis. The 45° polarizer performs a measurement in the +45°/-45° basis, sometimes called the diagonal basis.

Answer Key

A SURPRISING PHENOMENON

If you hold a +45° and -45° polarizer together, you should see that no light goes through.

What happens when we introduce a third polarizer into the experiment?



1. What do you observe when you place a vertical polarizer in front of the diagonal polarizers? Does this match your expectation?

   No light passes through, because the +45° polarizer absorbs all -45° polarized light, and the -45° polarizer absorbs all +45° light. This accounts for all of the light, and the vertical polarizer is left with no light to measure.

2. What do you observe when you place a vertical polarizer in between the two diagonal polarizers?

   Some light appears to pass through!

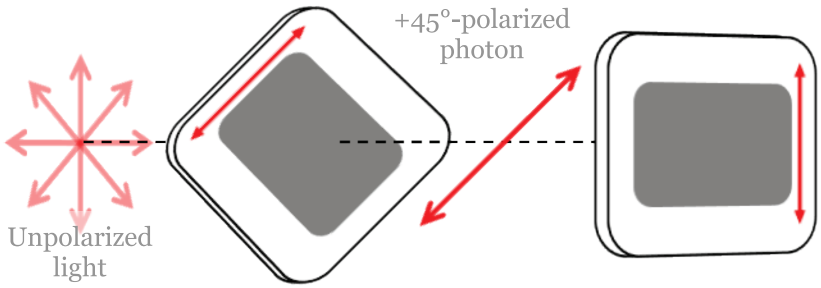
3. Can you explain this effect? With your group members, come up with a hypothesis.

   The first +45° polarizer polarizes all the light to +45°. By placing the vertical polarizer in the middle, we're performing a measurement in the HV basis. The +45° polarized light has an equal horizontal and a vertical component. The horizontal component is absorbed, and the vertical component passes through.

   After the middle polarizer, the light is vertically polarized, which has both +45° and -45° components. The final -45° polarizer absorbs the +45° component, but lets the -45° component through.

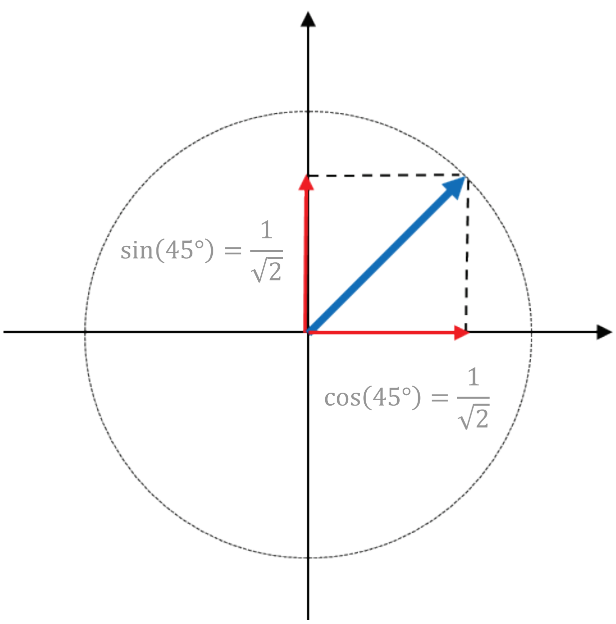
SUPERPOSITION AND MEASUREMENT

Let's analyze the situation below using what we've learned about photons, polarizers, and mutually exclusive states. In the first step, a +45° polarizer is used to prepare a +45°-polarized photon, which we then measure with a vertical polarizer.



We know that, after the first polarizer, all the photons will be +45°-polarized. The second polarizer performs a measurement in the HV basis. In other words, it asks the photons the question "**Are you in the horizontal or the vertical state?**"

The question cannot be answered immediately because the photon is +45°-polarized, which is neither horizontal or vertical. But any polarization state can be described using a combination of horizontal and vertical **components**.



We can describe +45° polarization as a unit vector, shown in blue.

It has an equal horizontal and vertical component, shown in red.

Let's investigate further using the Cartesian plane. We know that we can describe any point on the plane using only two coordinates: how much of "x" and how much of "y", or if you prefer, how much of a "**horizontal component**" and how much of a "**vertical component**".

