Answer Key

Activity

BB84: QUANTUM COINS

Created by the IQC Scientific Outreach team
Contact: iqc-outreach@uwaterloo.ca

Institute for Quantum Computing
University of Waterloo
200 University Avenue West
Waterloo, Ontario, Canada N2L 3G1

uwaterloo.ca/iqc

UNIVERSITY OF WATERLOO | IQC Institute for Quantum Computing

SC Schrödinger's Class

Answer Key

Outline

BB84: QUANTUM COINS

ACTIVITY GOAL:
Simulate the quantum key distribution protocol through role-playing.

LEARNING OBJECTIVES

- Measurement disturbance.
- Cryptography and the one-time pad.
- The BB84 quantum key distribution protocol.

ACTIVITY OUTLINE

In this hands-on activity, we'll simulate the BB84 quantum key distribution protocol using coins and boxes. Groups of four (at minimum) will take on the roles of Alice, Bob, Eve, and quantum nature itself.

Alice will encode information in one basis, which they will then send to Bob. Eve gets an opportunity to measure the system, which may disturb Alice's encoding. Bob then measures the bit. Alice and Bob post-process their data to either generate a secret key or detect the presence of an eavesdropper.

CONCEPT:
If an eavesdropper measures a quantum system, they will disturb the quantum state and can be detected.

PREREQUISITE KNOWLEDGE

The **quantum cryptography** lesson

SUPPLIES REQUIRED

Five coins
Two boxes labelled "HV" and "±45°"
Alice, Bob, and Eve worksheets
Groups of at least four students each

Activity

BB84: Quantum Coins

THE SET-UP

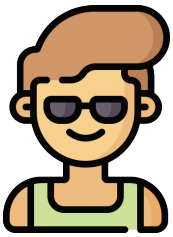
In this activity, you're going to demonstrate the basic ideas of the BB84 protocol for Quantum Key Distribution (QKD). Acting as Alice and Bob, you'll attempt to establish a secret key and test for the presence of an eavesdropper.

First, split your group into four teams:

Alice

Sends qubits

Starts with three coins and two boxes



Bob

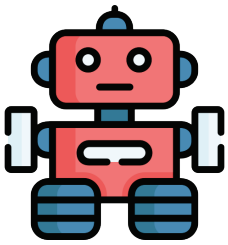
Measures qubits

Starts with one coin

Eve

Interrupts qubits

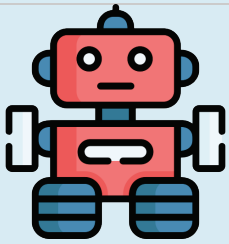
Starts with one coin



Moderator

Enforces quantum mechanics

Eve



Moderator

First, Eve must choose a strategy. Will they look at every bit, every other bit, or none at all? Your teacher may provide you with a strategy, or you may choose your own.

Teacher's Note: If you have multiple groups of four at the same time, we recommend assigning each Eve a different strategy and seeing if the Alice and Bobs can determine which Eve they have. Possibilities include:

- Eve always measures in a random basis
- Eve always measure HV or always measures ±45°
- Eve measures every other bit or every third bit
- Eve never measures anything at all

Eve should secretly tell the moderator their strategy so that they can communicate quietly about Eve's decisions during the key exchange.

Answer Key

THE PROTOCOL

To build the secret key, Alice, Bob, Eve, and moderator should repeat the following steps, filling out their tables as they go. Alice, Eve, and Bob should try to complete their tasks quietly, so that the others don't overhear their bits and basis choices.

Alice

Alice tosses her first coin, which tells her which bit to encode in her qubit.
If it lands heads, encode "0". If it lands tails, encode "1".

Alice's second coin tells her which **basis** to encode her qubit in.
If it lands heads, encode in HV. If it lands tails, encode in ±45°.

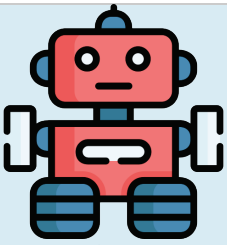
Alice then encodes her bit in the appropriate box gently with the first coin, and randomly shakes the second coin in other box.

Toss	Bit	Basis	HV box	±45° box
H, H	0	HV	Place coin heads-up	Place coin inside and shake to randomize
H, T	0	±45°	Place coin inside and shake to randomize	Place coin heads-up
T, H	1	HV	Place coin tails-up	Place coin inside and shake to randomize
T, T	1	±45°	Place coin inside and shake to randomize	Place coin tails-up

Write down your bit and basis on your worksheet.

Both boxes <u>together</u> represent a single photon. Each box individually represents a different possible measurement that could be made by Bob or Eve.

Shaking the box that is **not** used by Alice mimics the quantum uncertainty principle – since Alice knows exactly what the result is in one box / measurement, the result of the other must be completely random.



Moderator

Take both boxes **gently** to Eve, making sure that neither Eve nor Bob sees which box Alice shook up.

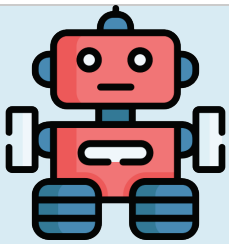
Next, give Eve the option to look in one (and only one!) of the boxes.

Eve

Eve chooses to open one box **or** neither box. If they wish, they may use their coin to help make their decisions random.

