Lesson

WAVE-PARTICLE DUALITY, REVISITED

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SC
Schrödinger's
Class

Outline

WAVE-PARTICLE DUALITY, REVISITED

ACTIVITY GOAL:
Consider and test both wave and particle behaviour in a single experiment.

LEARNING OBJECTIVES

Wave and particle behaviour of quantum objects.

Quantum interference.

Probabilities and photons.

Using superposition to build quantum sensors.

ACTIVITY OUTLINE

We start by describing the **beam splitter**, and how we expect it to behave with a wave or particle picture of light.

CONCEPT:
Different models result in different behaviour.

We then test our model of **coincidence counting** first with a group activity and then by comparing real lab data, showing that the photon model is necessary to explain it.

CONCEPT:
When measured, a photon must be in one place or another.

Finally, we introduce the **Mach-Zehnder Interferometer**, showing that the wave model is necessary to explain other experiments.

CONCEPT:
Whether we see wave-like or particle-like behaviour depends on how we measure and analyze our data.

PREREQUISITE KNOWLEDGE

Light as an electromagnetic wave
Probabilities

SUPPLIES REQUIRED

Two coins
Wave and Particle Worksheets
Mach-Zehnder interferometer model with waves

Lesson

WAVE-PARTICLE DUALITY, REVISTED

WAVE-PARTICLE EXPERIMENTS

In the classical world, we think of particles and waves as two very different things physically, defined by certain properties:

PARTICLES	WAVES
Exist at one place (localized)	Exist over a large space (delocalized)
Have well-defined properties like mass and volume	Have well-defined properties like wavelength and frequency
Have kinetic collisions	Show wave interference
Are countable	Are continuous

Our understanding of the wave-particle duality of light stems from a few major experiments. We see interference in the double-slit experiment, indicating that light behaves like a wave. We see that photons provide energy to electrons one-at-a-time in the photoelectric effect experiment, indicating that lights behaves like a particle. Other experiments, like Newton's rings or Compton scattering, also show either wave or particle behaviour.

These experiments are elegant and cornerstones in the development of quantum mechanics, but at a glance they seem completely unrelated to each other. It's easy to conclude from this that light behaves like a wave in some experiments, and like a particle in others.

In this lesson, we'll investigate one experiment that shows **both** wave and particle behaviour, the **Mach-Zehnder Interferometer**. Which behaviour we see depends on what kind of **quantum measurement** we make. This experiment reveals that quantum objects like photons are not "sometimes a particle, and sometimes a wave". Rather, the classical idea that waves and particles are distinct things has to be completely re-imagined.

THE BEAM SPLITTER: QUANTUM vs CLASSICAL

We'll start with a common tool in optics called a **beam splitter**. As its name suggests, a beam splitter "splits" a beam of light. If you shine light on a beam splitter, some of it will be transmitted, and some of it will be reflected. There are two input ports and two output ports.

A special example of a beam splitter is the 50/50 beam splitter, seen below. This beam splitter splits the incoming intensity in half: 50% of the incoming light is transmitted and 50% is reflected.
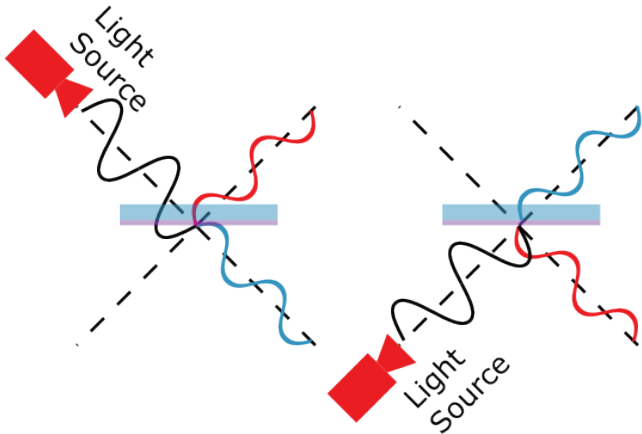

A 50/50 beam splitter splits the intensity of light in half; from either input port, half of it is transmitted, **and** the other half reflected.

(Note: The wave colours are to track reflected and transmitted beams, and do not represent the colour of the light)

When considering how laser light behaves at the beam splitter, we can use the wave picture of light. The wave has energy, which must be conserved. Half of the energy is transmitted, and half of the energy is reflected. We're then left with two beams, each with half of the original energy.

