see arXiv:1108.5329

Reliable Quantum State Tomography

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joint work with Renato Renner

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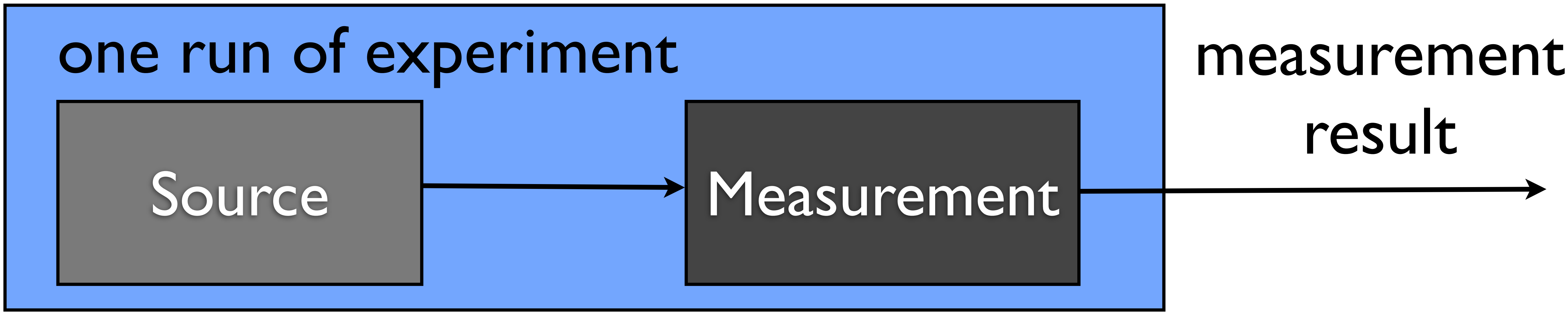
QSIT Quantum Science and Technology
National Centre of Competence in Research

Outline

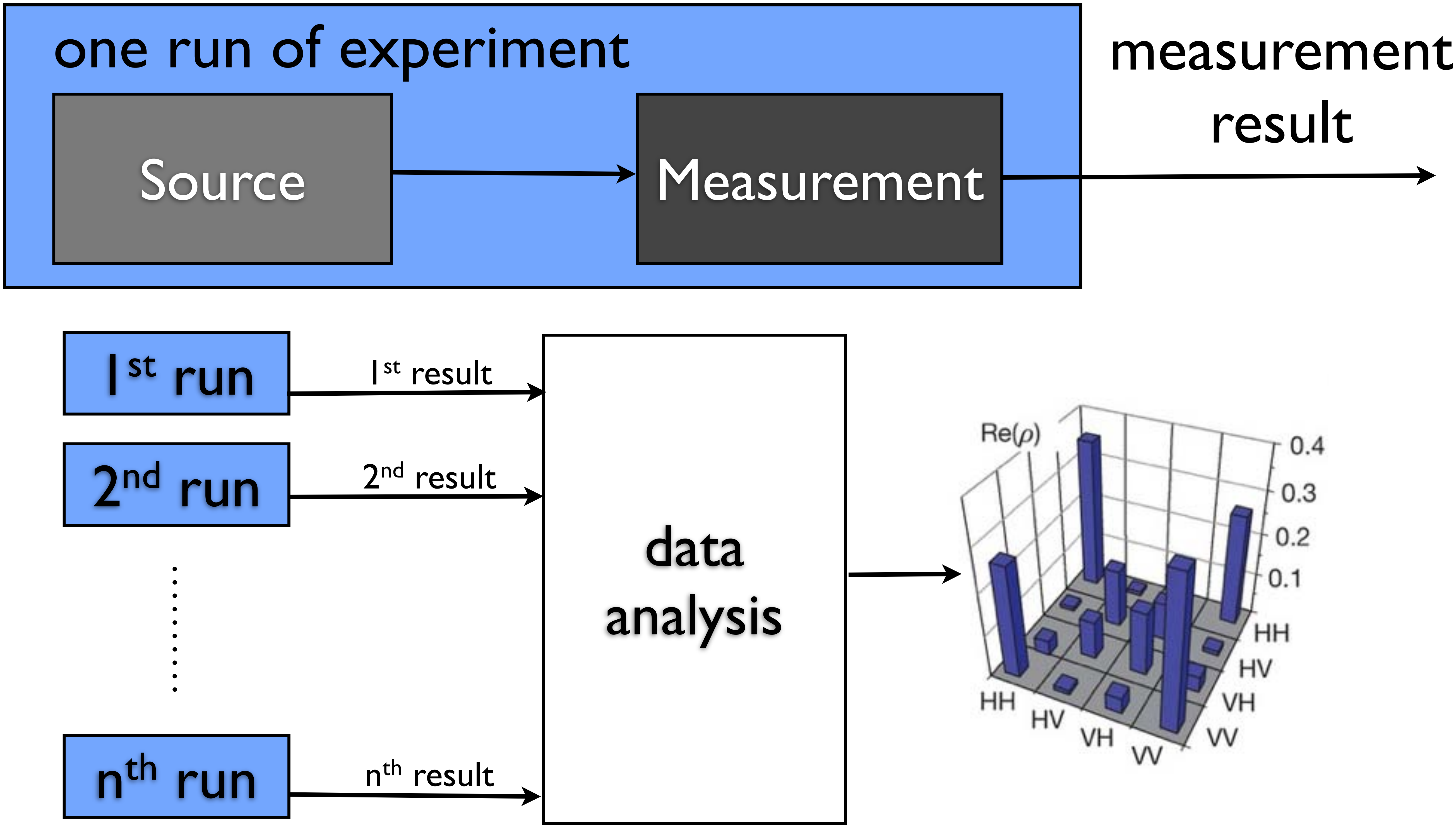
- Motivation
- Main Result
- Technique
- Features