We can directly relate polarization to the Cartesian plane. Think of the polarization state of the photon as a vector of length one starting at zero, or as a point on the unit circle centred on the origin.

We can describe polarization in any direction by decomposing that vector into horizontal and vertical components. For example, +45° polarization has a horizontal component of cos(45°), and a vertical component of sin(45°).

By breaking it into components, it is possible to describe +45°-polarized light as a combination of horizontally and vertically polarized light:

$$\nearrow = \frac{1}{\sqrt{2}} \rightarrow + \frac{1}{\sqrt{2}} \uparrow$$

A wave can have components oscillating in different directions at the same time. Recall, however, that the light is made of individual, indivisible photons. For the following questions, think about the situation as if we sent one single photon to the polarizers.

Answer Key

1. Using Malus' law, what is the probability that a +45°-polarized photon will make it through the vertical polarizer? What is the probability that it is absorbed?

   There is a 50% probability that it is either absorbed or passes through.

2. Can you find a mathematical relationship between the vertical component of the +45° vector and the probability of passing through the vertical polarizer?

   The probability is the component squared, $\frac{1}{2}=\left(\frac{1}{\sqrt{2}}\right)^2$.

3. If a photon makes it through the vertical polarizer, what is its polarization state afterward?

   Vertical, as only the vertical component passes through.

Because it has both a horizontal and a vertical component, we call +45° polarization a **superposition** of horizontal and vertical polarizations. However, since there is only one photon, it can only be horizontal or vertical when we ask it which it is with the vertical polarizer.

When we measure the superposition in the HV basis, the results are impossible to predict! It will randomly **collapse** to either horizontal or vertical polarization, with the probabilities given by Malus' law.

We can summarize this with two rules that describe the behaviour of photons, electrons, and all objects that obey quantum mechanics: the **two golden rules of quantum mechanics**.

Rule #1: Superposition

A quantum object can be in a superposition of multiple states at once.

For example, a +45°-polarized photon is in a superposition of horizontal and vertical polarization states

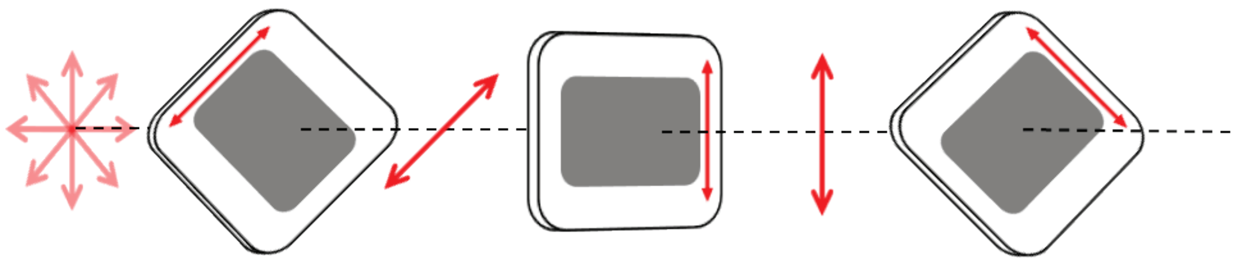
Rule #2: Measurement

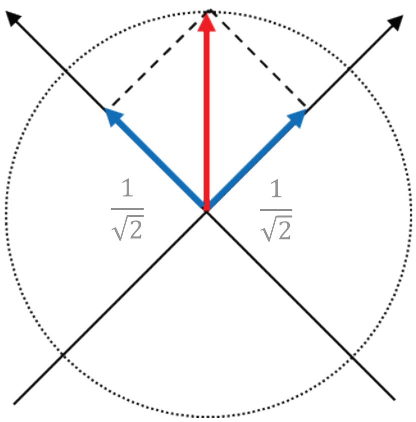
Rule #1 works as long as you don't look!

The act of measuring the superposition will collapse it and change the state.

For example, when we measure the +45°-polarized photon in the HV basis, it must collapse to either horizontal or vertical polarization

The third polarizer





The light that makes it through the first two polarizers is now vertically polarized. The final polarizer performs a measurement in the ±45° basis. In other words, it asks the photons the question: "**Are you in the +45° or -45° state?**"

We are in a situation similar to before, except this time we need to decompose the vertical polarization into +45° and -45° components, as in the plane on the left.

