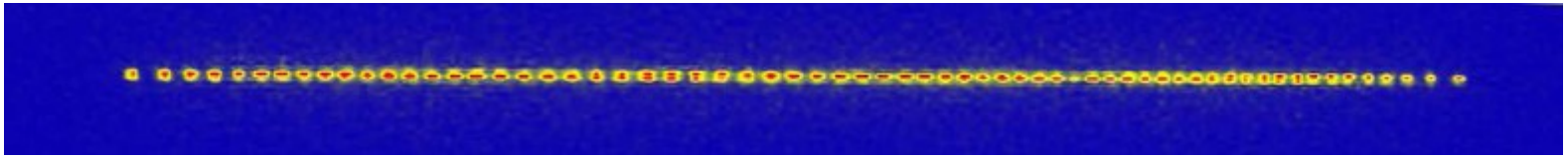


Quantum Metrology with highly entangled states and realistic decoherence

Bill Coish McGill

Department of Physics, McGill University, Montréal QC

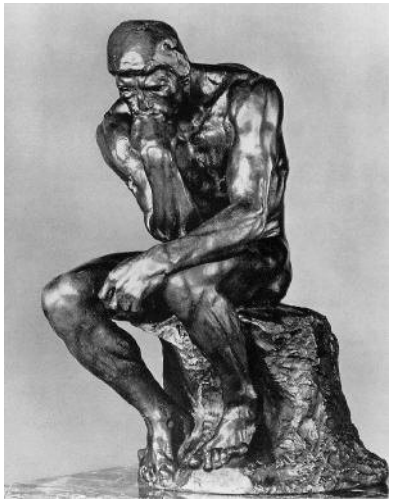
2012 June 11; IQC Colloquium,
Waterloo, ON

Work done with Maxime Hardy (co-op Université de Sherbrooke)

Collaboration with Innsbruck: R. Blatt, T. Monz, P. Schindler, J. T. Barreiro, ...

CIFAR CANADIAN INSTITUTE for ADVANCED RESEARCH INTRIQ FQRNT NSERC CRSNG

General Philosophy



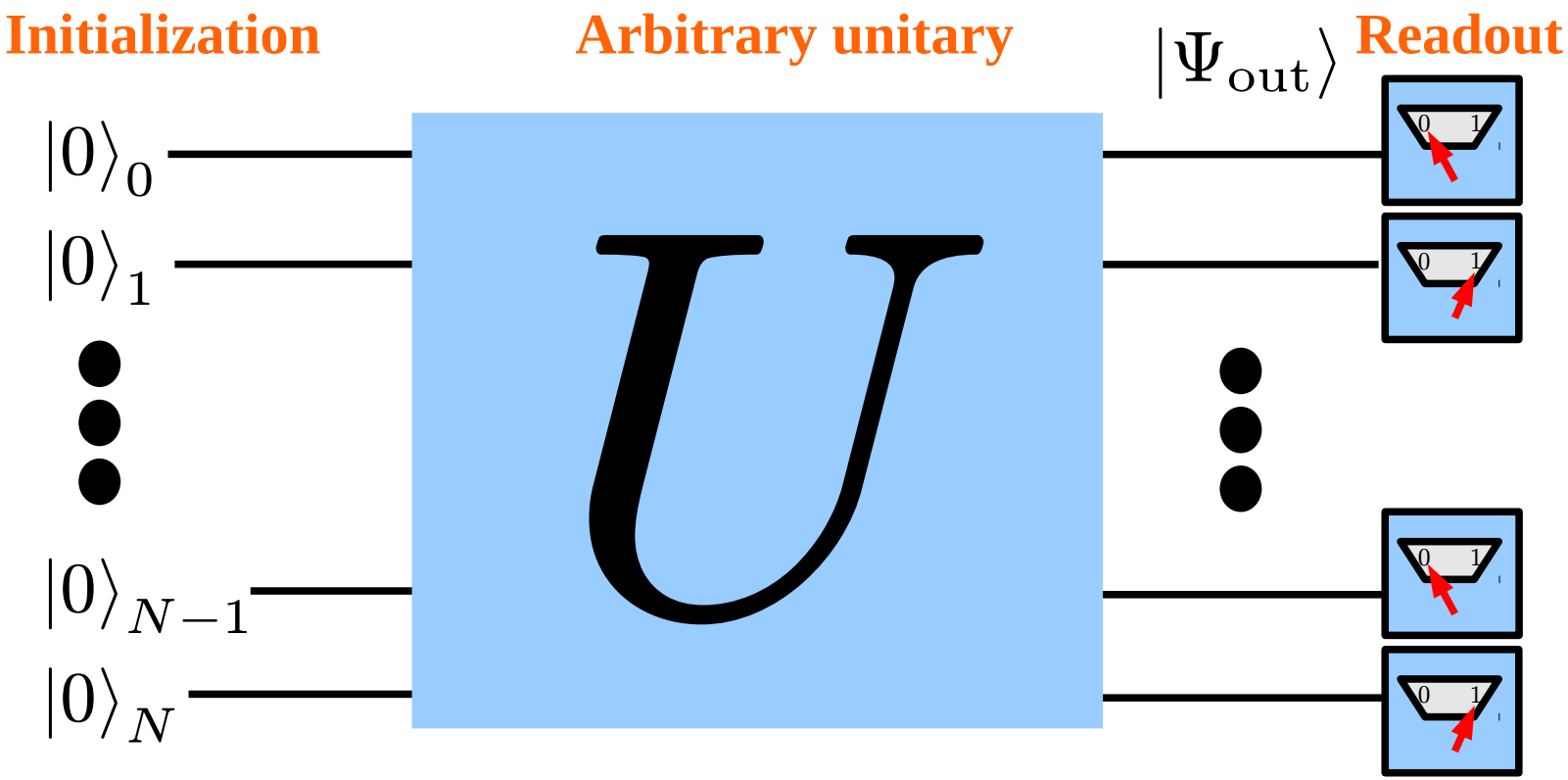
Abstract models are excellent for fast progress, well defined questions...

BUT: too many to choose from.

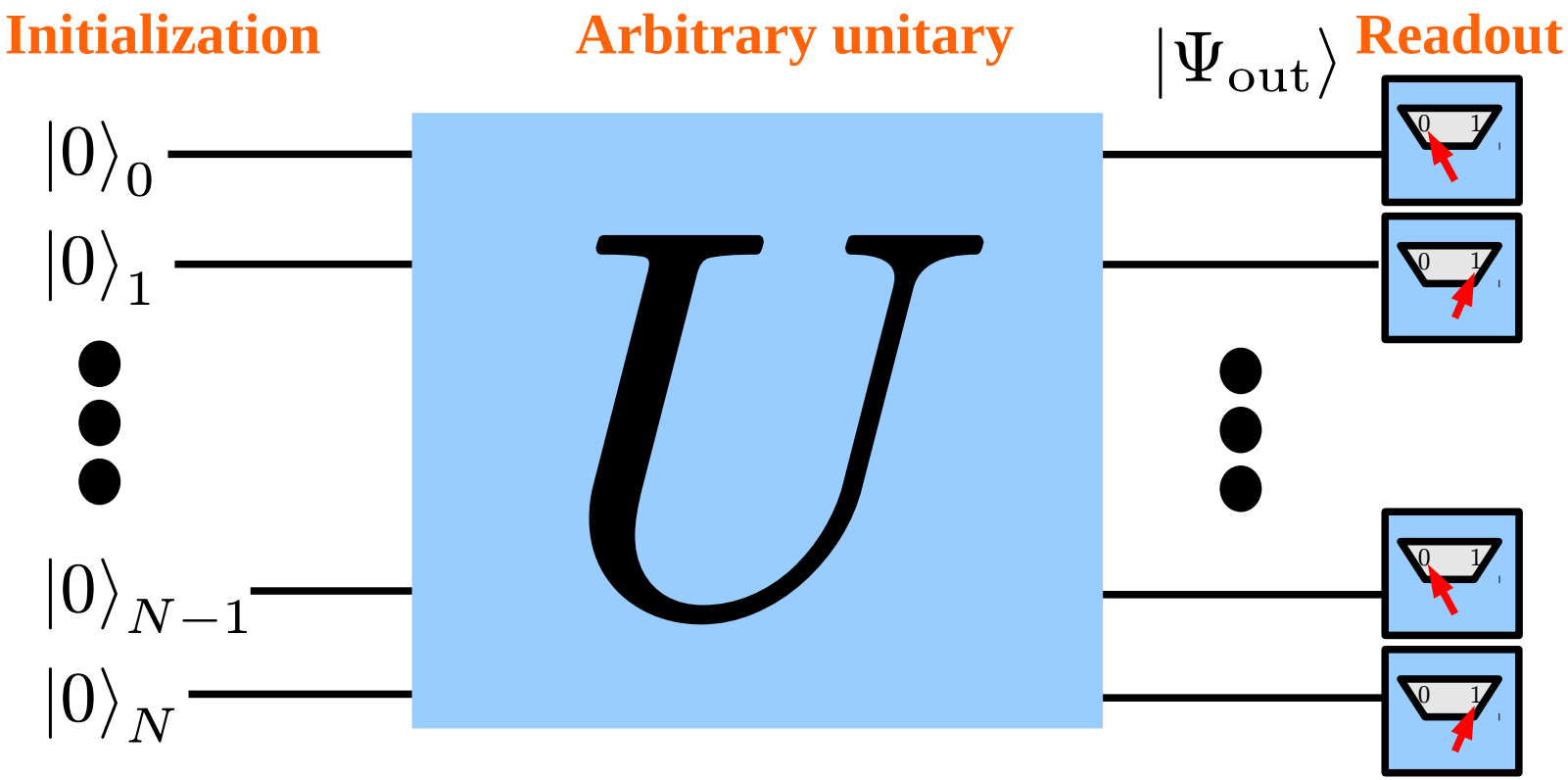
Physical considerations often show the way to go.