Motivation

The Question



The Question



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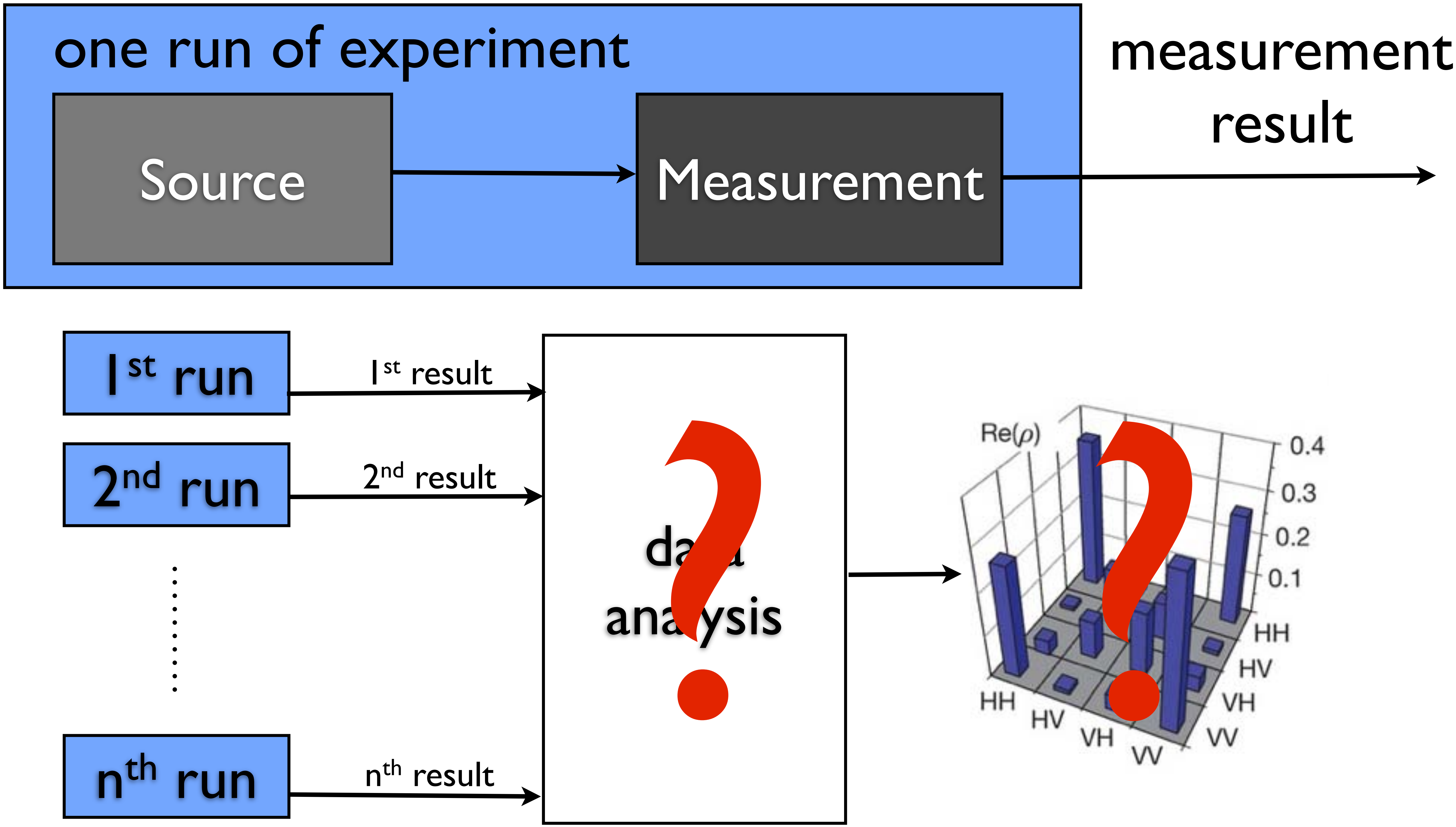


Existing methods

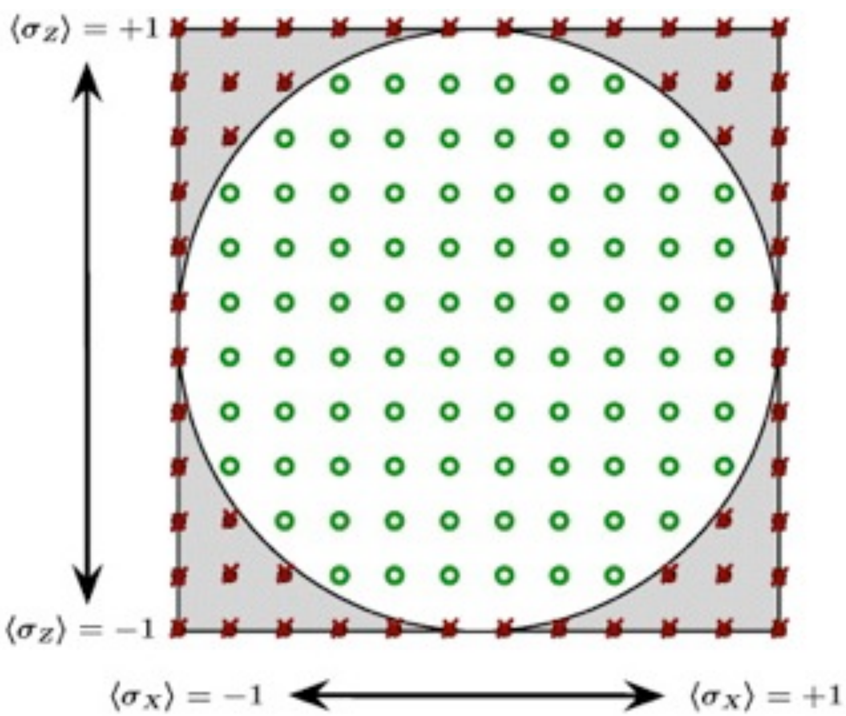
Linear Inversion: Solve $\mathrm{tr}\rho E_i = \frac{f_i}{n} \Rightarrow \rho$