Just as with the HV basis in the previous step, we can describe vertically polarized light in terms of its components in the ±45° basis as:

$$\uparrow = \frac{1}{\sqrt{2}} \nearrow - \frac{1}{\sqrt{2}} \searrow$$

1. Using the quantum superposition rule (Rule #1), can you relate vertical polarization to its decomposition in the ±45° basis?

   Vertical polarization can be thought of as a superposition of +45° and -45°.

2. What is the probability of the vertical photon making it through the -45° polarizer, and how does it relate to its -45° vector component?

   The probability is 50%, and is the component squared, $\frac{1}{2}=\left(\frac{1}{\sqrt{2}}\right)^2$.

3. If a photon makes it through the -45° polarizer, what is its polarization state afterward? Can you relate this to the measurement principle (Rule #2)?

   The state would collapse to -45° polarization. When we make a ±45° measurement, the state must collapse to either +45° or -45° polarization.

4. Inserting the vertical polarizer between the +45° and the -45° polarizers suddenly allowed light to pass through. Using the measurement principle (Rule #2), explain why the presence of an extra measurement changed the experiment.

   When we measured the +45°-polarized photon in the HV basis, we forced it to collapse to either horizontal or vertical. This measurement collapse changed the state, which we observed by seeing light pass through the final polarizer, even though it was perpendicular to the first polarizer.

5. The negative sign in front of the -45° component tells us the **phase** between the two components. For polarization, we can think of the negative sign flipping the arrow's direction.

   What polarization state does the following superposition represent?

$$\frac{1}{\sqrt{2}}\nearrow + \frac{1}{\sqrt{2}}\searrow = ?$$

   The positive superposition represents horizontal polarization, which we can see by adding the two vectors together on the Cartesian plane.

We saw that 45° polarized light can be seen as a superposition of both horizontal and vertical polarizations. Similarly, we found that vertical polarization can be seen as a superposition of 45° and -45° polarization. This suggests that t**he concept of superposition is relative and depends on the context of the measurement.**

For example, 45° polarization must be seen as a superposition of horizontal and vertical if we are performing a measurement in the HV basis. But, if we were to perform a measurement in the ±45° basis instead, the 45° polarization state would not be considered a superposition!

Similarly, vertical polarization is not considered a superposition when we measure it in the HV basis, but must be considered a superposition when measured in the ±45° basis.

These effects can be seen in both the photon and the classical electromagnetic wave theories of light. Indeed, there is nothing "quantum" about superposition; **superposition is a natural property of waves**.

When a single photon exists in superposition, it behaves as if it is in two states at the same time. If we measure a 45°-polarized photon in the HV basis, its state is not well-defined, which is a **wave behaviour**. But once it is measured, it is definitely either horizontal or vertical. Its state becomes well-defined, which is a **particle behaviour**. This mixture of wave and particle behaviours in one quantum system is the hart of **wave-particle duality**.

FURTHER EXPLORATION

Let's generalize the concept of superposition.

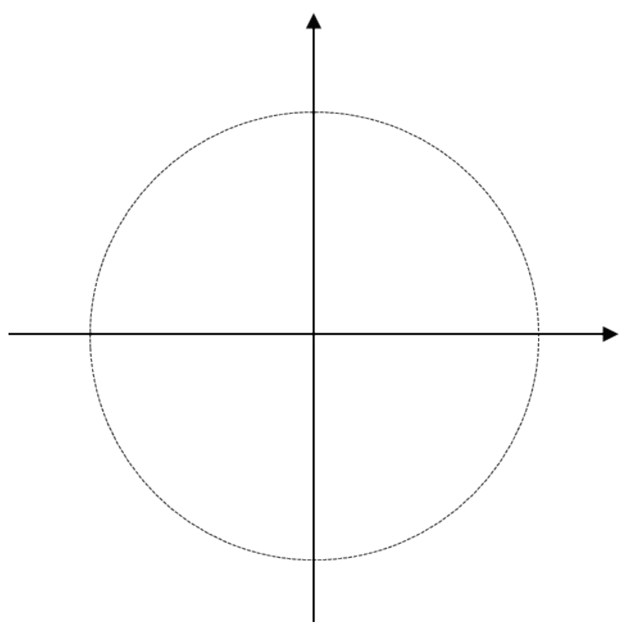
1. Suppose a single photon has a polarization vector with an angle θ with respect to the horizontal, such that $\theta = 0°$ corresponds to horizontal and $\theta = 90°$ corresponds to vertical polarization. You are measuring using a horizontal polarizer.