If Eve opens a box, write down the bit you see (heads = 0, tails = 1) and which box you opened.

If Eve choose to not open a box, pretend to so Alice and Bob remain suspicious, and write down an "X" to indicate a missed bit.



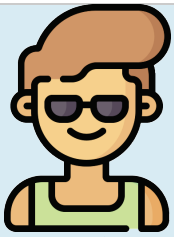
Moderator

If Eve **opened a box**, shake the box that Eve did **not** open to randomize it.

If Eve did nothing, do not shake either box.

Take both boxes **gently** to Bob, giving him the open to look in one box of his choosing. Do not let Bob look in the second box!

The moderator is playing the role of quantum mechanics in this simulation. By shaking the box, they're simulating the disturbance caused by Eve's measurement!



Bob

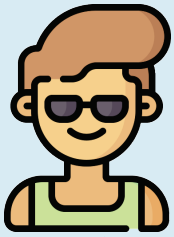
Bob flips a coin to choose which **basis** to measure in.
If heads, open the HV box. If tails, open the ±45° box.

Write down the bit value and which box you chose on the worksheet.

Repeat up to fifty times to build a secret key!

POST-PROCESSING

Alice



Bob

Basis reconciliation

Once all “photons” have been exchanged, Alice announces loudly the basis she used for each step, being careful not to reveal the bit.

For each basis announced, Bob will say “keep” if he measured in the same basis, and “discard” if he measured in a different basis.

Alice, Bob, and Eve will keep the bits they measured if Alice and Bob’s measurements agree, and cross out the bits where Alice and Bob disagreed.

The bits that remain are the sifted key! Alice and Bob should write it out separately.

Error estimation and eavesdropper detection

Alice and Bob publicly compare one third of the bits from their sifted key, chosen at random (for example, every third bit).

If all of these bits are the same, they can conclude that the eavesdropper was not listening in. They discard the bits they compared publicly and use the rest as their **secret key** to encrypt a message (try sending a binary number perhaps).

If those bits have errors, they can conclude that Eve was listening in! Their secret key isn’t so secret after all.

Quantum Concepts

1. In quantum mechanics, why can't we measure in two bases (open both boxes) at the same time?

   In quantum mechanics, when we measure in one basis, the state collapses to be one of the two (for example, H or V if we measure in the HV basis). The results of the next measurement depend on what state we measured after the last measurement, not on what it may have originally been.

   Since there is only one quantum system (e.g. one photon), we can't make two measurements on it at the same time. We have to make them one after the other, and whenever we make one measurement, we disturb the potential result of the other.

2. Eve was allowed to freely choose which box/basis to measure in, but Alice and Bob had to choose randomly. What could Eve have done if they knew which basis/box Alice and Bob would choose?

   If Eve knew a pattern to Alice or Bob's basis choices, Eve could always choose the same measurement as one of them. If Eve makes the same measurement as Alice and Bob, they won't disturb the system, and thus won't be detected. It is therefore important that Alice and Bob's choices are completely unpredictable.

Answer Key

Written by Martin Laforest
Published by the **IQC Scientific Outreach team**
Contact: iqc-outreach@uwaterloo.ca

Institute for Quantum Computing
University of Waterloo
200 University Ave. W.
Waterloo, ON, Canada, N2L3G1

About IQC
The Institute for Quantum Computing (IQC) is a world-leading research centre in quantum information science and technology at the University of Waterloo. IQC's mission is to develop and advance quantum information science and technology through interdisciplinary collaboration at the highest international level. Enabled by IQC's unique infrastructure, the world's top experimentalists and theorists are making powerful new advances in fields spanning quantum computing, communications, sensors and materials. IQC's award-winning outreach opportunities foster scientific curiosity and discovery among students, teachers and the community.

uwaterloo.ca/institute-for-quantum-computing

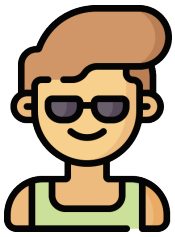
Worksheet (Alice)

	Communication		Post-processing
	Raw key		
	Bit (0, 1)	Basis (H/V, ±45°)	Keep bit?
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

	Communication		Post-processing
	Raw key		
	Bit (0, 1)	Basis (H/V, ±45°)	Keep bit?
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			
36			
37			
38			
39			
40			
41			
42			
43			
44			
45			
46			
47			
48			
49			
50			

Sifted Key: _____

Secret Key: _____



Answer Key

Worksheet (Bob)

	Communication		Post-processing
	Raw key		
	Bit (0, 1)	Basis (H/V, ±45°)	Keep bit?
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

	Communication		Post-processing
	Raw key		
	Bit (0, 1)	Basis (H/V, ±45°)	Keep bit?
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			
36			
37			
38			
39			
40			
41			
42			
43			
44			
45			
46			
47			
48			
49			
50			

Sifted Key: _____

Secret Key: _____

Worksheet (Eve)

	Communication		Post-processing
	Raw key		
	Bit (0, 1)	Basis (H/V, ±45°)	Keep bit?
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

	Communication		Post-processing
	Raw key		
	Bit (0, 1)	Basis (H/V, ±45°)	Keep bit?
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			
36			
37			
38			
39			
40			
41			
42			
43			
44			
45			
46			
47			
48			
49			
50			

Sifted Key: _____

Secret Key: _____