What if we send a single photon into the beam splitter instead? Unlike a laser beam, a photon can't split in two, with half a photon going in each direction. Instead of dividing in energy, a single photon divides in **probability**. At a 50/50 beam splitter, the photon has a 50% probability of being reflected, and a 50% probability of being transmitted.

How can measure this effect in an experiment? Let's put a single-photon detector in both the transmitted and reflected port. Single-photon detectors don't measure the intensity of light, but instead fire whenever they see any light. We then send in a stream of single photons, one at a time, and look for **coincidence detections**, meaning that both detectors fire at the exact same time.
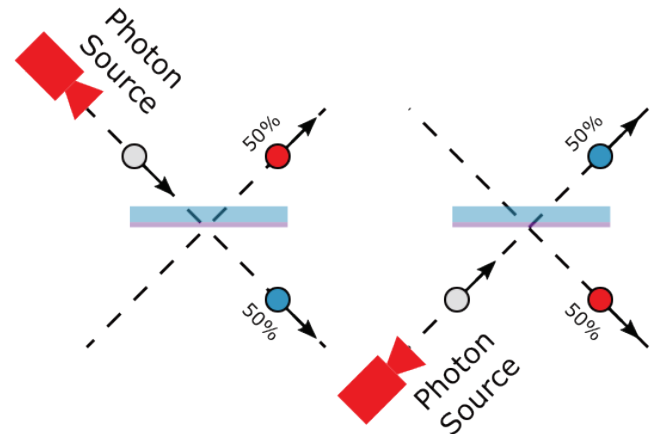
If we send a single photon of light into the beam splitter, it is **either** reflected or transmitted, with a 50% **probability** each.
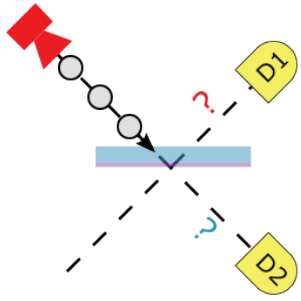
1. If we indeed only send one photon in at a time, how often will both detectors fire at the exact same time?

2. If we instead think of the light as a wave, and the detectors are sensitive to even the smallest amount of light energy, how often should the detectors fire at the same time?

A train of single photons are split at a beam splitter and detected in coincidence.

If a photon is split at a beam splitter followed by two detectors, we should **never** see a coincidence detection where both detectors fire at the same time. Since there is only one photon, it is either transmitted **or** reflected, not split in two. We can't predict which detector will fire (each has a 50% probability of firing), but can guarantee that only one of them will.

For a fair comparison, we need to consider other possible models for how our photon detectors respond to waves. Let's say that the wave was indeed split in two, and the amount of energy in the wave is just high enough that the detector has a 50% chance of firing. What kind of coincidence detections do we expect to see, and how does the real data from the experiment look?
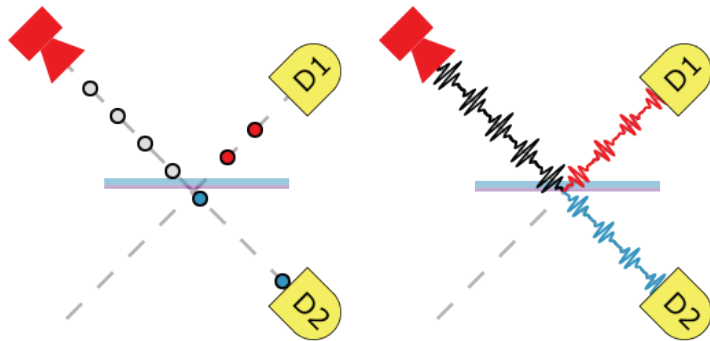
A comparison of photon and wave models of the beam splitter experiment, where we send a stream of pulses to the beam splitter.

In the photon model (left), the photon randomly hits either Detector 1 (D1) **or** Detector 2 (D2)

In the wave model (right), the pulse is split into two pulses with half the energy each, which go to **both** Detector 1 and Detector 2.