True dynamics/decoherence often **more complex** than initial models suggest.

Goal: Quantum Information Processing



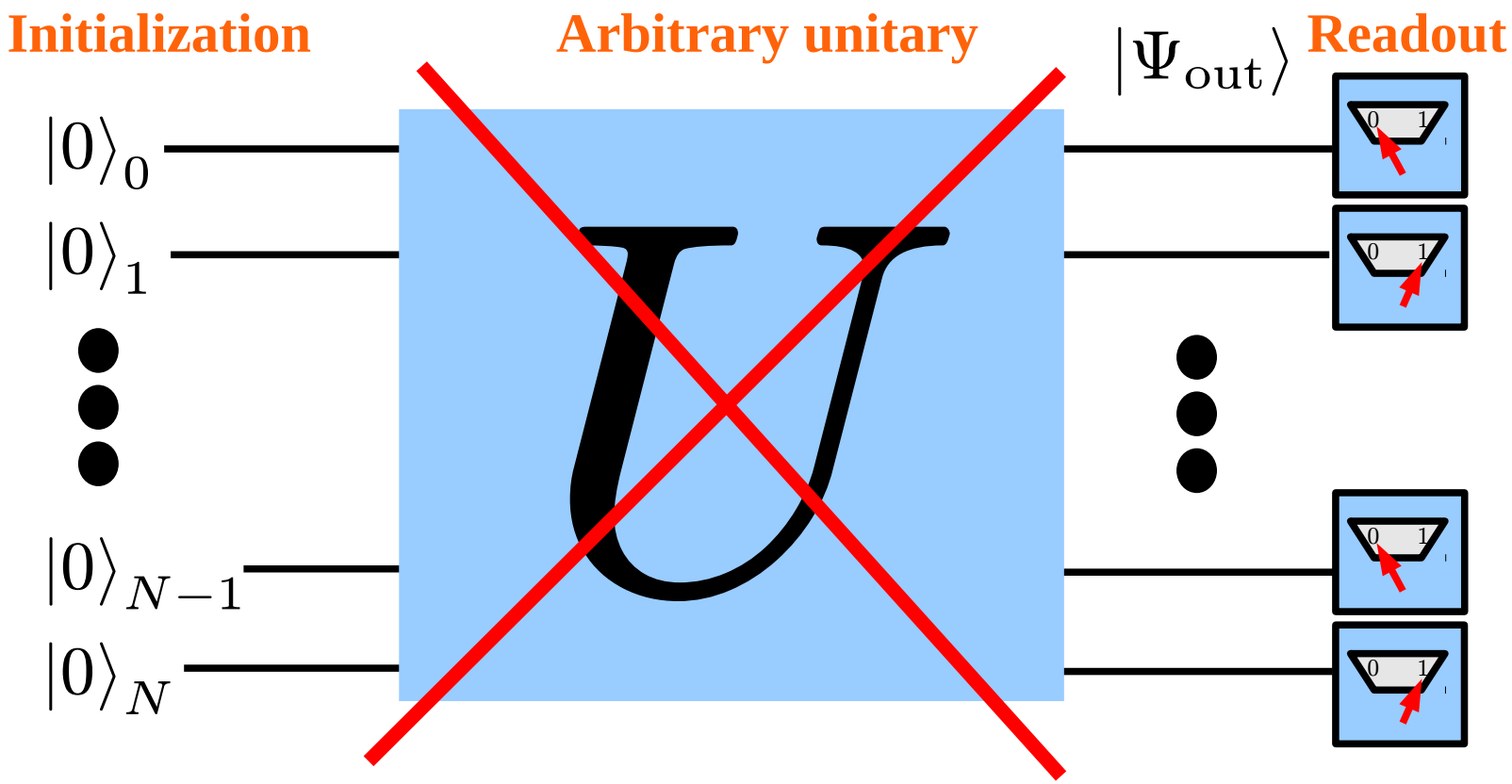
Goal: Quantum Information Processing



Physical Implementation:

$$U = \mathcal{T}\exp\left\{-i\int_0^t dt' H(t')\right\} \qquad H \in \mathcal{H}_S$$

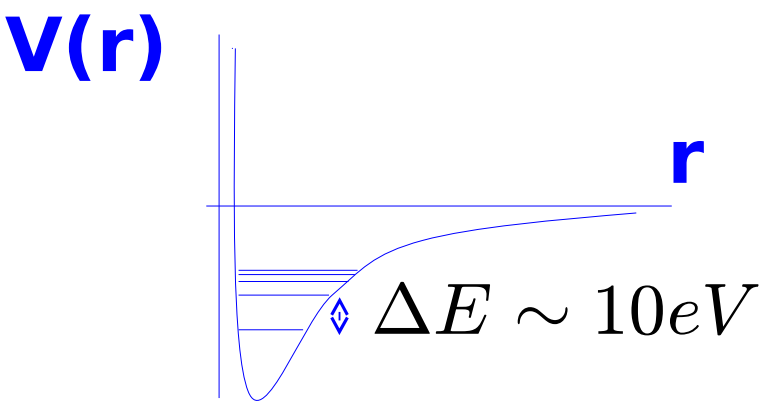
Reality: Imperfections

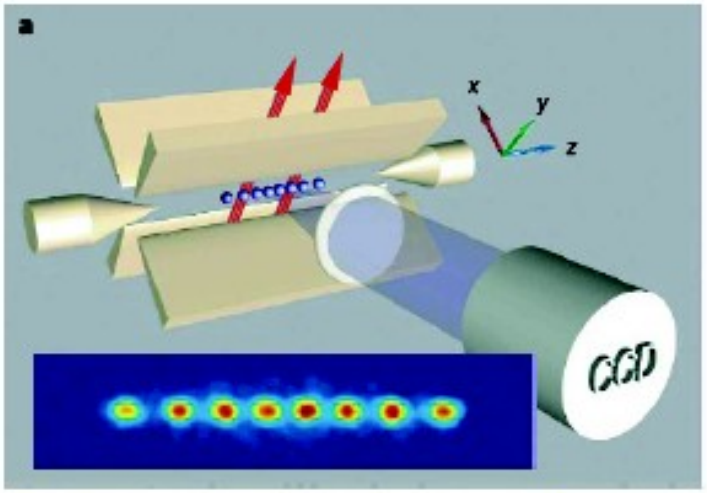


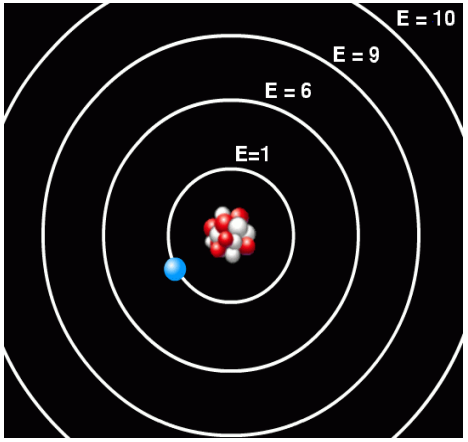
$$\tilde{U} = \mathcal{T}\exp\left\{-i\int_0^t dt'\,(H(t') + \delta H(t'))\right\} \qquad \delta H \in \mathcal{H}_S \otimes \mathcal{H}_E$$

Qubit encoding: Single ions ($^{40}Ca^{+}$)

R. Blatt and D. Wineland, Nature (2008)





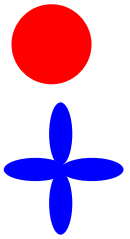


$\sim 10^{-10}$ m

encoding:

$|s\rangle \rightarrow |0\rangle$

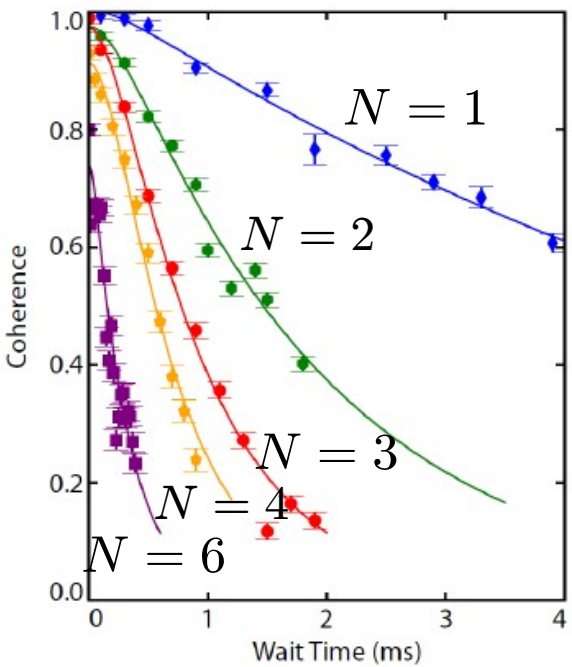
$|d\rangle \rightarrow |1\rangle$



Collective dephasing

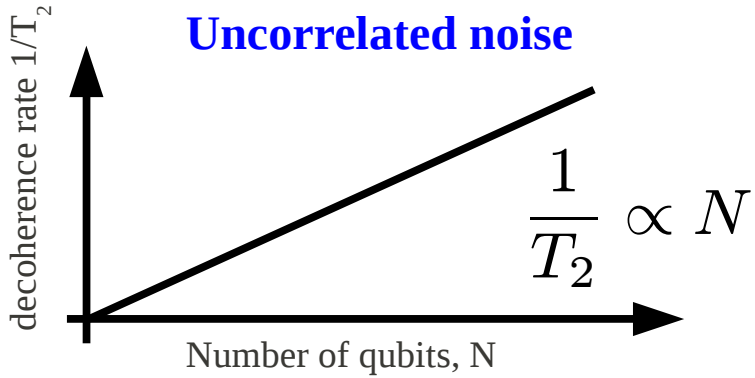
$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(\underbrace{|0000...\rangle}_{\text{N qubits}} + |1111...\rangle\right) \quad \text{(GHZ)}$$

$$F(t) = \left\langle |\langle\psi(0)|\psi(t)\rangle|^2 \right\rangle_{\text{av.}}$$