POVM elements

observed frequencies

may not be positive semi-definite

Example:
Qubit, Pauli measurements



$$\mathrm{tr}\rho\sigma_x = \frac{f_x(\uparrow) - f_x(\downarrow)}{n}$$

$$\mathrm{tr}\rho\sigma_z = \frac{f_z(\uparrow) - f_z(\downarrow)}{n}$$

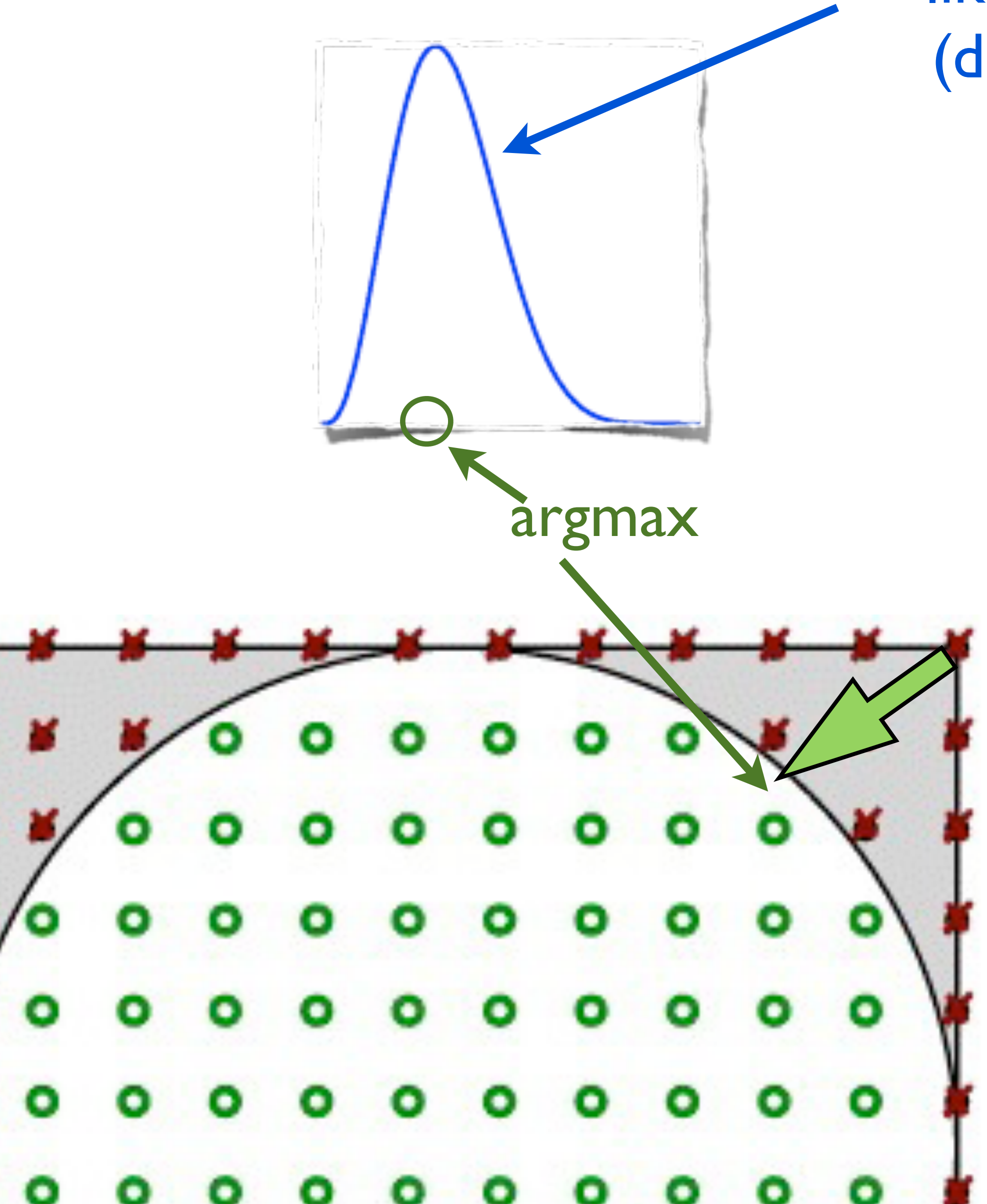
R. Blume-Kohout, NJP 2010

Existing methods

Maximum Likelihood Estimate

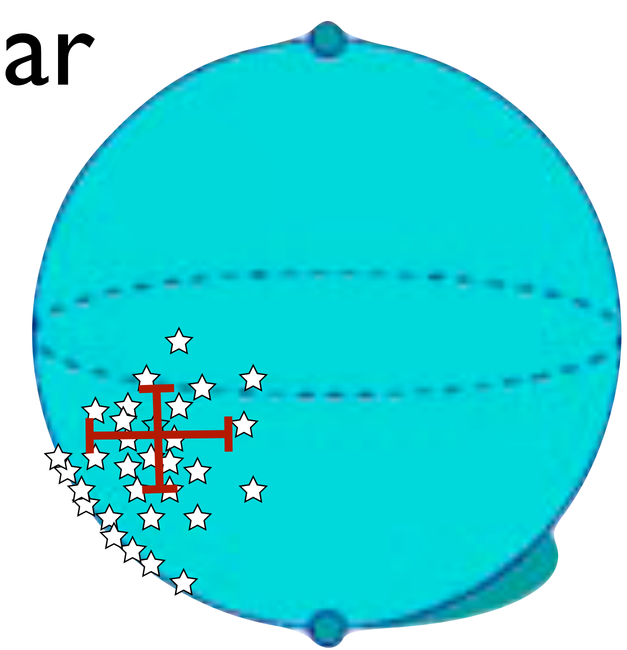
$$\operatorname{tr} \underbrace{E_1^{\otimes f_1} \otimes \cdots \otimes E_k^{\otimes f_k}}_{B^n} \rho^{\otimes n}$$

likelihood function
(depends on data)



estimate error bar
by resampling

unreliable



Existing methods

Bayesian Update $P_{\text{posterior}}(\rho)d\rho \; \alpha \; \text{Prob}[f|\rho]P_{\text{prior}}(\rho)d\rho$

From posterior
compute estimate
and error bar

Existing methods

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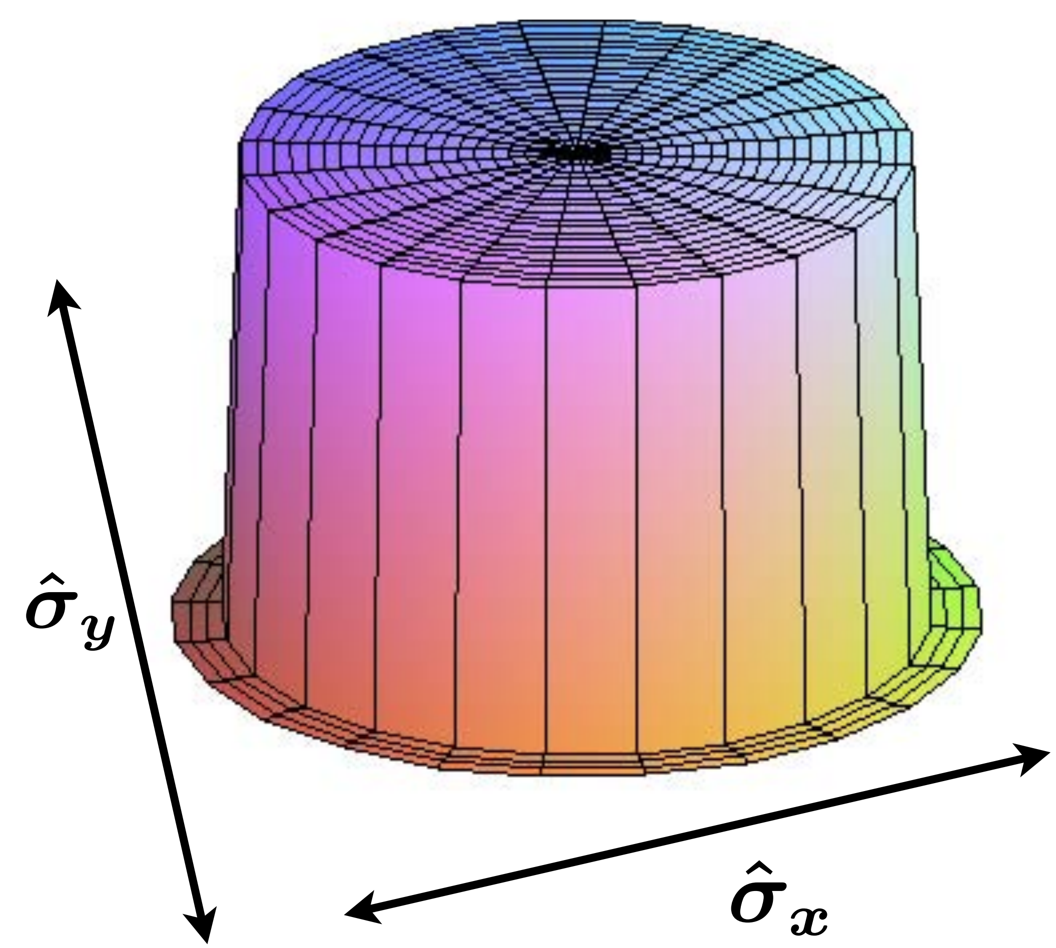

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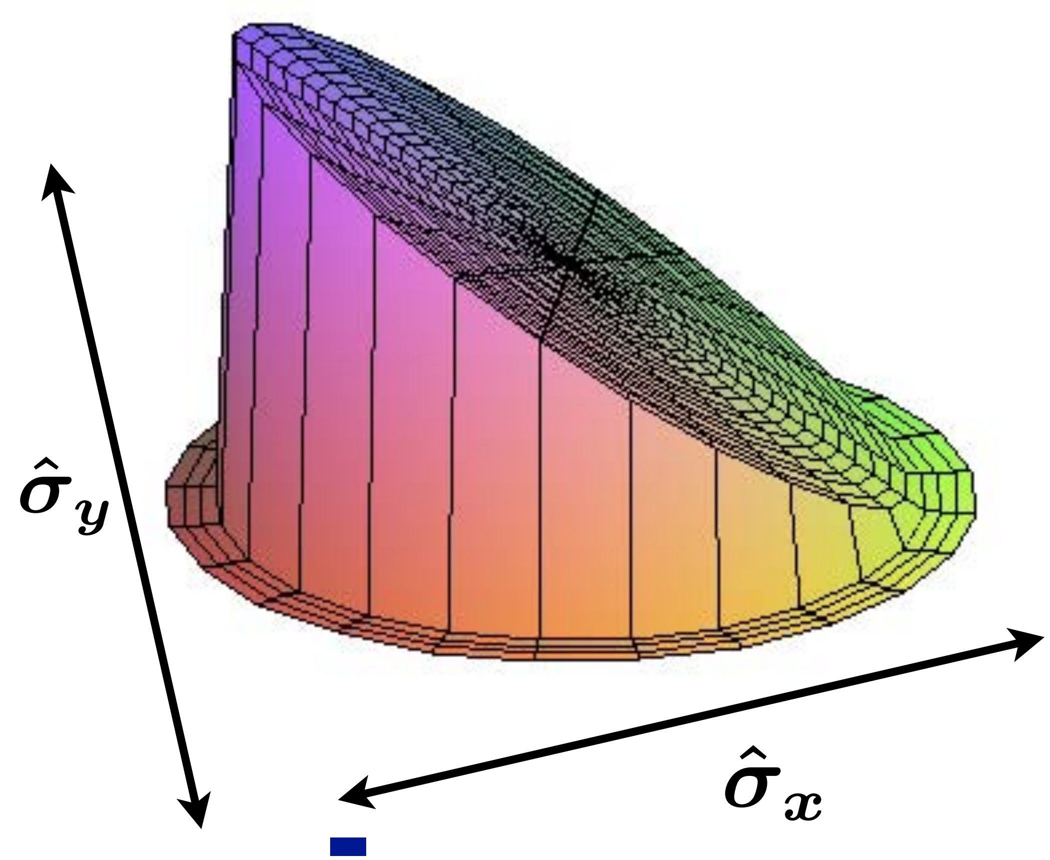