ACTIVITY: COINCIDENCE COIN FLIPPING

For the following activity, use the Coincidence Detections worksheets and two coins. In each worksheet, we'll fire 30 pulses at the beam splitter, and look for coincidences.
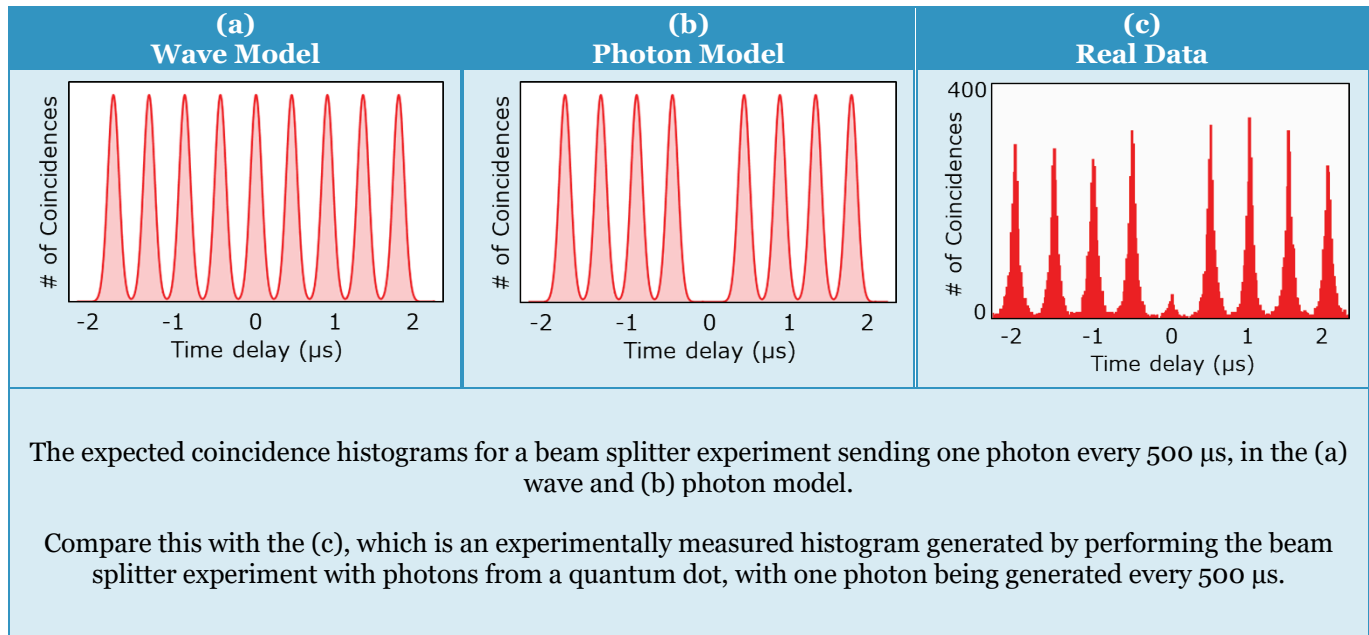
1. In the **wave model**, each detector has a 50% chance of firing. For each pulse, flip two coins. If the first coin is heads, mark an "X" in Detector 1 column. If tails, leave it blank. If the second coin is heads, mark an "X" in the Detector 2 column. If tails, leave it blank.
   Repeat for all 30 pulses.

2. Check for **coincidences** using the columns on the right. In each column, mark an X when:
   a   Detector 1 fires and then, two pulses later, Detector 2 fires
   b   Detector 1 fires and then Detector 2 fires on the next pulse
   c   Detector 1 and 2 fire at the same time
   d   Detector 2 fires and Detector 1 fires on the next pulse
   e   Detector 2 fires and then, two pulses later, Detector 1 fires.

3. Tally the coincidences in each column and make a histogram with respect to time delay.

4. In the **photon model**, each detector has a 50% chance of firing, but only one can fire at a time. Model this by flipping **one** coin. If its heads, Detector 1 fires; if its tails, Detector 2 fires. Repeat for 30 pulses, and count coincidences and make a histogram just as in the wave model.

5. What is the probability that the two detectors fire at the same time in the wave model?
   What about in the photon model?

6. What is the probability that Detector 1 fires and, on the next pulse, Detector 2 fires in the wave model?
   What about the photon model?

7. In the photon model, what would the probability be of both detectors firing at the same time if we sent a pulse containing two photons (i.e. two photons arrive at the beam splitter at the same time)?

After the activity, you should have two histograms showing coincidence detections as a function of time delay, one for the wave model and one for the particle model. The should look something like images (a) and (b) below, assuming the photon source emits a pulse every 500 µs.

In plot (c), we show real data from an experiment that generated single photons ever 500 µs from a special light source called a **quantum dot**, and measured for coincidences after a beam splitter.

(a) Wave Model	(b) Photon Model	(c) Real Data

The expected coincidence histograms for a beam splitter experiment sending one photon every 500 µs, in the (a) wave and (b) photon model.

Compare this with the (c), which is an experimentally measured histogram generated by performing the beam splitter experiment with photons from a quantum dot, with one photon being generated every 500 µs.

