Lesson

QUANTUM COMPUTING WITH INTERFEROMETERS

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SC Schrödinger's Class

Outline

QUANTUM COMPUTING WITH INTERFEROMETERS

ACTIVITY GOAL:
Use the Mach-Zehnder interferometer's quantum properties to solve a logical problem efficiently.

LEARNING OBJECTIVES

- Measuring quantum interference.
- Classical vs. quantum algorithms.
- Computational efficiency.

ACTIVITY OUTLINE

We start by outlining the Deutsch-Josza problem, a logic game whose computational efficiency can be analyzed.

CONCEPT:
Different problems may be fundamentally more or less difficult for a computer to solve.

We then put the Deutsch-Josza problem in a quantum context, showing that a quantum computer can solve it with fewer steps.

CONCEPT:
Quantum computers can be used to solve problems in different ways than classical computers.

Finally, we take a step back to consider what features of quantum mechanics were needed to obtain the advantage.

CONCEPT:
Quantum computers take advantage of superposition for their advantage, but also need to be measured the right way.

PREREQUISITE KNOWLEDGE

The *Wave-Particle Duality Revisited* lesson
Mach-Zehnder Interferometers

SUPPLIES REQUIRED

Mach-Zehnder interferometer model with waves (optional)

Lesson

QUANTUM COMPUTING WITH INTERFEROMETERS

THE DEUTSCH-JOSZA PROBLEM

In quantum computing, we try to harness the remarkable behaviours of quantum mechanics to build quantum algorithms that solve problems in a more efficient way, e.g., using less memory, or by performing fewer operations. To understand the inner workings of a quantum algorithm, one requires a deeper understanding of the mathematics of quantum mechanics. Fortunately, we can use the Mach-Zehnder interferometer to give us a taste. In this activity, you'll investigate how the single particle Mach-Zehnder interferometer can be used to solve a simple problem more efficiently than the best possible classical algorithm.

The Deutsch-Josza quantum algorithm, though of very little practical use, is one of the first, and simplest, quantum algorithms that shows the power of harnessing quantum mechanics for computational use. It solves the following problem:

The Deutsch-Josza Problem

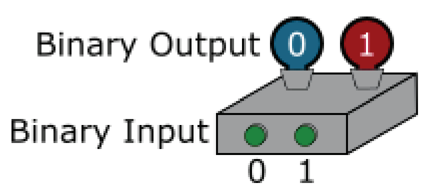
Your friend Alice knows a function $f(x)$ that accepts a single bit (0 or 1) as input, and produces a single bit (0 or 1) as output.

Alice refuses to tell you what $f(x)$ is, but she's willing to let you "query" it: you can give Alice any input bit x, and she'll give you back the corresponding output bit $f(x)$.

Your task is to determine whether the function $f(x)$ is **constant**, meaning it always outputs the same bit regardless of the input, or not.

The Deutsch-Josza Problem is defined in terms of an abstract function that accepts one bit of input and provides one bit of output. This means that you have one of two possible input choices, labelled "0" and "1", and depending on your choice, one of two possible things will happen, also labelled "0" and "1".

The analysis of the problem shouldn't depend on exactly how the function is built, so we can imagine it occurring with the box seen to the left, which has two buttons on the front that you can push to turn on one of two light bulbs.



A model of the Deutsch-Josza setup with binary input (two buttons) and binary output (two lights).

With two inputs and two outputs, there are only four possible functions:

<table>
<tr><th>Function</th><th>$f_1(x)$</th><th>$f_2(x)$</th><th>$f_3(x)$</th><th>$f_4(x)$</th></tr>
<tr><td>Truth Table</td>
<td><table><tr><th>x</th><th>$f_1(x)$</th></tr><tr><td>**0**</td><td>0</td></tr><tr><td>**1**</td><td>0</td></tr></table></td>
<td><table><tr><th>x</th><th>$f_2(x)$</th></tr><tr><td>**0**</td><td>0</td></tr><tr><td>**1**</td><td>1</td></tr></table></td>
<td><table><tr><th>x</th><th>$f_3(x)$</th></tr><tr><td>**0**</td><td>1</td></tr><tr><td>**1**</td><td>0</td></tr></table></td>
<td><table><tr><th>x</th><th>$f_4(x)$</th></tr><tr><td>**0**</td><td>1</td></tr><tr><td>**1**</td><td>1</td></tr></table></td>
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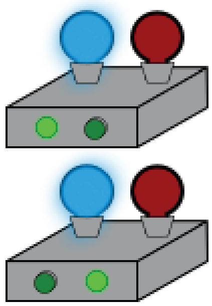
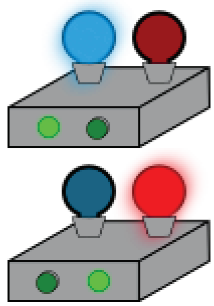
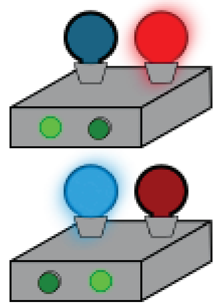
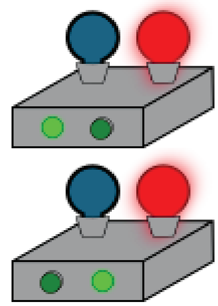
<tr><td>Box Diagram</td>
<td></td>
<td></td>
<td></td>
<td></td>
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In the Deutsch-Josza game, Alice programs one of these four functions into a box. It is your task to figure out which of the four functions Alice programmed.

1. What is the minimum number of queries (button pushes) that you would need to test to determine **exactly** which function Alice programmed into the device?

2. Which functions are constant? Which are not?

3. If you query the function with the input “0” and find the output “1”, can you conclude whether the function is constant or not?

4. What is the minimum number of queries that you’ll need to determine whether the function is constant or not?

THE QUANTUM APPROACH

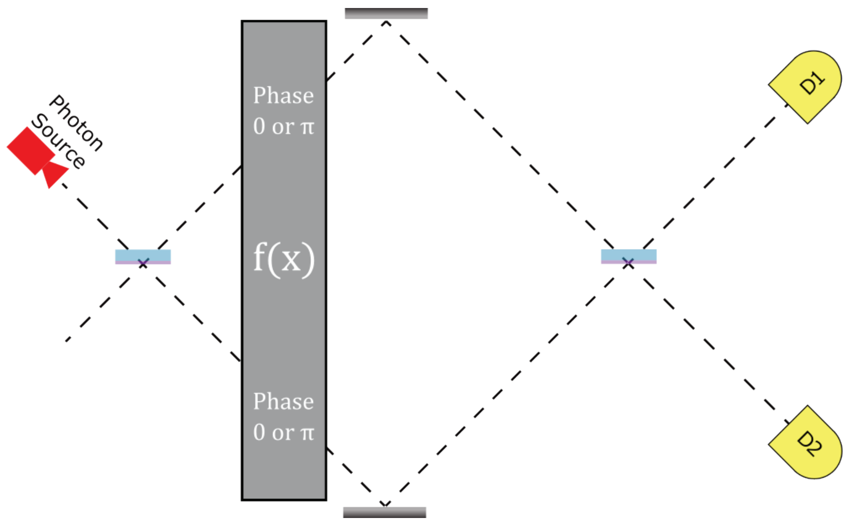
By analysing the problem, we found that we needed two queries to answer whether the function was constant or not. But, with two queries, we can answer even more than that: we can tell exactly which function Alice programmed into the box.

Is there a way to answer whether the function is constant or not without knowing everything about the function? Classically, this is impossible, but if we build the box in such a way that we can take advantage of **quantum mechanics**, it may be possible.

Let's say we convince Alice to build the function as a box with two possible phase shifts, which we can query with in a Mach-Zehnder interferometer. One query corresponds to sending **one** photon into the interferometer.

In the **top** arm, Alice places a π phase shift if $f(0) = 1$, and nothing if $f(0) = 0$.

In the **bottom** arm, Alice places a π phase shift if $f(1) = 1$, and nothing if $f(0) = 0$.

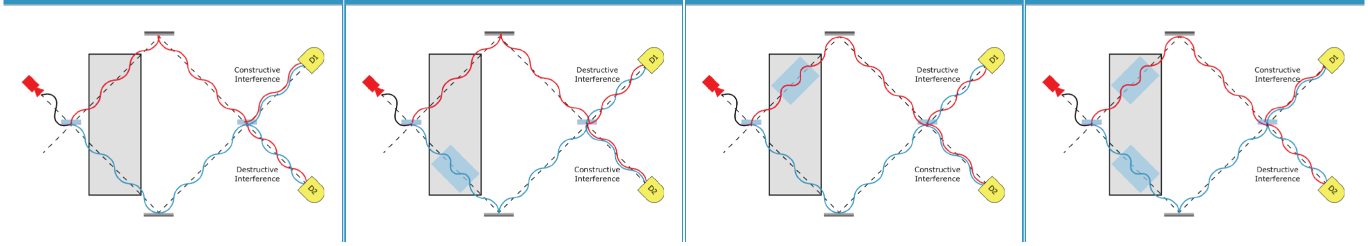


Use your Mach-Zehnder Interferometer model with π phase shifts to answer these questions.