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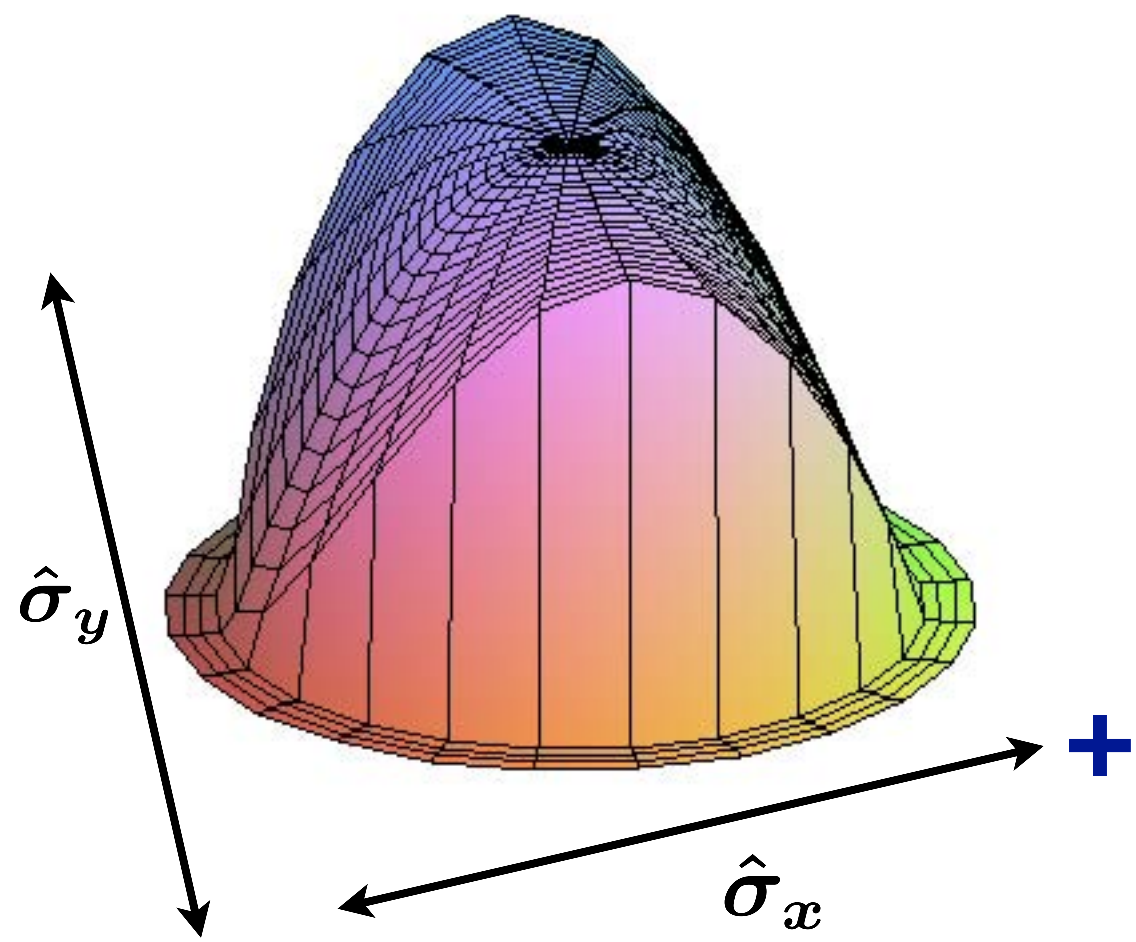

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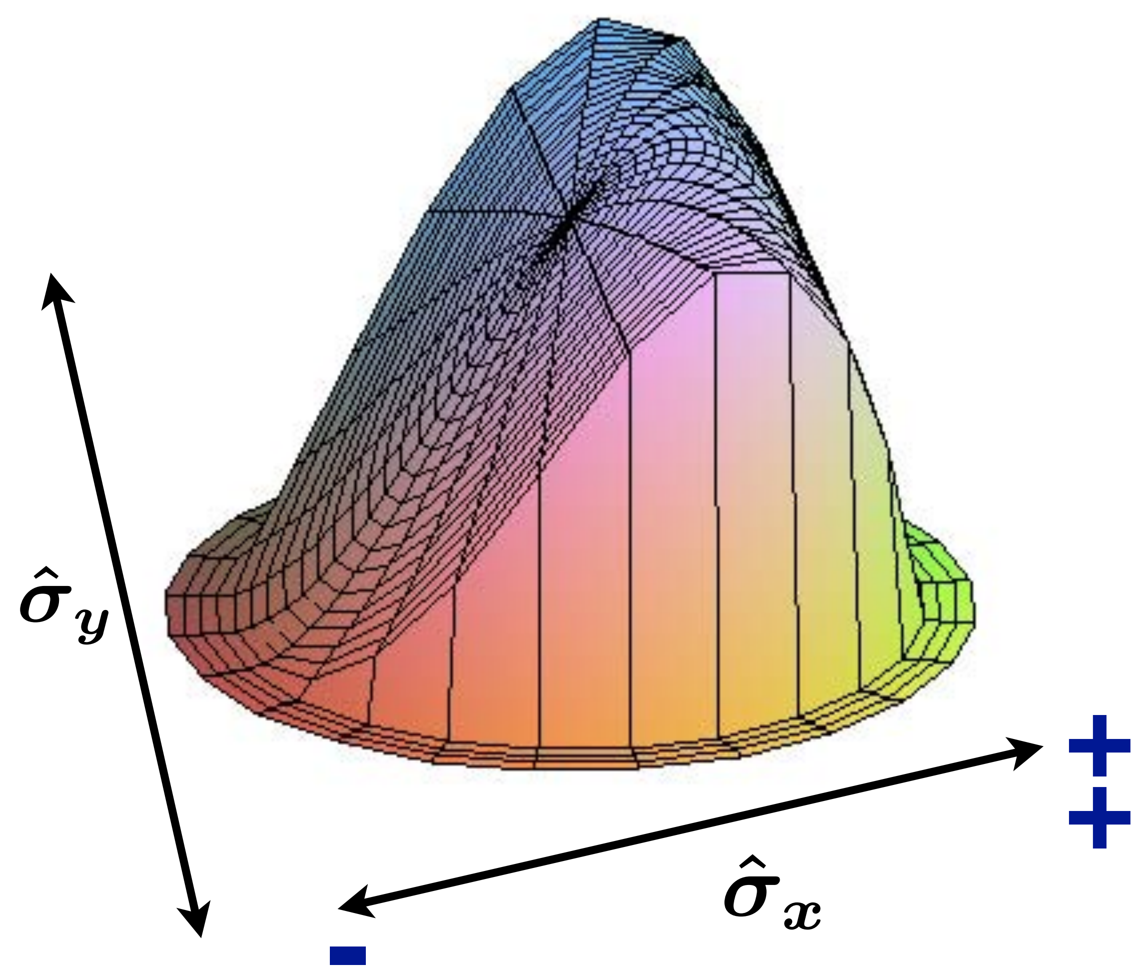