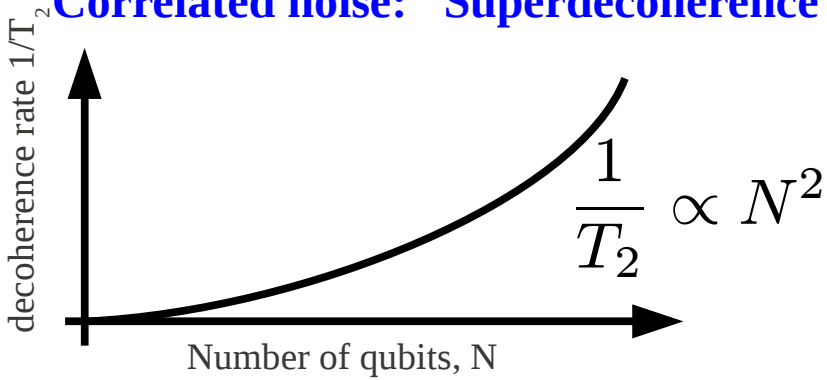


Expectation

Uncorrelated noise



Correlated noise: "Superdecoherence"



T. Monz et al., PRL (2011)

G. Palma et al., Proc. Roy. Soc. Lond. A (1996)

Collective dephasing

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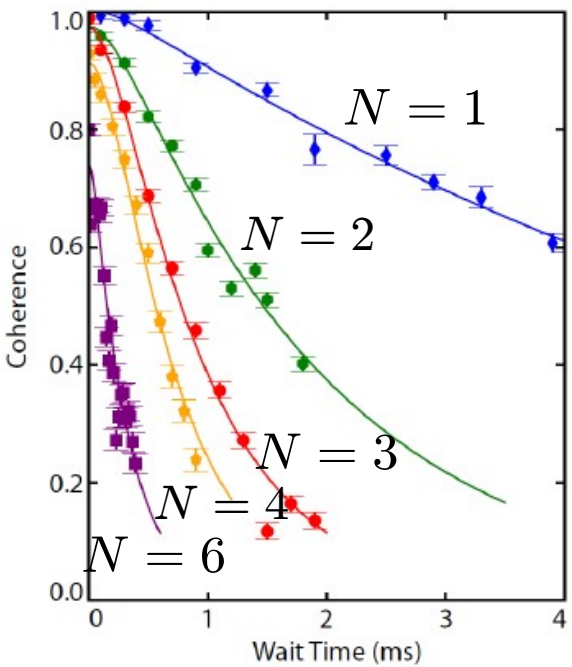
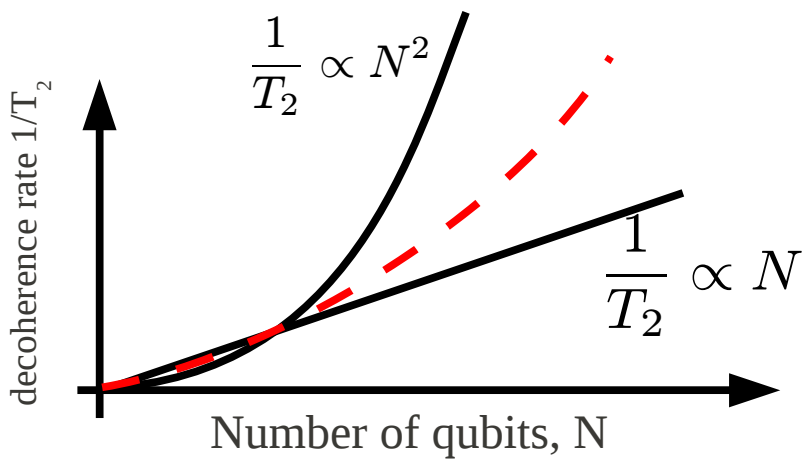


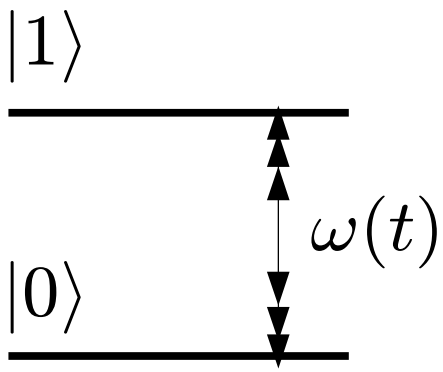
Fits

something in between?



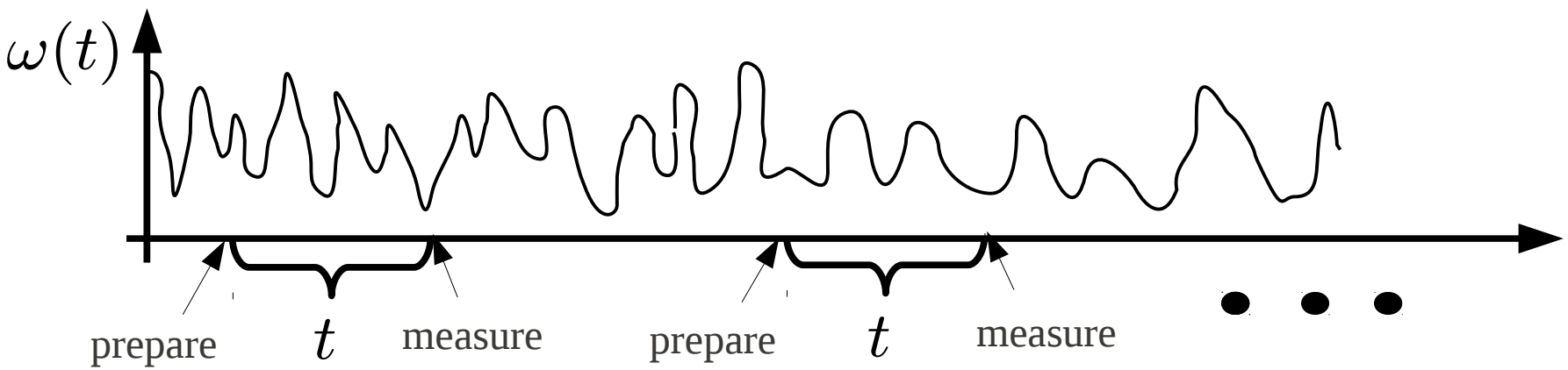
T. Monz et al., PRL (2011)

Dephasing: General



$$H(t) = \omega(t)\sigma_z/2 \qquad \dot{\rho} = -i\left[H(t), \rho\right]$$

$$\langle\sigma_+(t)\rangle = e^{i\phi(t)}\langle\sigma_+(0)\rangle \qquad \phi(t) = \int_0^t dt'\omega(t')$$



Average over noise realizations:

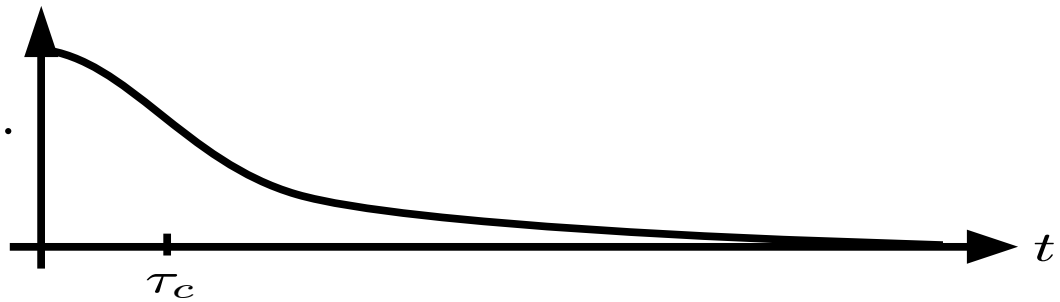
$$\langle\sigma_+(t)\rangle_{\text{av.}} = \left\langle e^{i\phi(t)}\right\rangle_{\text{av.}} \langle\sigma_+(0)\rangle$$

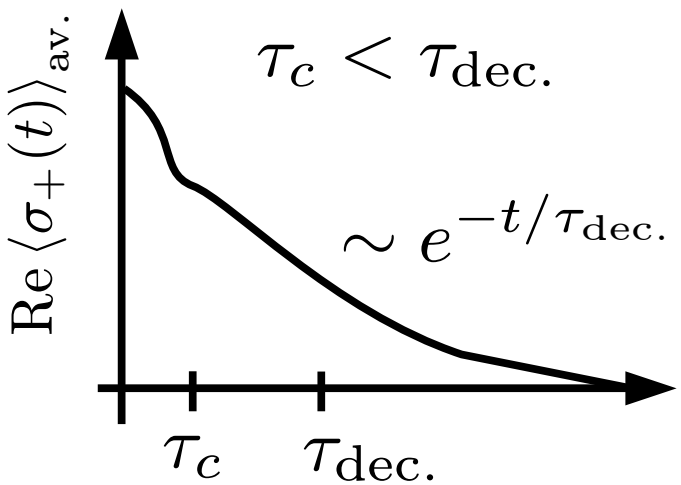
Dephasing: General

$$\langle \sigma_+(t) \rangle_{\text{av.}} = \left\langle e^{i\phi(t)} \right\rangle_{\text{av.}} \langle \sigma_+(0) \rangle = e^{-\frac{1}{2}\left\langle \phi^2(t) \right\rangle_{\text{av.}}} \langle \sigma_+(0) \rangle$$