1. From the real data, can you conclude if the wave or photon model is correct? Which wave or particle behaviours does it demonstrate?

2. Why are there coincidence detections when the time delay is not zero in the data?

3. Make a hypothesis for why the coincidences aren't exactly zero in the real data at 0 µs delay.

PHASE AFTER A BEAM SPLITTER

We saw that, in the coincidence-counting experiment, the photon model provided a better explanation for the data. But is that the only experiment we can do with a beam splitter?

When testing for wave behaviour, we need to consider both the amplitude and frequency of the wave as well as its **phase**, which tells us which part of the cycle the wave is in. In particular, we need to understand what happens to the phase of light after a beam splitter.

When light is **transmitted**, it keeps its phase; if it was on its way to becoming a peak, it continues to do so. When light is **reflected**, the phase depends on the **index of refraction** of the material is travels through and surface it reflects from. If the waves are reflected from surface with a higher index of refraction than the one they are travelling in (for example, a reflection off of glass in air), they experience a phase shift of π (180°), turning peaks into troughs. Otherwise, there is no phase shift.

A beam splitter is made of a very thin, semi-transparent dielectric coating deposited on a piece of glass. The index of refraction of the dielectric, n_d, is higher than that of air, n_a, and smaller than that of glass, n_g.

When light hits the glass first, the reflection is from the coating into the glass. Since $n_d < n_g$, there is no phase shift. When light hits the coating first, the reflection is from the coating into air. Since $n_d > n_a$, the light picks up a π phase shift.

No matter how the beam splitter is constructed, there will be a π phase difference between a reflected beam from the top port, and a reflected beam from the bottom port.
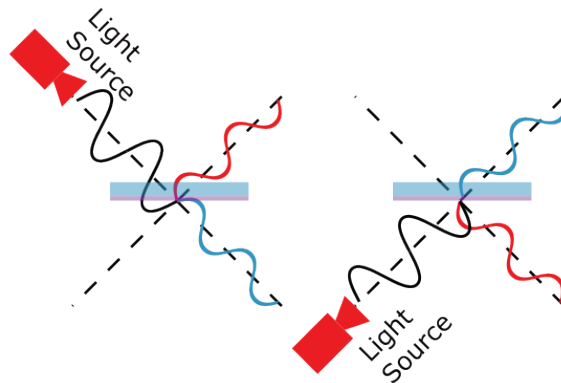
No matter what port the light comes from, the transmitted light always passes in phase.

If the light hits the glass (blue) before the dielectric (purple), as in the left image,
it will experience no phase change ($n_d < n_g$).

If the light hits the dielectric directly from air, as in the right image,
it will pick up a π phase shift ($n_d > n_a$).

To see the phase shift clearly, note that in both images, the incoming light wave (black) has finished a trough and is "going up" when it hits the beam splitter. In the left image, the reflected wave (red) continues to become a peak after reflection. However, in the right image, it flips and becomes a trough after reflection.

THE MACH-ZEHNDER INTERFEROMETER

For the next experiment, we will use two beam splitters and two detectors in the following configuration:
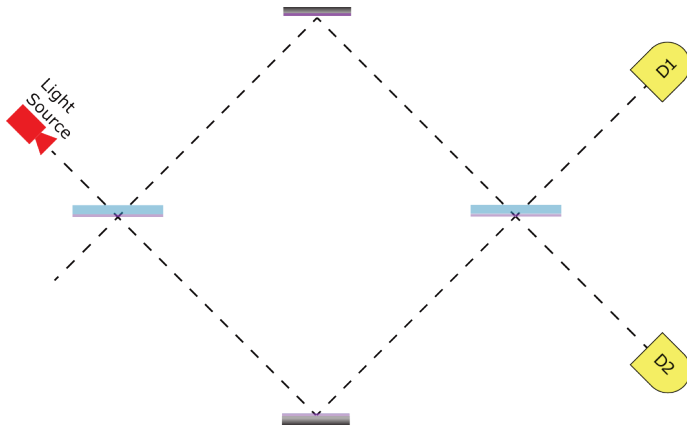
The Mach-Zehnder Interferometer (MZI), consisting of two beam splitters, two mirrors, and two photon detectors D1 and D2.

This is known as the Mach-Zehnder interferometer (MZI). The light is split at the first beam splitter, and mirrors are use to redirect both the transmitted and reflected light to a second beam splitter. The light is then detected after the second beam splitter.

The phase of the light in the interferometer can be adjusted by either moving the bottom mirror or inserting a piece of glass, sometimes called a **phase shifter**.