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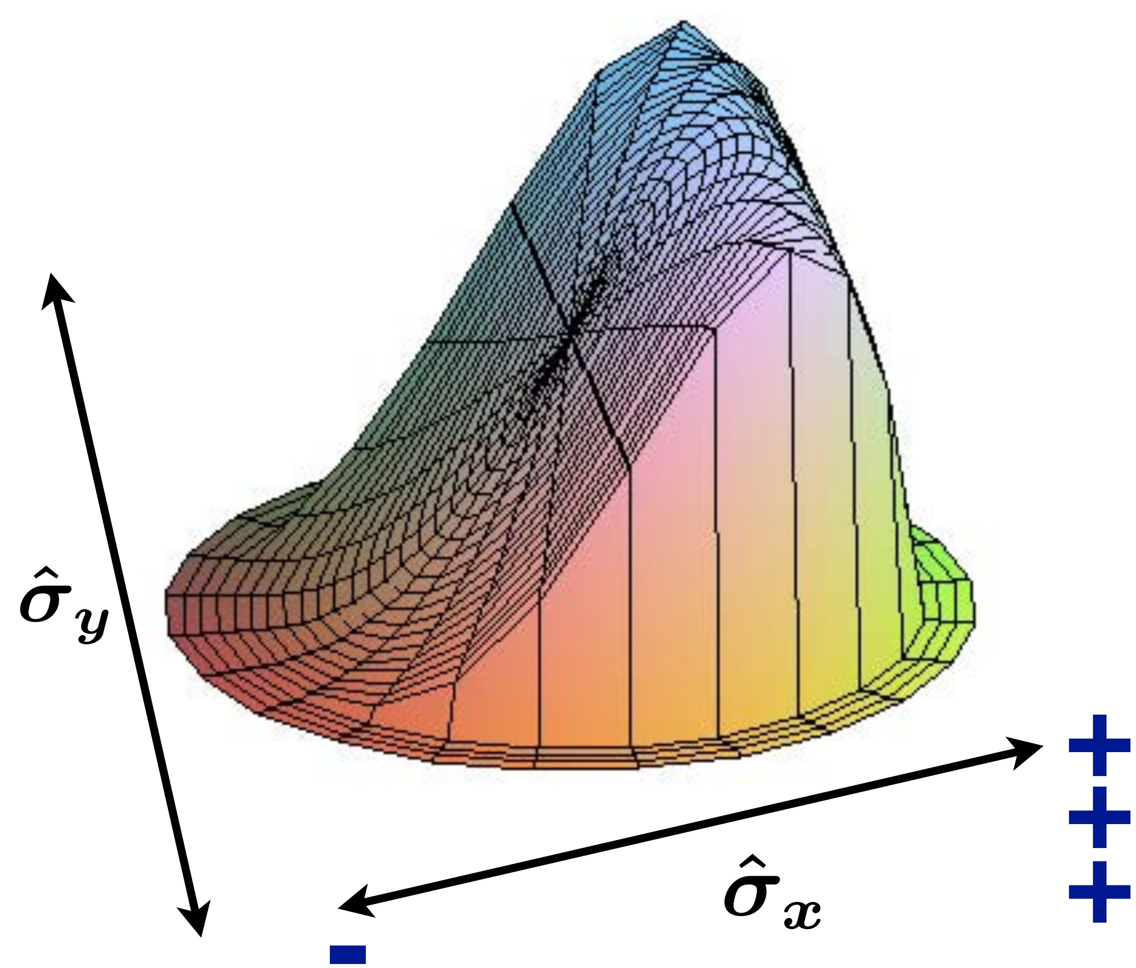

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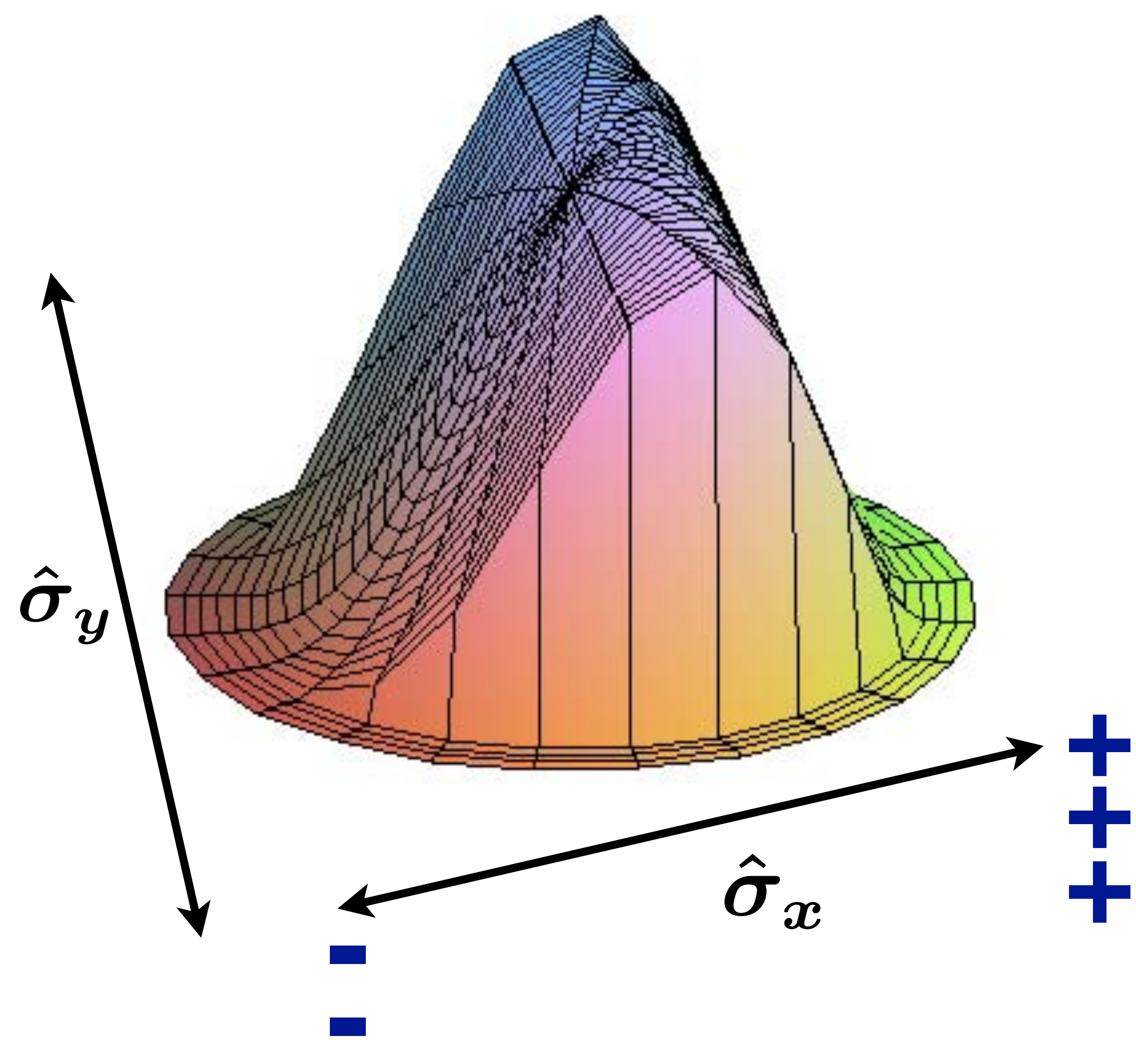