   Fill in the table below, using Malus' law to calculate probabilities. You can use the graph below the table to help find components.

	$\theta = 0°$	$45°$	$-45°$	$-30°$	$60°$	$90°$
H component	1	$1/\sqrt{2}$	$1/\sqrt{2}$	$\sqrt{3}/2$	$1/2$	0
V component	0	$1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/2$	$\sqrt{3}/2$	1
Superposition	$\rightarrow$	$\frac{1}{\sqrt{2}} \rightarrow + \frac{1}{\sqrt{2}} \uparrow$	$\frac{1}{\sqrt{2}} \rightarrow - \frac{1}{\sqrt{2}} \uparrow$	$\sqrt{3}/2 \rightarrow - \frac{1}{2} \uparrow$	$\frac{1}{2} \rightarrow + \frac{\sqrt{3}}{2} \uparrow$	$\uparrow$
"H" probability	1	½	½	¾	¼	0
"V" probability	0	½	½	¼	¾	1

2. From observation and with the help of Malus' law, what is the relationship between the horizontal/vertical components and the probabilities of measuring the photon in the horizontal or vertical state?

   The probability is equal to the amplitude of the component squared.



3. For a photon with generic polarization angle θ, what is the probability of it being measured in the horizontal or vertical states? Give general formulas in terms of θ.

   $P(H) = \cos^2\theta$ and $P(V) = 1 - P(H) = \sin^2\theta$

In general, it is possible to decompose any polarization state using any two mutually exclusive states. The probability of measuring each of the exclusive states is given by the absolute-value squared of the superposition component. We refer to that component as a **probability amplitude**.

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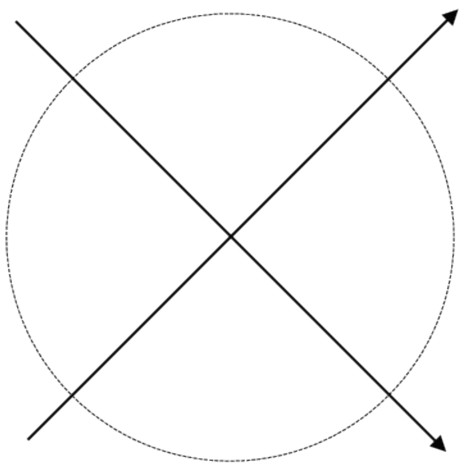
4. Suppose that you measure with a polarizer aligned to the +45° diagonal instead. As before, the angle θ in each column corresponds to a different polarization state.

Fill in the table below, once again using the axes below to help find components.

	$\theta = 0°$	$45°$	$-45°$	$-30°$	$60°$	$90°$
45° component	$1/\sqrt{2}$	1	0	$\cos 75°$	$\cos 15°$	$1/\sqrt{2}$
-45° component	$1/\sqrt{2}$	0	1	$\sin 75°$	$-\sin 15°$	$-1/\sqrt{2}$
Superposition	$\frac{1}{\sqrt{2}}\nearrow + \frac{1}{\sqrt{2}}\searrow$	$\nearrow$	$\searrow$	$\cos 75° \nearrow + \sin 75° \searrow$	$\cos 15° \nearrow - \sin 15° \searrow$	$\frac{1}{\sqrt{2}}\nearrow - \frac{1}{\sqrt{2}}\searrow$
45° probability	½	1	0	$\cos^2 75°$	$\cos^2 15°$	½
-45° probability	½	0	1	$\sin^2 75°$	$\sin^2 15°$	½

5. For a photon with generic polarization angle θ relative to the horizontal, what is the probability of it being measured in the +45° or -45° states? Give general formulas in terms of θ.
$P(45°) = cos^2(\theta - 45°)$, and
$P(-45°) = 1 - P(45°) = sin^2(\theta - 45°)$



THE QUANTUM WAVE FUNCTION

In this activity, we investigated the **quantum wave function**. We just saw how thinking of different polarization states in terms of their components allows us to write them as **superposition states**.