1. Assuming that light is a particle, and is transmitted or reflected with a 50% probability at each beam splitter, what is the probability of measuring a photon at D1? What about D2?

2. Using the wave model of light, what percentage of the original light intensity do you predict will arrive at D1 and D2? Will it be equal?

If you have printed Mach-Zehnder interferometer models, follow the instructions on the next page.

If you do not have printed Mach-Zehnder interferometer models, skip to a step-by-step walkthrough on Page 11.

ACTIVITY: THE MACH-ZEHNDER MODEL

1. Verify your prediction by placing wave sections on the model Mach-Zehnder Interferometer. Use "Path 1" for the bottom path, and remember:
   - There is no phase change for transmitted light.
   - If the wave hits the glass first then the coating, it has no phase shift.
   - When the wave hits the coating in air, it experiences a π phase shift.
   - At a mirror, there is also a π phase shift.
   - Waves can interfere with each other.

2. What percentage of the light reaches D1? What about D2?

3. Repeat the experiment but use "Path 2" and "Path 3" instead. Explain your results.

4. Phase shifts can also be seen when inserting a piece of glass in one path. Repeat the experiment using the π phase shift in the top path to replace one of your wave pieces. What do you notice?

5. Replace a second wave piece in the top path with a π phase shift. What do you notice?

6. What happens if you insert two π phase shifts in different arms of the interferometer (one in the top and one in the bottom path)?

THE MACH-ZEHNDER INTERFEROMETER, STEP BY STEP

At the first beam splitter, the light is either transmitted or reflected. Let's examine what happens when we block either the transmitted or reflected "arm" of the interferometer, being careful to flip the phase by π whenever reflecting in air:
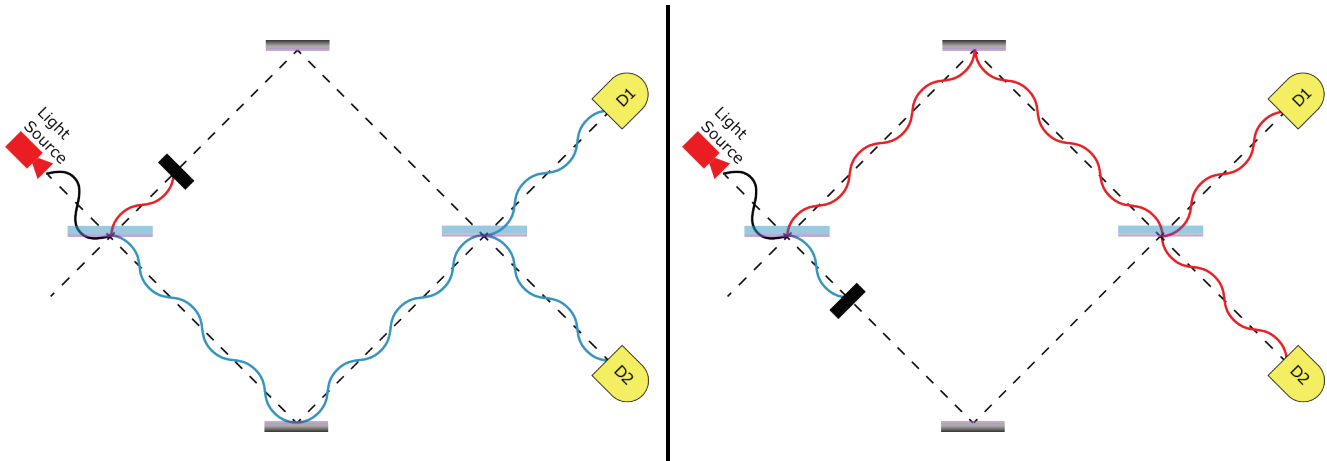
In both cases, the light then reaches the second beam splitter, where it is split 50/50 again. Half of the intensity reaches D1, and half of the intensity reaches D2.