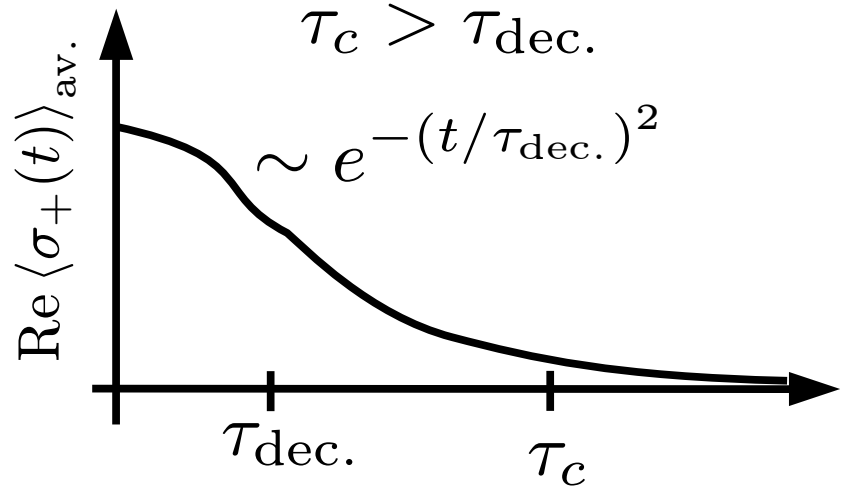
$$\left\langle \phi^2(t) \right\rangle_{\text{av.}} = \int_0^t dt'(t-t') \left\langle \delta\omega(t')\delta\omega(0) \right\rangle_{\text{av.}}$$ (Gaussian, stationary)

$\langle \delta\omega(t)\delta\omega(0) \rangle_{\text{av.}}$





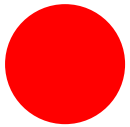
(Markovian)

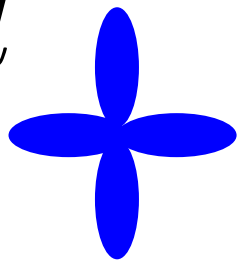


(Non-Markovian)

Sources of dephasing in ion traps

- Global magnetic field fluctuations (slow)

s  $= |0\rangle$

d  $= |1\rangle$ (orbital Zeeman)

- Fluctuating global phase reference (laser stability, also slow)

AMO Physics: Usually assume fast, local noise.

Gaussian dephasing model:

$$H(t) = \delta B(t) \sum_k S_k^z \qquad S_k^z = \left(|0\rangle\langle 0|_k - |1\rangle\langle 1|_k\right)/2$$

$$\langle \delta B(t) \delta B(0) \rangle = \left\langle \delta B^2 \right\rangle e^{-t/\tau_c}$$

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} \left(\underbrace{|0000...\rangle}_{\text{N qubits}} + |1111...\rangle \right) \qquad \text{(GHZ)}$$

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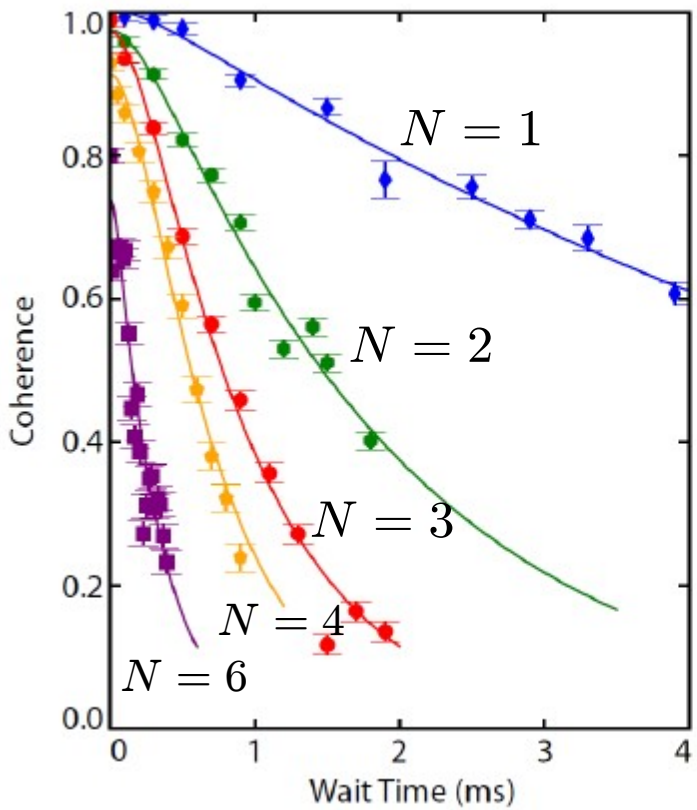
$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} \left(\underbrace{|0000...\rangle}_{\text{N qubits}} + |1111...\rangle \right) \qquad \text{(GHZ)}$$

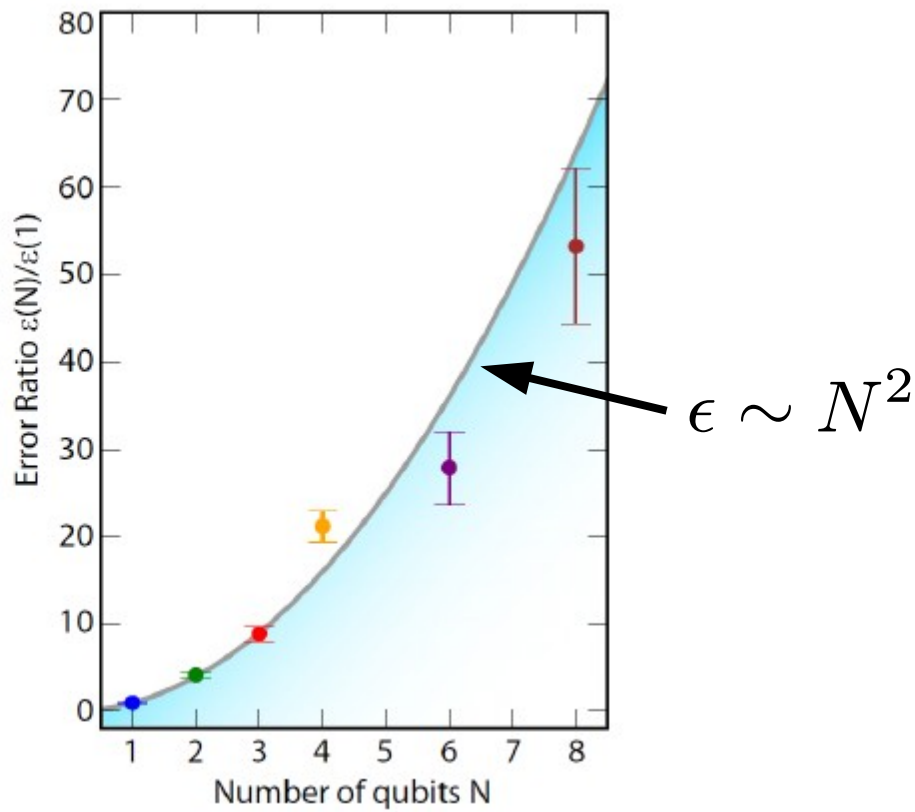
$$F(t) = \left\langle \left| \langle \psi(0) | \, \psi(t) \rangle \right|^2 \right\rangle_{\text{av.}} = \frac{1}{2} \left(1 + \exp\left[-2\epsilon(N,t) \right] \right) \simeq 1 - \epsilon(N,t)$$

$$\epsilon(N,t) = \frac{N^2}{2} \int_0^t d\tau (t-\tau) \langle \delta B(t) \delta B(0) \rangle$$

"Superdecoherence"

Revised noise model, accounting for a finite correlation time (non-Markovian)



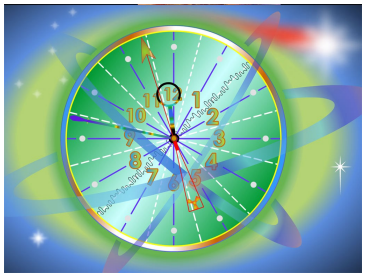


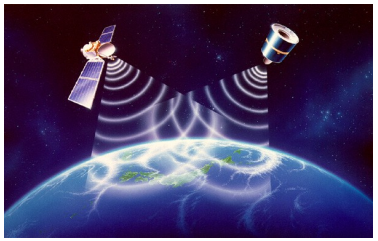
Dominant noise source (B-field) identified; **N extended to 14 qubits**!