Classical waves also have components, but they represent intensity along a certain direction. In the quantum case, the components represent **probabilities** instead. If you square the coefficient (or probability amplitude) of a given component, you get the probability of measuring the photon in that state.

This idea extends beyond the polarization of photons. For example, a photon can be found in a specific location "x", and each possible position is mutually exclusive with all others (if we find it at "x", it is certainly not at position "y"). The position of the photon must have some **quantum uncertainty**, which is another way of saying it is in a superposition of many positions! The position wave function is defined by a function $\psi(x)$, and the square of the wave function gives the probability distribution of where to find the photon.

It also extends beyond photons to other quantum objects. Electrons have a property called **spin** which, like polarization, can be in one of two exclusive states (spin-up or spin-down). And just like polarization, these states can be put into superposition, and different measurement contexts can reveal different properties of the electrons.

APPLICATIONS AND TECHNOLOGY

Many quantum technologies rely on the fact that we can measure quantum systems in different ways.

Quantum computing works by using quantum systems built of two mutually exclusive states, like polarization, as the binary bits of a computer. These quantum bits, or **qubits**, can sometimes perform computational tasks more efficiently than classical computers by taking advantage of the fact that we can measure them in different bases (along with other properties, like quantum entanglement).

When using the three polarizers, we saw that introducing a measurement changed the state, allowing light to pass through. Let's apply this idea to information security! If two people communicate using the polarization of single photons, and someone tries to eavesdrop by measuring in the middle, they'll change the state of the photon, which can be measured. This idea is the key to **quantum key distribution**, which allows people to guarantee that their communications are not being read by anyone else.

Answer Key

QUANTUM QUIZ

1. Are each the following examples of mutually exclusive states? Why or why not?
   a. Whether a coin is heads or tails
   YES, since if it is in one state, it is definitely not in the other
   b. Whether a cup has a volume of 250 mL or is made of glass
   NO, since it is possible for the cup to be both
   c. Whether a photon is diagonally polarized or horizontally polarized
   NO, since we cannot measure both polarizations at they same time (not distinguishable)
   d. Whether a photon is +75°-polarized or -15°-polarized
   YES, since the two polarization states are perpendicular

2. 1000 horizontally polarized photons are sent to a polarizer at +30°. How many do you expect will make it through? Is this number exact?
Using Malus' law, the probability for each photon is $(\cos(30° - 0))^2 = 0.75$. 75% of the photon will pass through, so approximately 750 photons will pass. This is an average because the process is random.

3. What are the horizontal and vertical components of a photon polarized at +30° relative to the horizontal? Write the state as a superposition of horizontal and vertical components.
The horizontal component is $\cos(30° - 0) = \sqrt{3}/_2$ and the vertical component is $\cos(30° - 90°) = {}^1/_2$. Recall that the probability is calculated from the square of the component. The superposition state can be written as $(\cos 30°) \rightarrow +(\sin 30°) \uparrow$.

4. What are the +45° and -45° components of a photon polarized at +30°? Use the Cartesian plane to be sure whether the components are positive or negative, and write the photon's state as a superposition of +45° and -45° polarizations.
The +45° component is $\cos(30° - 45°) \approx 0.97$ and the vertical component is $\cos(30° + 45°) \approx 0.26$. The superposition state is written as $(\cos 15°) \nearrow +(\sin 15°) \searrow$.

QUANTUM CONCEPTS

1. Noting that a laser beam is made up of many photons, relate Malus' law for the intensity of a laser beam through a polarizer to the probability rules for a single photon through a polarizer.
Each photon in the laser beam is either absorbed or transmitted with a probability given by the quantum rules. Since the intensity is proportional to the number of photons, Malus' law gives the average number of photons that make it through the polarizer.

2. Explain how wave-particle duality is connected to the superposition and measurement principles.
Superposition is a wave property, and the definitiveness of measurements is a particle property.