If we remove the blocks, the light originally transmitted meets the light that was originally reflected at the second beam splitter. Overlaying the two possibilities (as below on the left), we see **constructive interference** leading to D1, and **destructive interference** leading to D2. Therefore, D1 will detect 100% of the original light energy, and D2 will detect nothing.
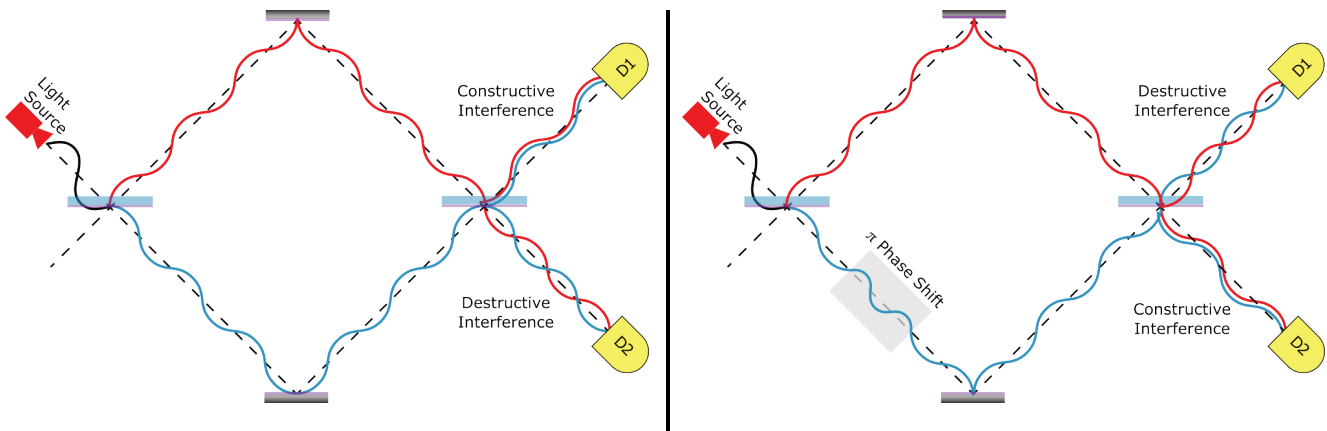
We can change the **phase** of the light by moving one of the mirrors, or by inserting pieces of glass into one of the arms of the interferometer. In the right image, we insert a glass π phase shifter, which adds an extra half of a wave to the transmitted arm. We see that this changes which detectors experience constructive and destructive interference; now 100% of the light arrives at D2, and none at D1.

PHOTONS AND INTERFEROMETERS

In our model, we assumed that light behaves like a wave. But what happens if we send photons in one-at-a-time? As before, we'll compare with real experimental data. In this experiment, researchers sent a single photon into a Mach-Zehnder Interferometer, and measured the number of photons detected at D1 and D2, while adjusting the path difference to control the phase:
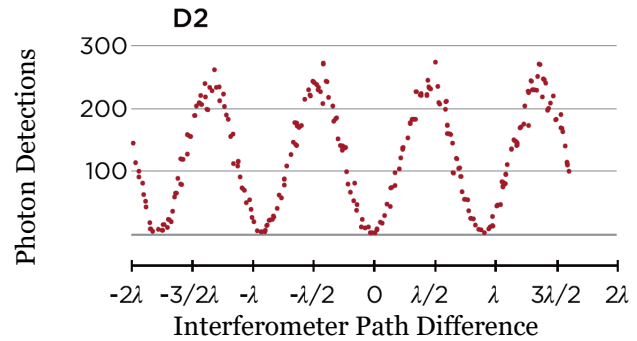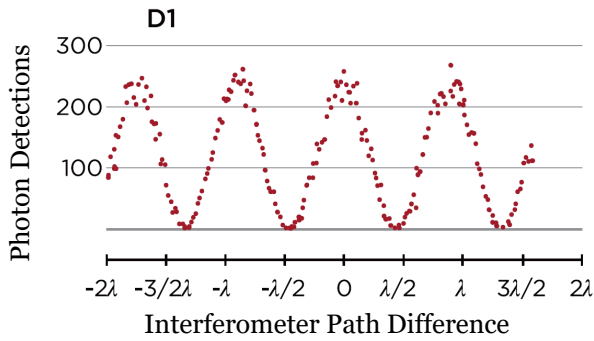

1. Do these results show wave or particle behaviour? Explain why, using elements from the table in the background section.

2. What is the probability of measuring a photon at detector D2 when the probability of measuring a photon at D1 is maximized or minimized? Explain why.

3. Discuss the role played by interference in the probability of measuring light at each detector.

4. Since we sent in one photon at a time, will D1 and D2 ever both fire at the same time?

5. Combining the results of the beam-splitter coincidence experiment and the single-photon Mach-Zehnder interferometer experiment, can you conclude if light is a wave or particle?