T. Monz, ... WAC, ..., R. Blatt, PRL (2011)

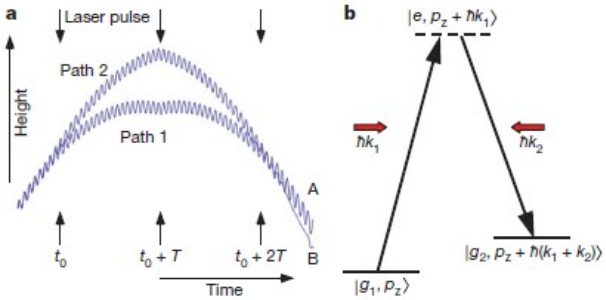
Quantum Metrology

Frequency standards





Fundamental tests of gravitation



Mueller, Peters, Chu, Nature (2010)

Parameter estimation for Qm. Inf. Proc.

e.g., Rafal Demkowicz-Dobrzanski et al., arXiv (2012)

Quantum Metrology

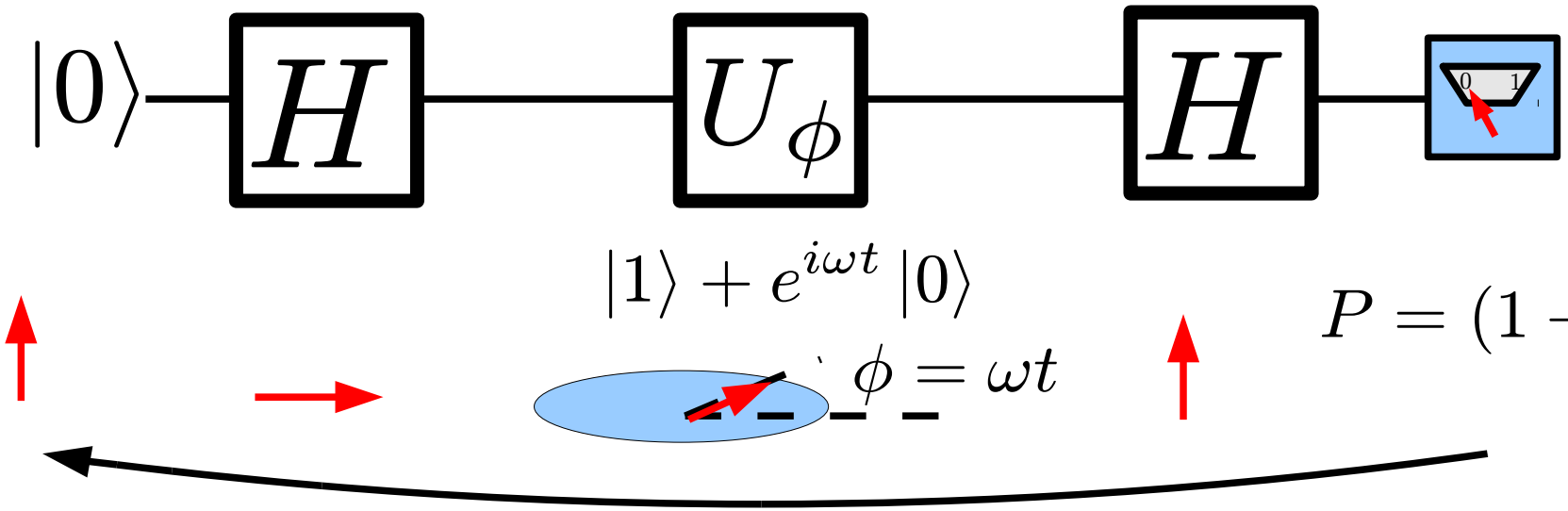
REVIEW ARTICLES | FOCUS
PUBLISHED ONLINE: 31 MARCH 2011 | DOI: 10.1038/NPHOTON.2011.35
nature photonics

Advances in quantum metrology

Vittorio Giovannetti[1]*, Seth Lloyd[2] and Lorenzo Maccone[3]

The statistical error in any estimation can be reduced by repeating the measurement and averaging the results. The central limit theorem implies that the reduction is proportional to the square root of the number of repetitions. Quantum metrology is the use of quantum techniques such as entanglement to yield higher statistical precision than purely classical approaches. In this Review, we analyse some of the most promising recent developments of this research field and point out some of the new experiments. We then look at one of the major new trends of the field: analyses of the effects of noise and experimental imperfections.

Precision measurements?



$$P = (1 + \cos\omega t)/2$$

Repeat N times...

Classical:

$$\delta\omega_{\text{class.}} = \frac{\sqrt{P(1-P)/N}}{|dP/d\omega|} = \frac{1}{\sqrt{NTt}} \propto \frac{1}{\sqrt{N}}$$

T: total experiment time

Quantum (GHZ state): $|1111...\rangle + e^{iN\omega t}|0000...\rangle$

$$\delta\omega_{\text{quant.}} = \frac{1}{N\sqrt{Tt}} \propto \frac{1}{N}$$

Problem! Decoherence

Markovian (exponential) dephasing, spatially uncorrelated noise:

$$P = (1 + \cos N\omega t)/2 \rightarrow (1 + e^{-N\gamma t} \cos N\omega t)/2$$

$$\delta\omega^{\text{opt.}} \propto \frac{1}{\sqrt{NT}} \propto \delta\omega_{\text{class.}}$$

$$\tau_{\text{dec.}} = 1/(\gamma N)$$

S. F. Huelga et al., PRL (1997)

With dephasing (in general):

$$\boxed{\delta\omega^{\text{opt.}} \propto \frac{1}{N\sqrt{\tau_{\text{dec.}} T}}}$$

For (Markovian) spatially correlated noise:

$$\tau_{\text{dec.}} \propto 1/N^2 \Rightarrow \delta\omega_{\text{quant.}} \propto \text{const.}$$

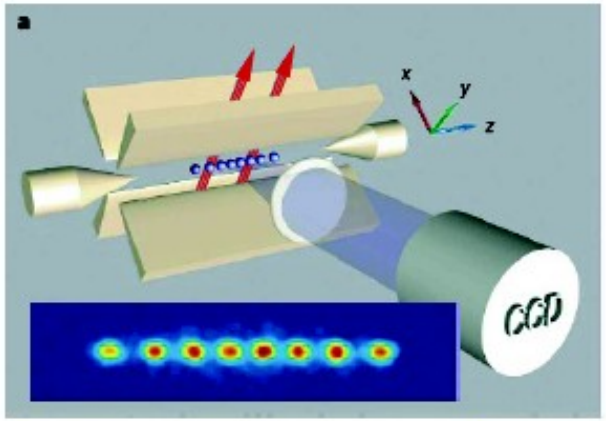
U. Dorner, New J. Phys. (2012)

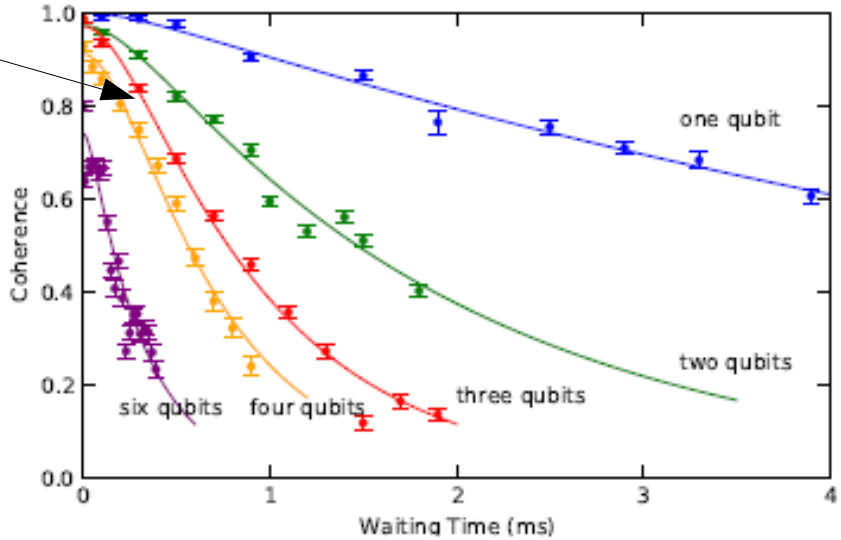
Even worse!

What kind of decoherence?

non-exponential
(long correlation time)