1. For each of the four possible functions, where would we detect a photon?
2. How many photons do we need to send in to answer whether the function is constant or not?

By firing one photon at the beam splitter, it transforms into a **quantum superposition state** and queries the box that Alice built **in superposition**. By using our wave-picture of a photon, we find the following can happen depending on the function that Alice used:

$f_1(0) = 0, f_1(1) = 0$	$f_2(0) = 0, f_2(1) = 1$	$f_3(0) = 1, f_3(1) = 0$	$f_4(0) = 1, f_4(1) = 1$



If the function is constant (functions 1 and 4), the two paths experience the same phase, and at the second beam splitter constructively interfere towards Detector 1. If the function is not constant, one of the two paths experiences a π phase shift, and the constructive interference occurs in Detector 2 instead.

Using any classical approach, the Deutsch-Josza problem required two queries to solve. However, by using a quantum algorithm, taking advantage of the superposition and measurement principles, we can solve the Deutsch-Josza problem with one single photon!

QUANTUM COMPUTERS OF THE FUTURE

The Deutsch-Josza algorithm was one of the earlier quantum algorithms that showed an advantage for solving a problem with quantum tools rather than classical ones. The problem is not necessarily a useful one, but thinking about it allows quantum computer scientists to come up with other quantum algorithms that use the same principles to solve real-world problems. For example, Shor's Algorithm was proposed two years later, tackling a much more relevant problem: finding the prime factors of very large numbers.

It's important to note that quantum computers are not better at solving **all** problems than classical computers. Quantum computers are exponentially better at finding the factors of numbers, but are no better at all at multiplying two numbers together. Finding problems that can be solved more efficiently by a quantum computer is still a very active field of research!

While photonic interferometers are one path toward building quantum computers, there are many other promising ways that are being researcher by scientists and engineers around the world. Some examples include superconducting circuits, the energy levels of trapped atoms, and the spin state of quantum dots. While most of the benefits of quantum computers are still in the future, significant progress has been made in the last twenty years in making the building blocks of this quantum technology.

QUANTUM CONCEPTS

1. If, in the quantum Deutsch-Josza algorithm, we detect a photon at D1, can we determine whether Alice programmed $f_1(x)$ or $f_4(x)$? Does the quantum algorithm give us arbitrary power to solve any possible problem Alice can pose to us?

1. Discuss how the quantum superposition principle and interference help you beat the best possible classical algorithm for the Deutsch-Josza problem by discussing which path the photon took.

2. If you measure the photon immediately before the second beam splitter, can you infer whether the function is constant or balanced? Discuss how the choice of quantum measurement is important in solving the Deutsch-Josza problem.

QUANTUM LEAP: CHALLENGE QUESTION

1. How could you ask Alice to build her function if you wanted to probe it with the polarization of single photons rather than in a Mach-Zehnder interferometer?

GLOSSARY

- A **constant function** is a function that always outputs the same answer no matter the input, for example $f(x) = 1$.
- A **query** is a question that we ask a function. For example, we query the function $f(x)$ with the value $x = 1$ to get the answer $f(1)$. A more intuitive example may be a search engine, where we query with a phrase (e.g. `cat videos`) and receive a desired result.
- When discussing algorithms, **efficiency** defines the number of steps or queries needed to solve a problem. The best possible efficiency may be different between classical and quantum computers. This is closely related to the idea of **complexity**, which describes how the problem grows as more inputs and outputs are added to it.
- A **quantum computational advantage** exists if we are able to answer a question more efficiently (with fewer queries) on a quantum computer than if we have a classical computer.

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About IQC
The Institute for Quantum Computing (IQC) is a world-leading research centre in quantum information science and technology at the University of Waterloo. IQC's mission is to develop and advance quantum information science and technology through interdisciplinary collaboration at the highest international level. Enabled by IQC's unique infrastructure, the world's top experimentalists and theorists are making powerful new advances in fields spanning quantum computing, communications, sensors and materials. IQC's award-winning outreach opportunities foster scientific curiosity and discovery among students, teachers and the community.

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