BEYOND LIGHT

The constructive and destructive interference seen in the Mach-Zehnder interferometer is not only for light. Quantum mechanics predicts that everything is a particle with wave-like properties. Quantum interference has been demonstrated for electrons, neutrons, bucky balls (C60) and even larger molecules. For the last decade, scientists have been trying to perform this interference experiment with ever larger molecules. Here are a few experimental results from researchers in Vienna, Austria and collaborators:
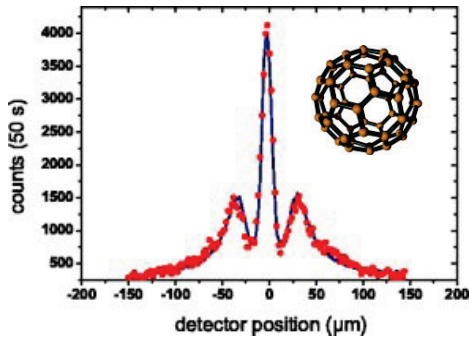
M. Arndt et al., *Wave-particle duality of Carbon-60 molecules*. Nature **401**, 680 (1999).
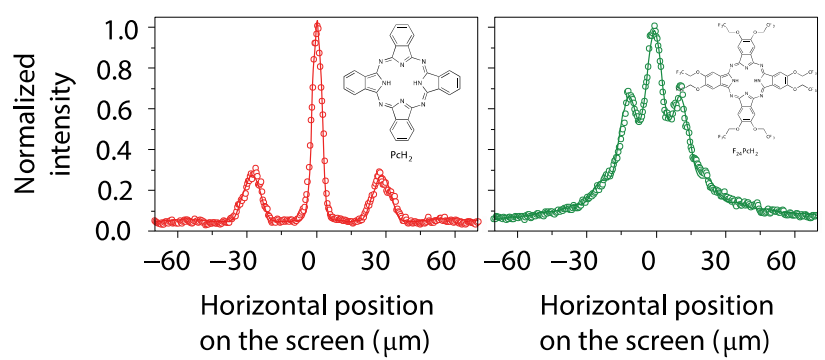
T. Juffman et al., *Real-time single-molecule imaging of quantum interference*. Nature Nanotechnology 7, 297 (2012).

APPLICATIONS AND TECHNOLOGIES

In this experiment, we saw that the way we measure a quantum system determines whether it displays wave or particle behaviour. While often times we're only concerned with one type of behaviour at a time, many quantum technologies require us to consider both at once. For example, **electron microscopes** use the wave properties of accelerated electrons to image with resolutions down to 50 picometres. But the focusing techniques for electron microscopes use shaped electric fields, still requiring us to think of electrons as charged particles.

We can connect the MZI experiments to our previous explorations of **quantum cryptography**, which required us to measure in two different **bases**. The two bases for the MZI are represented by the path the photon is in (top or bottom), and different phases of the superposition state. The beam splitter is the tool that lets us switch between these basis states, converting a definite position state (top or bottom) to a superposition state, and through interference, converting a superposition state back to a definite position state.

In the *A Simple Quantum Algorithm* activity, we'll see how these exact properties can be used to process information in entirely new ways, harnessing superposition and quantum measurement to enable **quantum computing**.

QUANTUM QUIZ

1. In the single beam splitter experiment, what are the differences between the expected behaviour if we model the light as a wave or as particles? When using a single-photon source, what model most closely resembles the data?

2. In the Mach-Zehnder interferometer experiment, what are the differences between the expected behaviour if we model the light as a wave or as particles? When using a single-photon source, what model most closely resembles the data?

QUANTUM CONCEPTS

1. We can cancel out the effect of a π phase shift by inserting a π phase shift into the other arm of the interferometer, as seen below. Why does putting a phase in the other arm allow this?
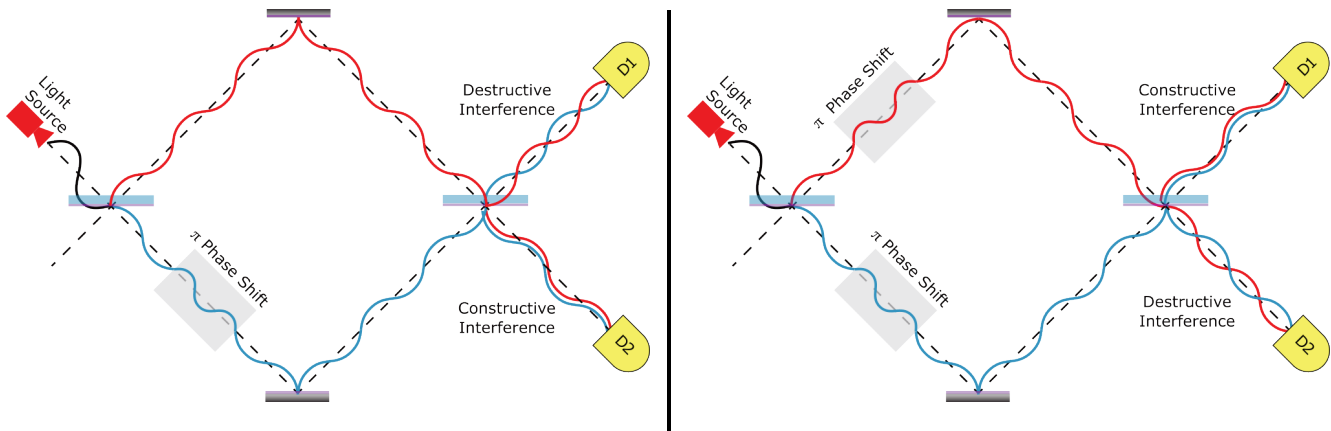
2. How can an MZI be used to measure the thickness or index of refraction of a transparent object? What is the sensitivity of the interferometer, approximately?