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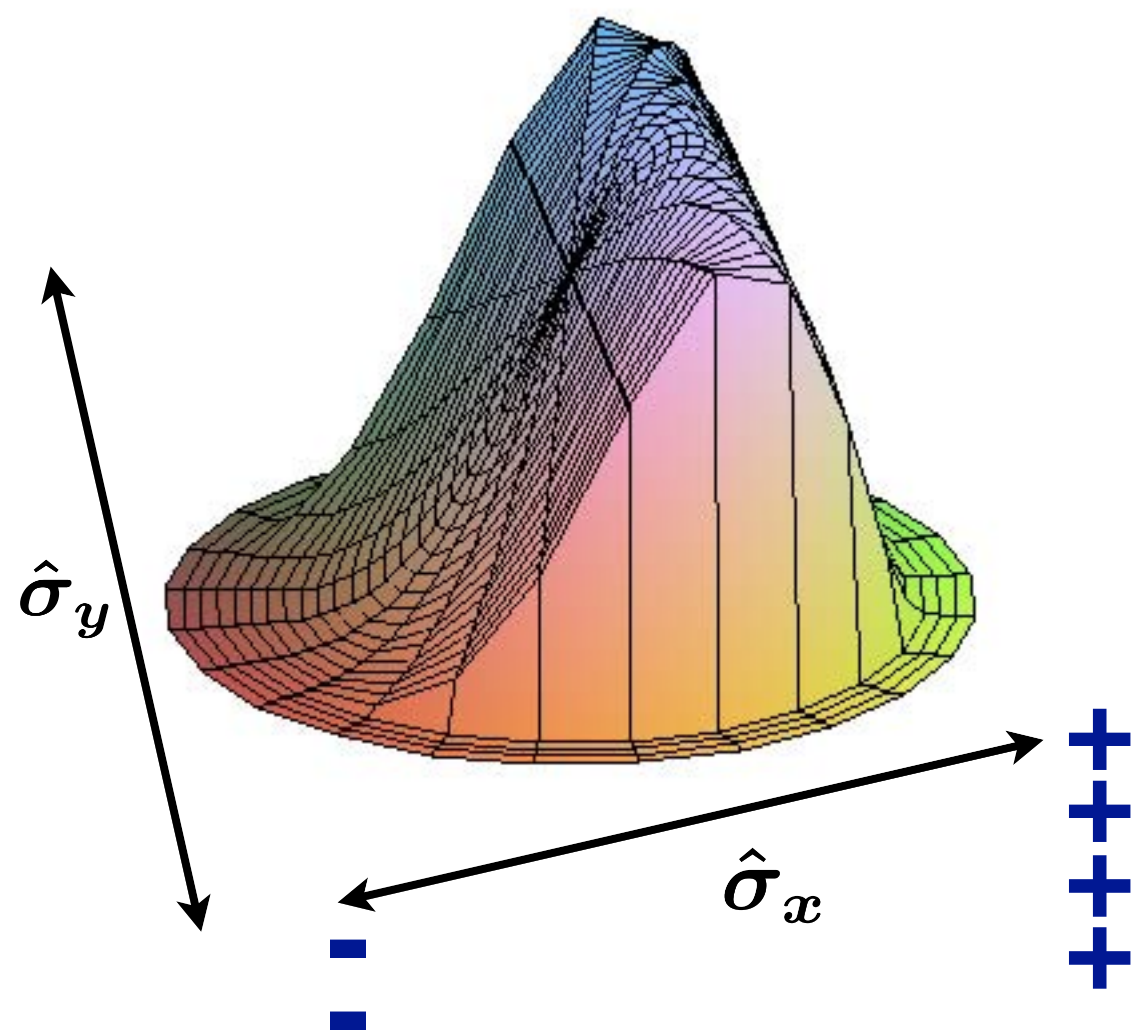
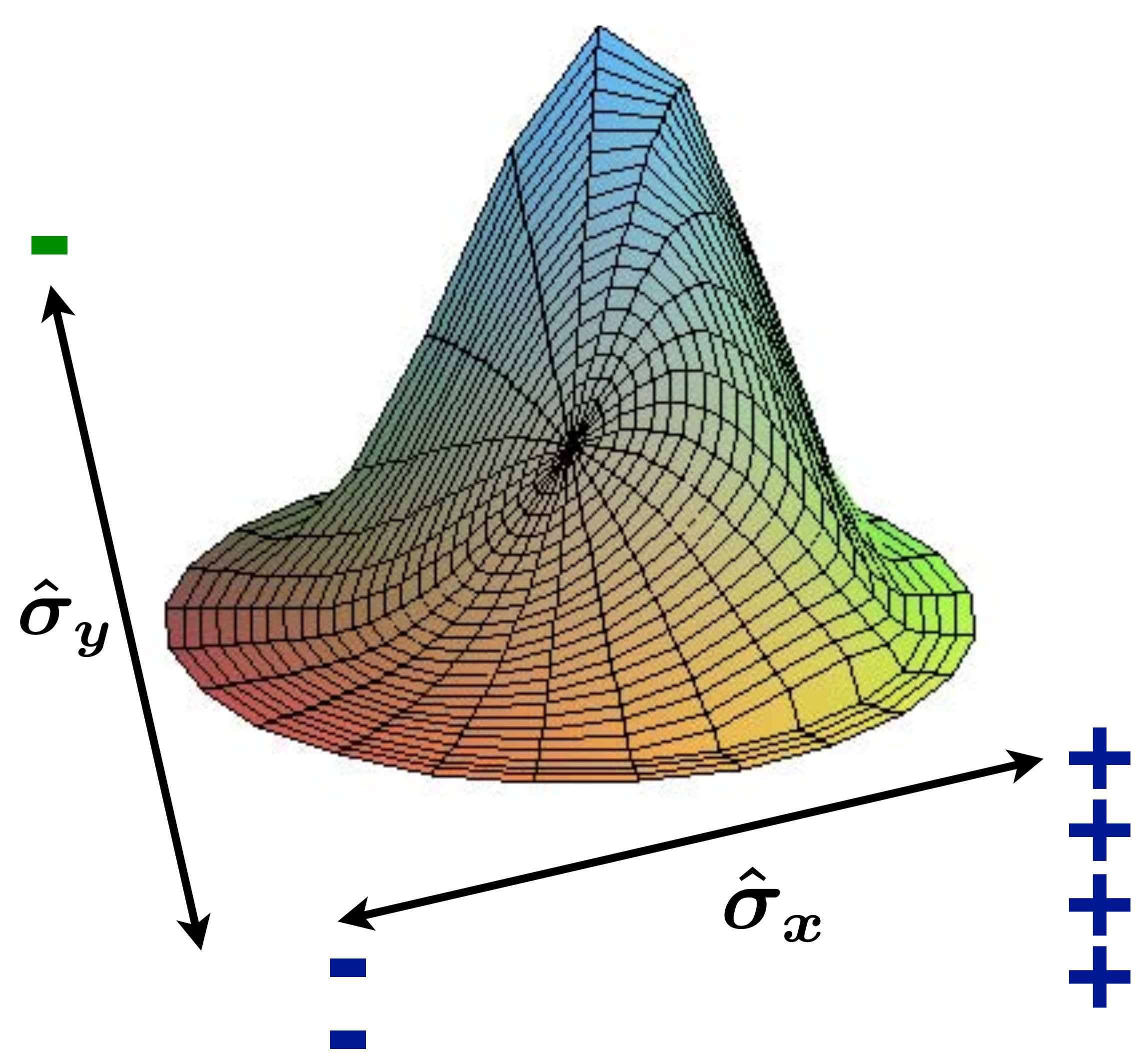