$$\tau_c > \tau_{\rm dec.}$$



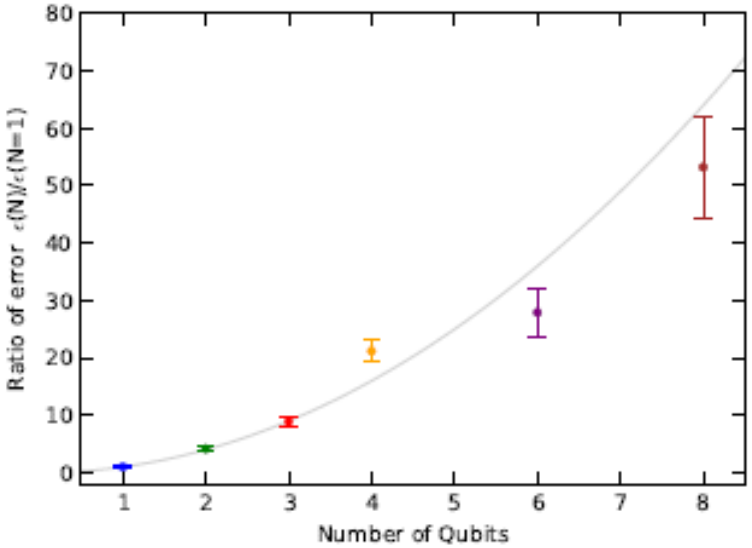


T. Monz, ..., WAC,..., R. Blatt PRL (2011)

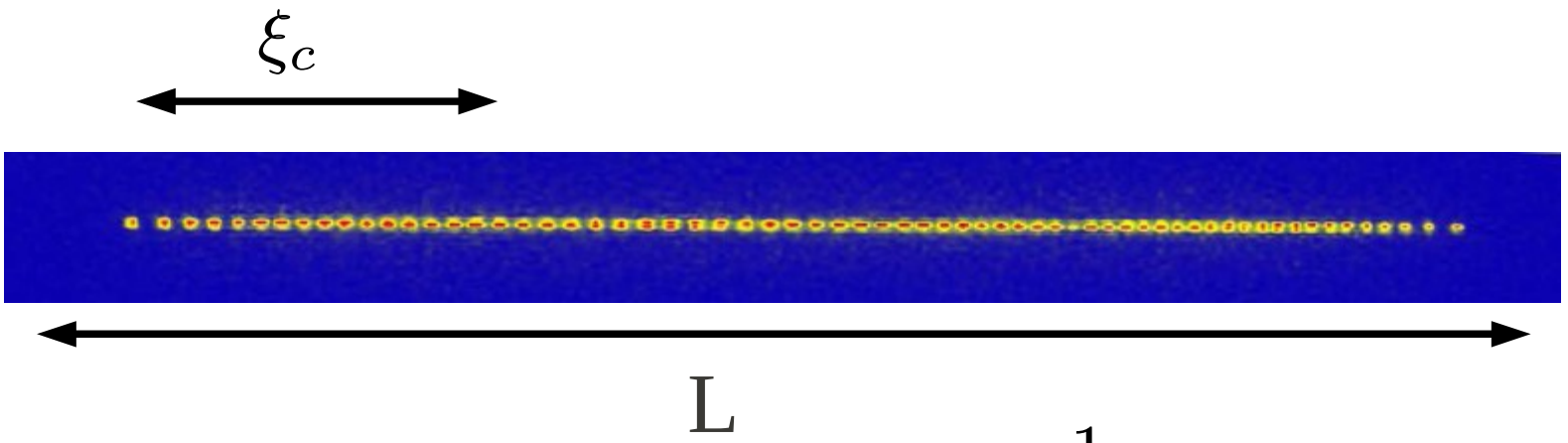
"Superdecoherence"
(long correlation length)

$$\xi_c > L \sim N$$

$$F = 1 - \epsilon \qquad \epsilon(N)/\epsilon(1) \propto N^2$$



Large N: dephasing becomes local



Generalized model:

$$H(t) = \sum_k \delta h_k(t) S_k^z$$

$$S^z = \frac{1}{2}\left(|1\rangle\langle 1| - |0\rangle\langle 0|\right)$$

Features of the environment (independent of N!)

Gaussian fluctuations:

$$\langle \delta h_k(t)\delta h_l(0)\rangle = \langle \delta h^2(0)\rangle \underbrace{e^{-|x_k - x_l|/\xi_c}}_{\text{Space}} \times \underbrace{e^{-t/\tau_c}}_{\text{Time}}$$

But: $L \propto N$ **(incr. with N)** $\tau_{\text{dec.}}$ **decr. with N!**

Large N: Quantum advantage?

$$\delta\omega^{\mathrm{opt.}} \propto \frac{1}{N\sqrt{\tau_{\mathrm{dec.}}T}}$$

$$\tau_c > \tau_{\mathrm{dec.}}; \quad \xi_c < L:$$

$$\boxed{\tau_{\mathrm{dec.}} \propto \frac{1}{\sqrt{N}} \Rightarrow \delta\omega \propto \frac{1}{N^{3/4}}}$$

Also see, e.g., Jones et al., Science (2009);
Matsuzaki, Benjamin, Fitzsimons PRA (2011)

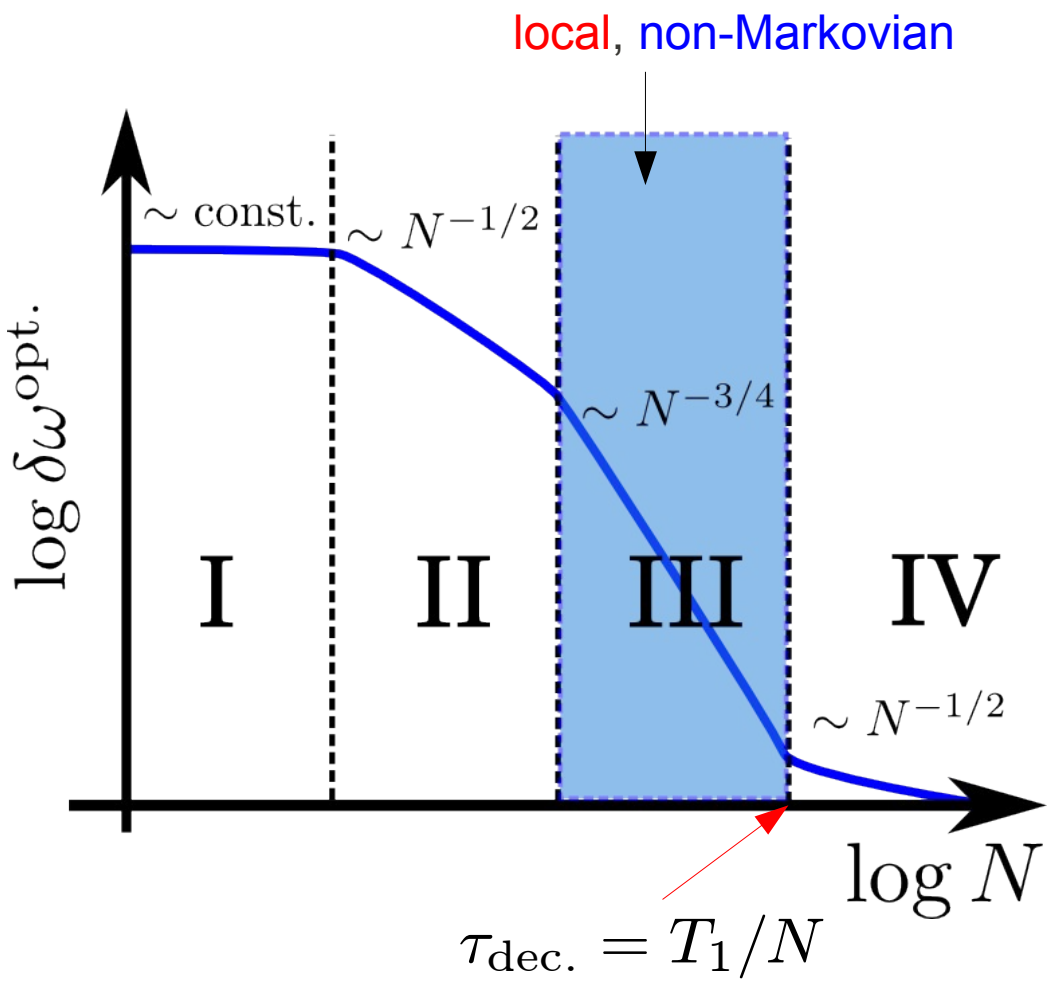
Large N: Quantum advantage?

$I : \tau_c < \tau_{\text{dec.}};\ L\,(\sim N) < \xi_c$

$II : \tau_c > \tau_{\text{dec.}};\ L\,(\sim N) < \xi_c$

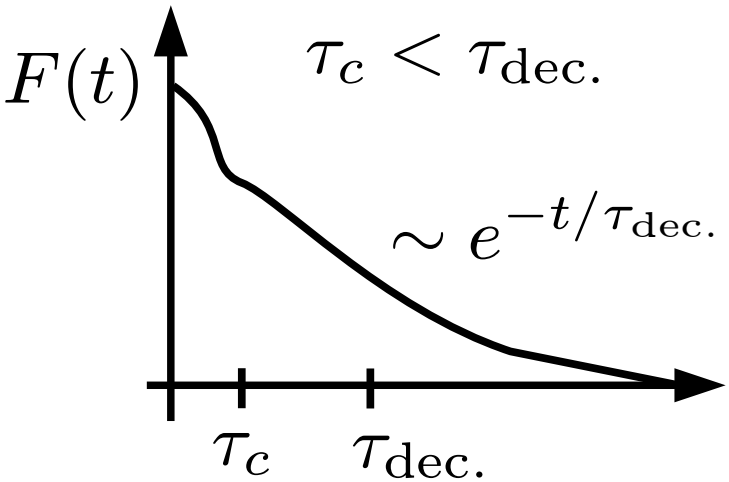
OR $\tau_c < \tau_{\text{dec.}};\ L\,(\sim N) > \xi_c$

$III : \tau_c > \tau_{\text{dec.}};\ L\,(\sim N) > \xi_c$



If the qubit frequency fluctuates locally in space and is approximately constant in time, quantum wins.

Model summary: Spatial and temporal correlations

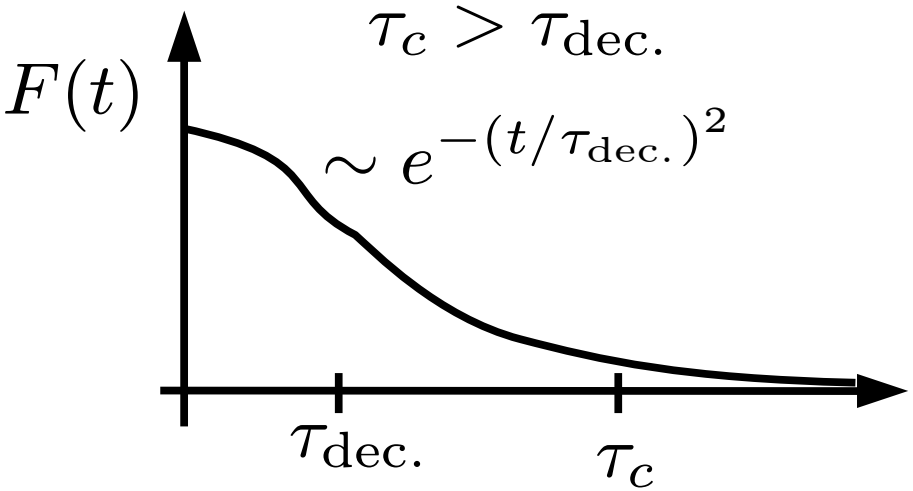


$\xi_c \gg L$

$\tau_{\rm dec.} \propto N^{-2}; \quad \delta\omega = \text{const.}$

$\xi_c \ll L$

$\tau_{\rm dec.} \propto N^{-1}; \quad \delta\omega \propto N^{-1/2}$



$\xi_c \gg L$

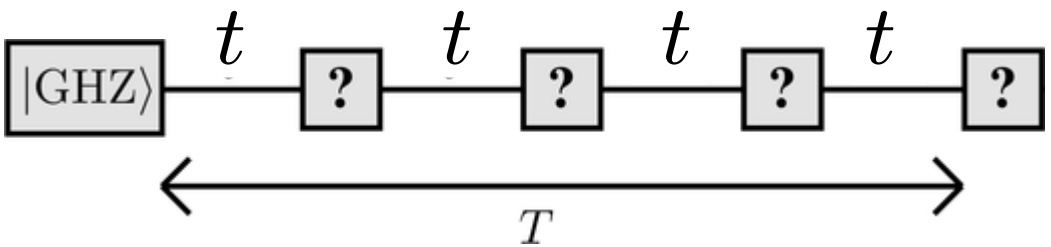
$\tau_{\rm dec.} \propto N^{-1}; \quad \delta\omega \propto N^{-1/2}$