3. Is a horizontally polarized photon in a superposition state? Explain why or why not.
It depends on the measurement context; in the horizontal-vertical context, it is not, but in any other context (such as 45° basis), we would need to think of it as a superposition.

4. Polarization can be used to encode one bit ("0" or "1") of information. Sketch how someone could two people could communicate by encoding photons and measuring their polarization.
The sender could encode a "0" as a horizontally polarized photon, and a "1" as a vertically polarized photon. If the receiver measures in the HV basis, they'll understand the message perfectly.

QUANTUM LEAP: CHALLENGE QUESITONS

1. Consider the two different experiments.

In Experiment A, we prepare a +45°-polarized photon and measure it with a -45° polarizer.

In Experiment B, we flip a coin and prepare either a horizontally or vertically polarized photon, which we measure with a -45° polarizer.

What is the probability of the photon passing through the -45° polarizer in each experiment? How does a superposition behave differently than randomly being in one state or another?
In Experiment A, using Malus' law, the probability of the photon passing through the -45° polarizer is 0. In Experiment B, we can consider each case independently. Both a horizontally and a vertically polarized photon both have a 50% chance of making it through the -45° polarizer. The probability of a random horizontal or vertical photon making it through is therefore also 50%.

The superposition gives the same probability as the random chance experiment if we measured in the HV basis, but by measuring in the 45° basis, we see different probability for superpositions. This is a form of **quantum interference**.

2. What is the probability that a horizontally polarized photon makes it through (in order) a 30° polarizer followed by a 60° polarizer, and vertical polarizer?

We can break this four-polarizer problem into three steps, just as we did when breaking the three-polarizer problem into two steps earlier.

In step one, a horizontal photon encounters a 30° polarizer, and transmits with a probability of $(\cos 30°)^2 = 0.75$, by Malus' law. If it passes through that photon is now 30°-polarized and encounters a 60° polarizer, transmitting with a probability of $(\cos(60° - 30°))^2 = 0.75$ again. Finally, if it makes it this far, the photon would be 60°-polarized and transmit through the vertical polarizer with probability $(\cos(90° - 60°))^2 = 0.75$ again.

The overall probability of it making it through is equal to the combined probability of it making it through each step, which is (0.75)*(0.75)*(0.75) ≈ 0.423 .

3. If we start with a horizontally polarized photon and pass it through N polarizers, starting at an angle of ${}^{90°}/_{N}$ and increasing in increments of ${}^{90°}/_{N}$ up to vertical (90°), what is the probability that the photon makes it through the vertical polarizer? What is the limit as N goes to infinity? This is called the "Quantum Zeno Effect".

The probability at each step is $\left(\cos {}^{90°}/_{N}\,°\right)^2$, and there are N steps. So the total probability is $\left(\cos {}^{90°}/_{N}\,°\right)^{2N}$. This probability goes to 100% in the limit of infinite polarizers.

Answer Key

GLOSSARY

- **Photons** are indivisible units of light. Each photon carries the smallest amount of electromagnetic energy possible, equal to
- **Polarization** is the direction the electric field oscillates in a beam of light. The polarization is always perpendicular to the direction the beam travels.
- A **state** defines the properties of an object. For example, a light switch can be in the "ON" state or the "OFF" state.
- A **quantum state** defines the properties of an object that obeys quantum mechanics, such as an electron or photon. Unlike a classical state, a quantum state may be fundamentally uncertain.
- **Mutually exclusive states** are two possible states that cannot both occur at the same time. More precisely, they are two states that can be perfectly distinguished by an ideal measurement. For example, "heads" and "tails" on a coin are mutually exclusive, as are the horizontal and vertical polarization states of a photon.
- **Measurement bases** or **measurement contexts** are different ways of measuring quantum objects that cannot be performed at the same time. For example, we cannot measure the position and the momentum of a photon at the same time, nor can we measure the horizontal/vertical and ±45° polarization at the same time. We can think of these as the different possible questions we can ask our quantum object.
- **Superposition** is the property of a quantum state having a component in more than one possible state in a set of mutually exclusive states. For example, a +45° polarized photon has both a horizontal and a vertical component, and can be considered a superposition of the two. All states are superposition states when considered in the context of different measurement bases.

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