Existing methods

Bayesian Update $P_{\rm posterior}(\rho)d\rho \;\alpha\; {\rm Prob}[f|\rho]P_{\rm prior}(\rho)d\rho$

From posterior compute estimate and error bar



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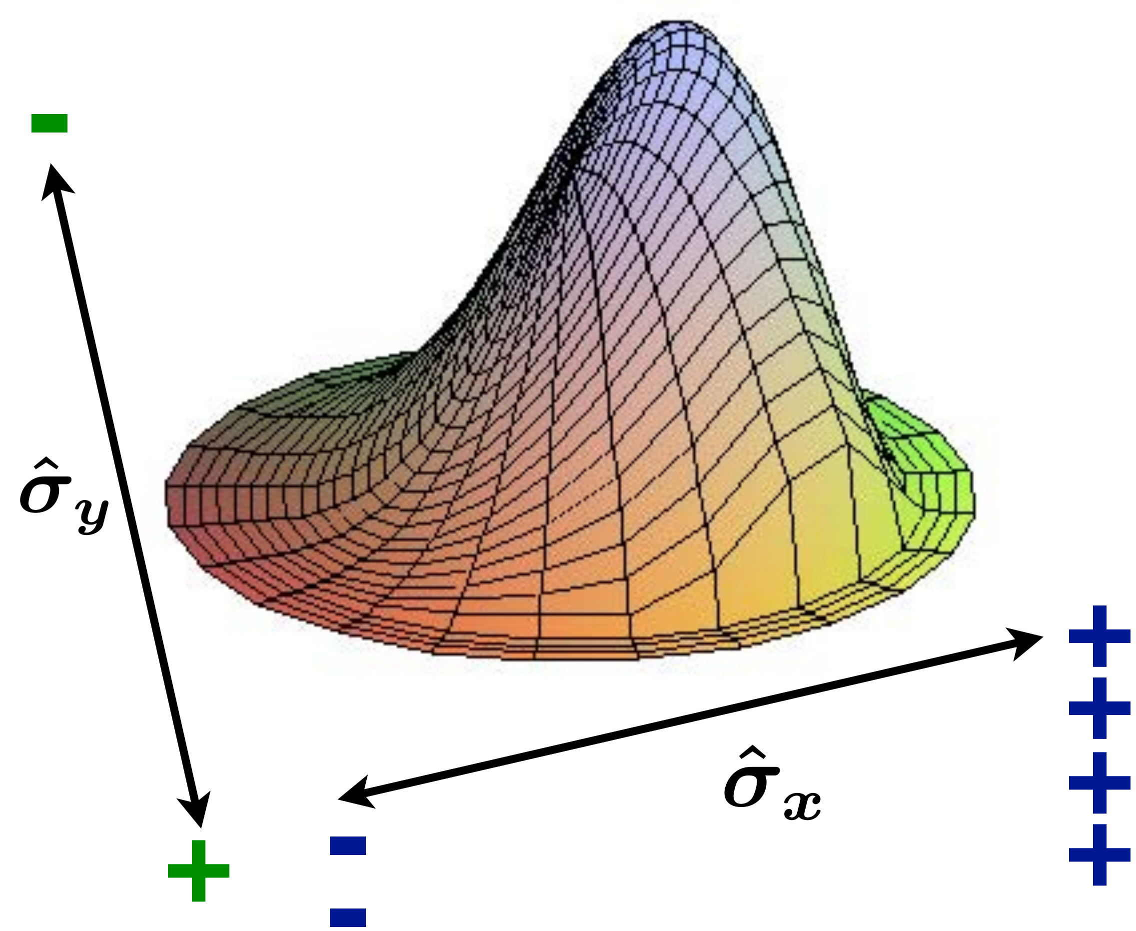

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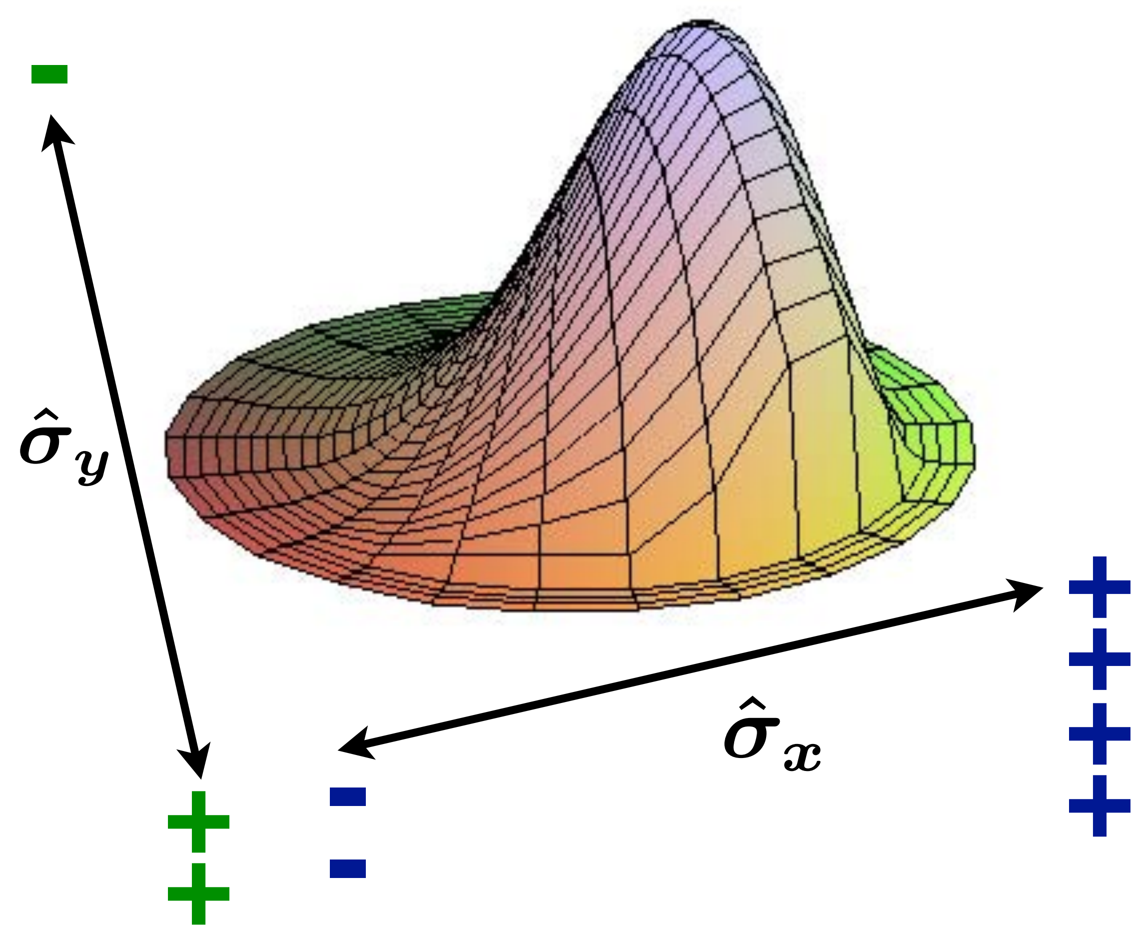