$\xi_c \ll L$

$\tau_{\rm dec.} \propto N^{-1/2}; \quad \delta\omega \propto N^{-3/4}$

See also:
Matsuzaki, Benjamin, Fitzsimons, PRA (2011)

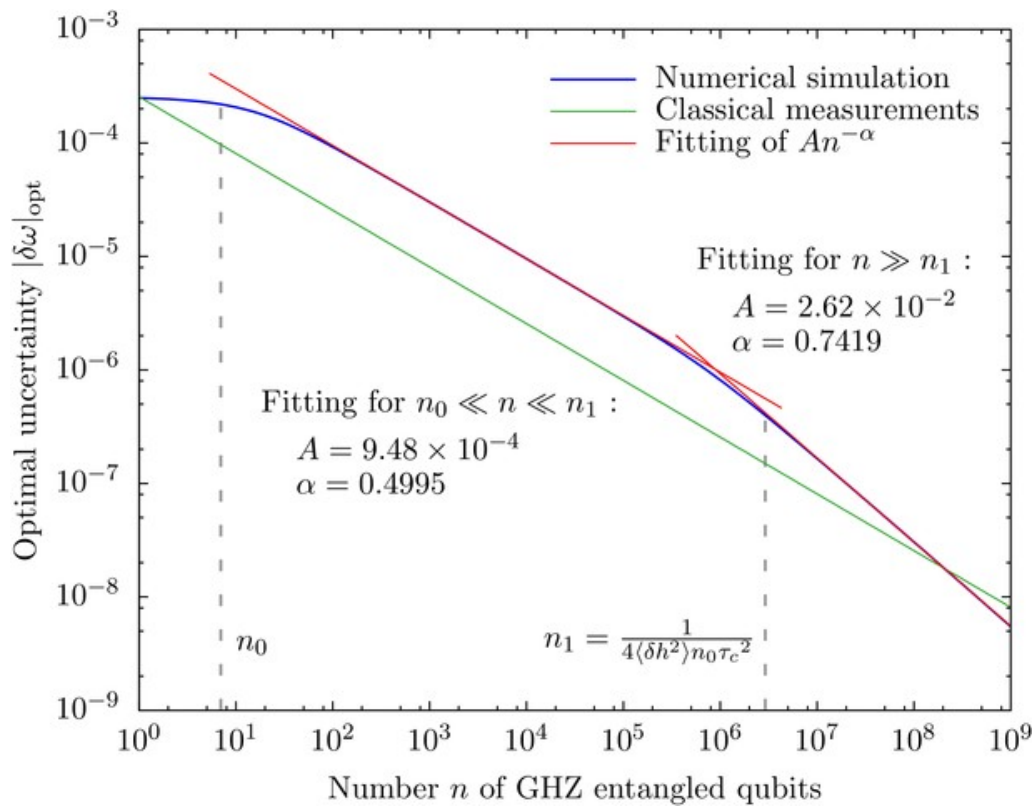
Scaling: Regimes



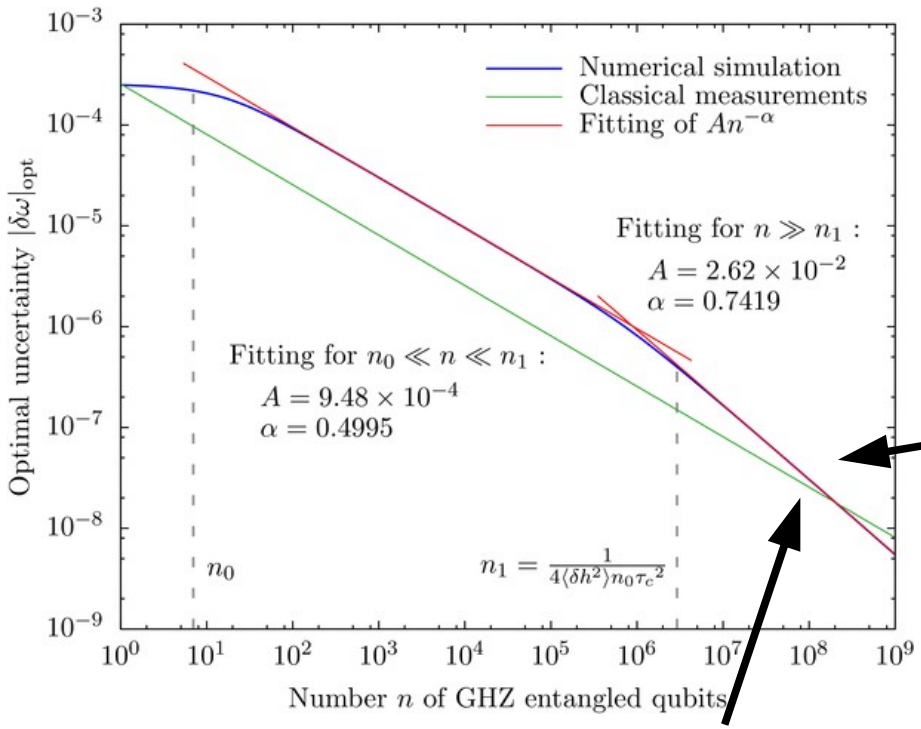
$$\xi_c \sim n_0 = 7$$

$$\tau_c = 1$$

$$\delta\omega_0 = 10^{-4}$$



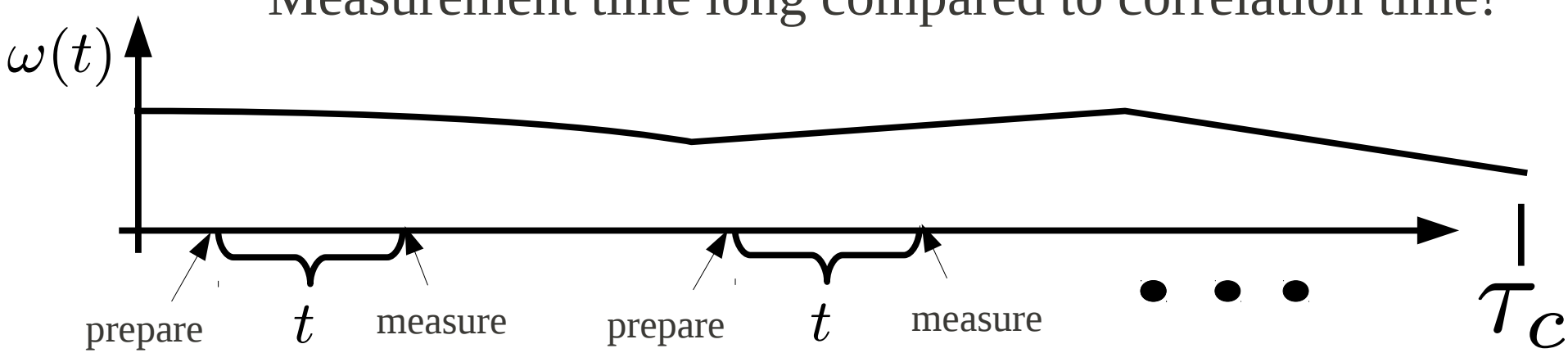
Can we do better?



Jones et al., Science (2009)
Matsuzaki et al., PRA (2011)

$$\frac{1}{N^{3/4}}$$

Measurement time long compared to correlation time!



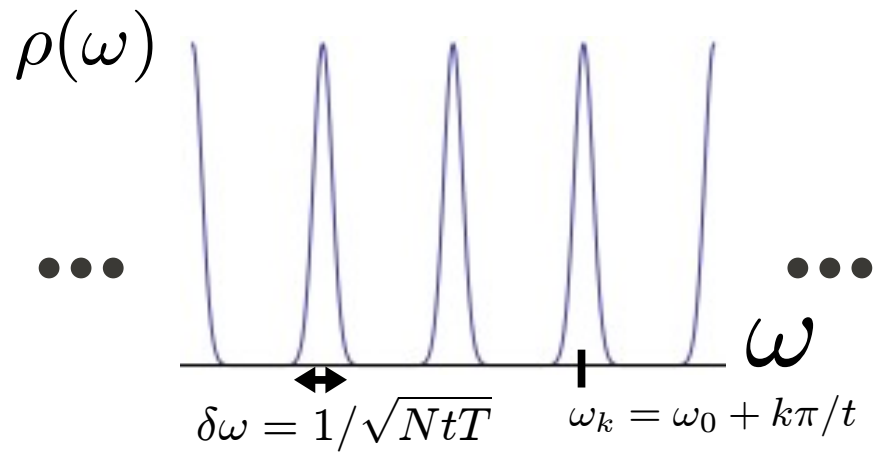
Parameter Estimation = "Instantaneous Measurement"

After many measurements, frequency still not precisely defined!