QUANTUM CONNECTIONS

Try the following questions if you also completed the *Two Golden Rules of Quantum Mechanics* lesson.

1. Which quantum state is mutually exclusive with a photon in the bottom arm of a Mach-Zehnder interferometer?

2. After the first beam splitter, the photon is in a superposition of the top and bottom paths. Which quantum state is mutually exclusive with this state? Remember that, for two states to be mutually exclusive, there must exist a measurement that can perfectly tell them apart.

3. Can you relate the photon's behaviour in the Mach-Zehnder interferometer to quantum superposition and quantum measurement (the Two Golden Rules of Quantum Mechanics)?

QUANTUM LEAP: CHALLENGE QUESTIONS

1. Let us reconsider an interferometer balanced such that the photon always ends up at D1. Suppose you place a detector, D3, on the bottom arm just before the second beam splitter. If you send a single photon through the interferometer, but D3 does not fire, which of D1 or D2 will detect the photon? Explain your answer.

2. Constructive and destructive interference are also key to the double-slit experiment. Discuss how the Mach-Zehnder Interferometer is related to the double-slit experiment.

3. If we send a pulse containing a specific number of photons "N" to a 50/50 beam splitter, what is the probability of measuring a coincidence detection? Recall that photon detectors don't measure the number of photons, but only whether there were any photons or no photons that hit the detector. Try starting with N=2 and N=3 photons and generalizing from there.

GLOSSARY

- A **beam splitter** is an optic that partially transmits and partially reflects light, like a semi-transparent mirror. We're often interested in **50/50 beam splitters**, where the probability of transmission and reflection are equal.

- A **photon detector** is a device that detects whether a photon is present or not. Most detectors cannot measure the number of photons present, and fire whenever they see any photons at all. The detector not firing implies that no photons are present.

- A **coincidence detection** or **coincidence count** occurs when two photon detectors fire at the same time, implying that a photon was detected by both at the same time.

- A **quantum dot** is a nano-engineered quantum system that behaves like a single atom. After exciting it to a higher energy state (like a higher orbital in an atom), it will decay back to the ground state, emitting precisely one photon in the process. In this way, we can use the quantum dot as a single-photon source.

- A **phase shift** is a delay of a wave expressed as a fraction of the wavelength, λ. A **π phase shift** delays the wave by half a period, converting constructive interference to destructive interference.

- **Interference** is a wave phenomenon where two or more wave sources add together, growing larger when their peaks and troughs line up in phase (constructive interference) and cancelling each other out when out of phase (destructive interference).

About IQC
The Institute for Quantum Computing (IQC) is a world-leading research centre in quantum information science and technology at the University of Waterloo. IQC's mission is to develop and advance quantum information science and technology through interdisciplinary collaboration at the highest international level. Enabled by IQC's unique infrastructure, the world's top experimentalists and theorists are making powerful new advances in fields spanning quantum computing, communications, sensors and materials. IQC's award-winning outreach opportunities foster scientific curiosity and discovery among students, teachers and the community.

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	Detector 1	Detector 2	Coincidences				
			(a) X . / . . / . X	(b) X . / . X	(c) . . / X X	(d) . X / X .	(e) . X / X .
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
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24							
25							
26							
27							
28							
29							
30							
	TOTAL:						

Build Histogram

	Detector 1	Detector 2	Coincidences				
			(a)	**(b)**	**(c)**	**(d)**	**(e)**
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
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25							
26							
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28							
29							
30							
		TOTAL:					

Build Histogram