"Instantaneous": $T \ll \tau_c$

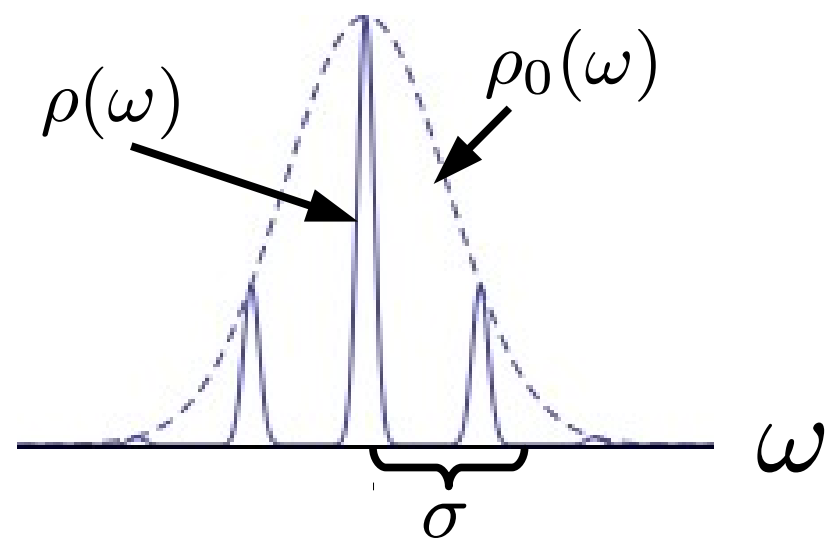
$$\rho(\omega) = P^{N_+}(1-P)^{N-N_+}/N_0$$

$$P = (1+\cos\omega t)/2$$



More realistic: Gaussian prior

$$\rho(\omega) = P^{N_+}(1-P)^{N-N_+}\rho_0(\omega)/N_0$$

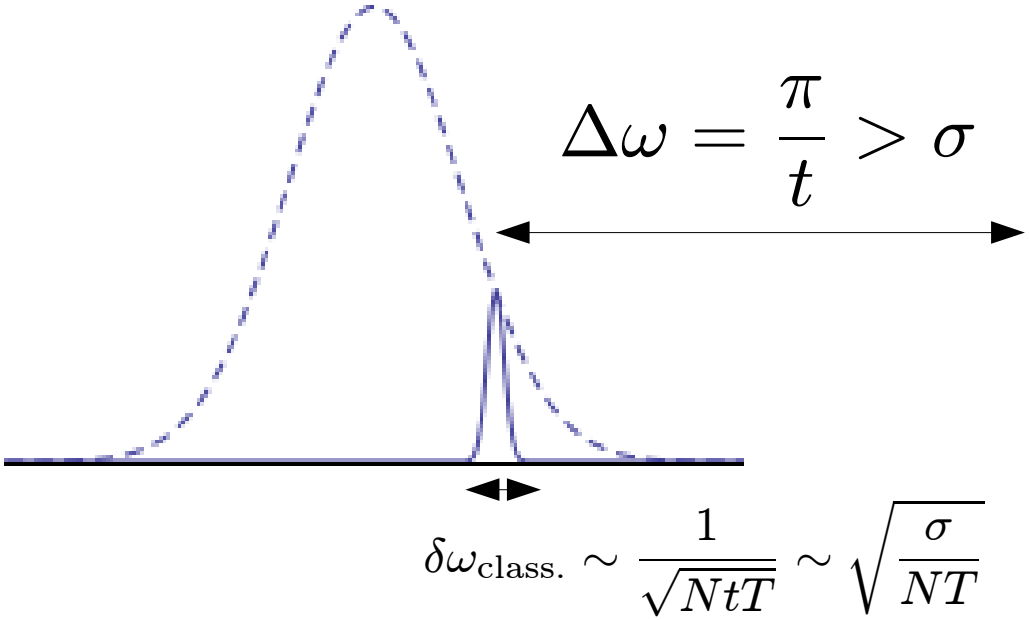


Measurement times t?

Classical:

Advantage in performing measurements at **short times**, even if the standard formula suggests larger t is always better:

$$\delta\omega \sim \frac{1}{\sqrt{tT}}$$



Quantum:

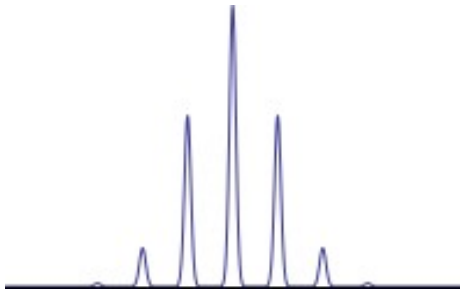
Peak spacing smaller: $\Delta\omega = \frac{\pi}{nt}$ $\quad \Delta\omega > \sigma \Rightarrow t < 1/n\sigma$

This protocol gives the same scaling! (not optimal?):

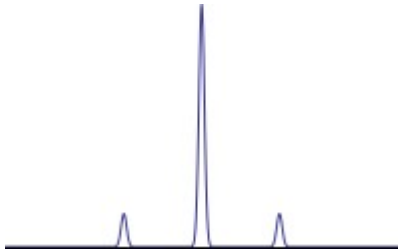
$$\delta\omega_{\text{quant.}} \sim \sqrt{\frac{\sigma}{NT}}$$

Improved measurement strategy

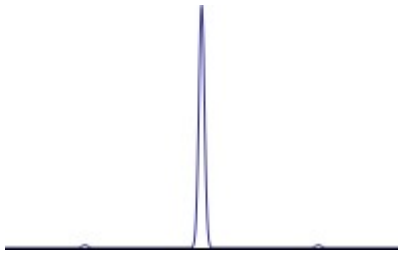
Measurement time t



Measurement time $t/2$



Measurement time $t/4$



Measurement time overhead:

$$t_0 = \sum_k \frac{1}{2^k} t = 2t$$

Result:

$$\delta\omega_{\text{class.}} = \frac{\sqrt{2}}{\sqrt{N}\sqrt{tT}}$$

$$\delta\omega_{\text{quant.}} = \frac{\sqrt{2}}{N\sqrt{tT}} \quad \text{(GHZ)}$$

$$\boxed{T \ll \tau_c}$$

Summary

$$T \ll \tau_c$$

A static random frequency can always be found with Heisenberg-limited precision using GHZ states, provided the prior distribution has finite width.

$$\delta\omega_{\mathrm{quant.}} \sim \frac{1}{N\sqrt{Tt}}$$

This beats the ~$1/N^{3/4}$ scaling found previously [Jones et al., Science (2009), Matsuzaki et al., PRA (2011)], even for Gaussian decay of P(t).

$$t \ll \tau_c \ll T?$$

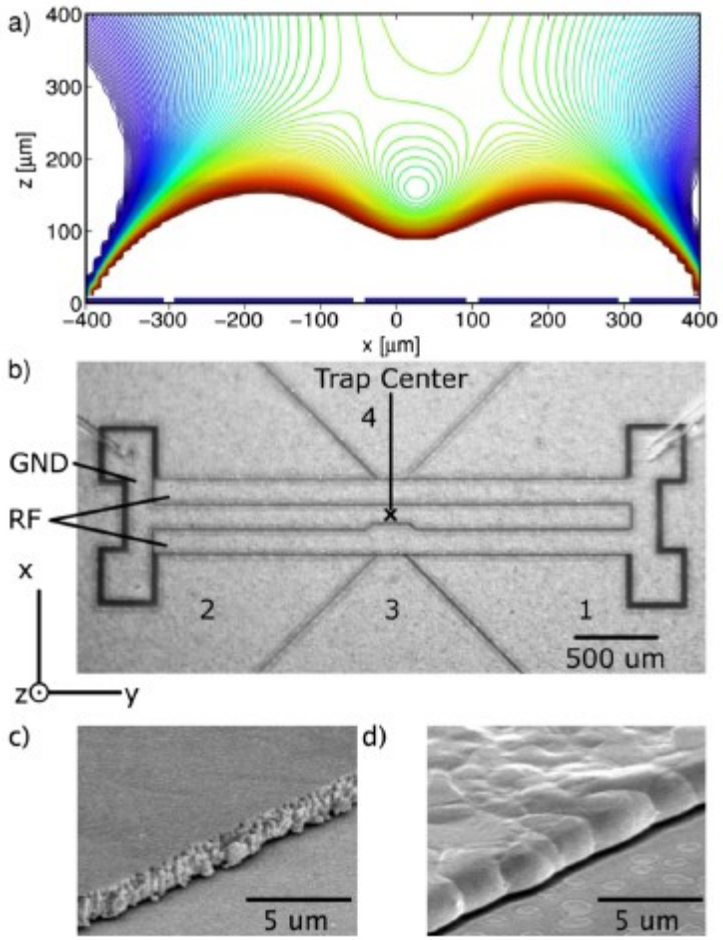
In this regime, frequency drifts between measurements; problem still open?

Conclusions

- “Superdecoherence”: a short-term problem for ion-trap and other implementations.

- Quantum-enhanced precision measurements still possible in spite of dephasing.

How large can the quantum region be?



Open questions:

Physical dephasing mechanisms:

Charge traps (fluctating electric field)

Surface spins (magnetic field)

Power-law correlations in space/time