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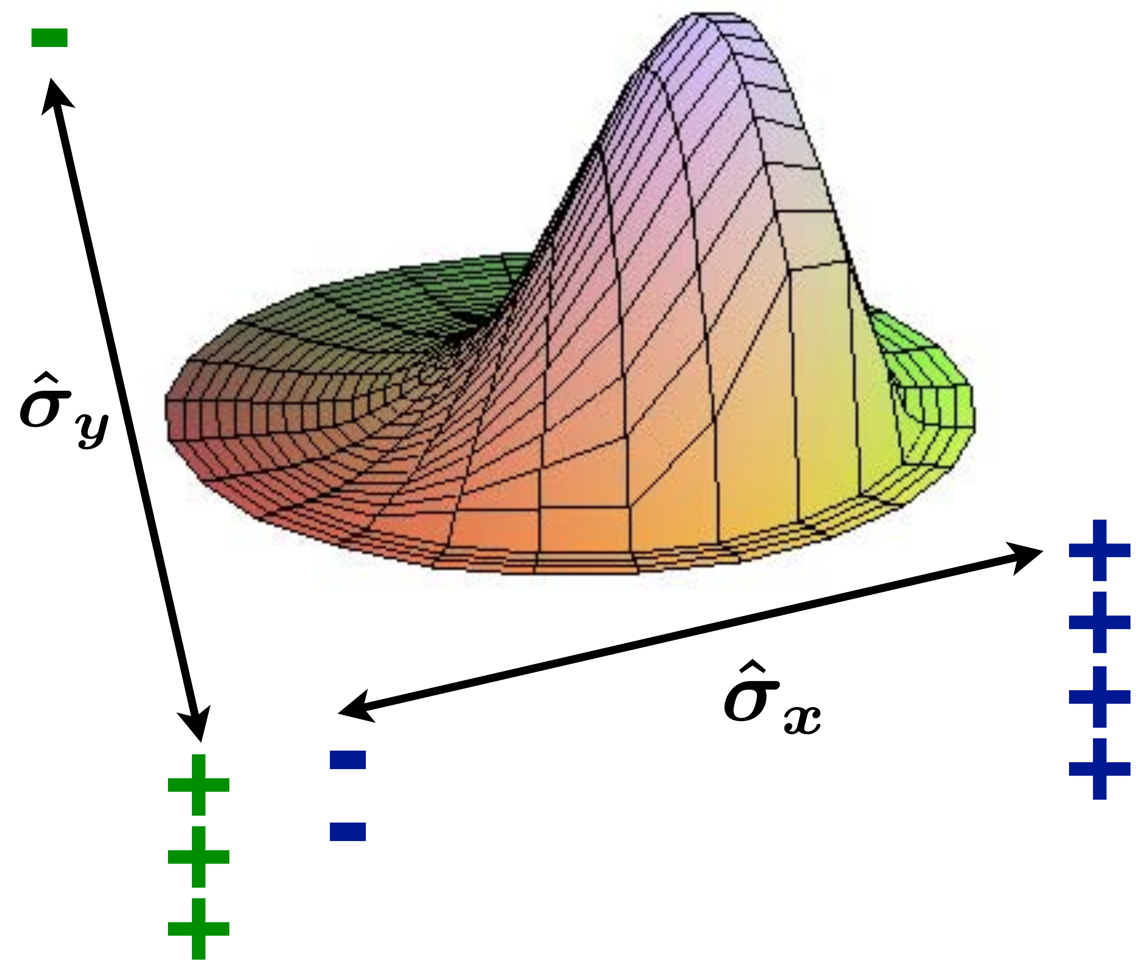

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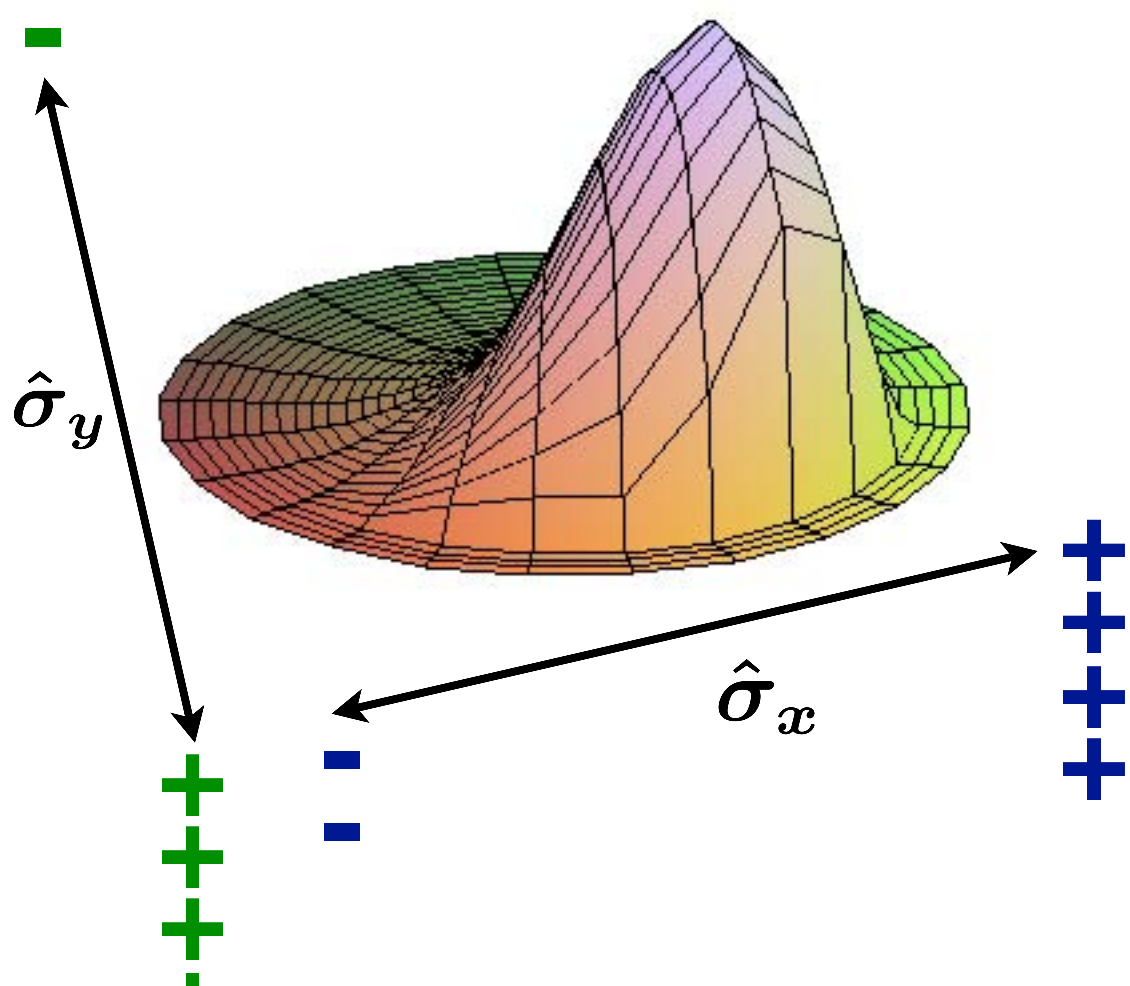

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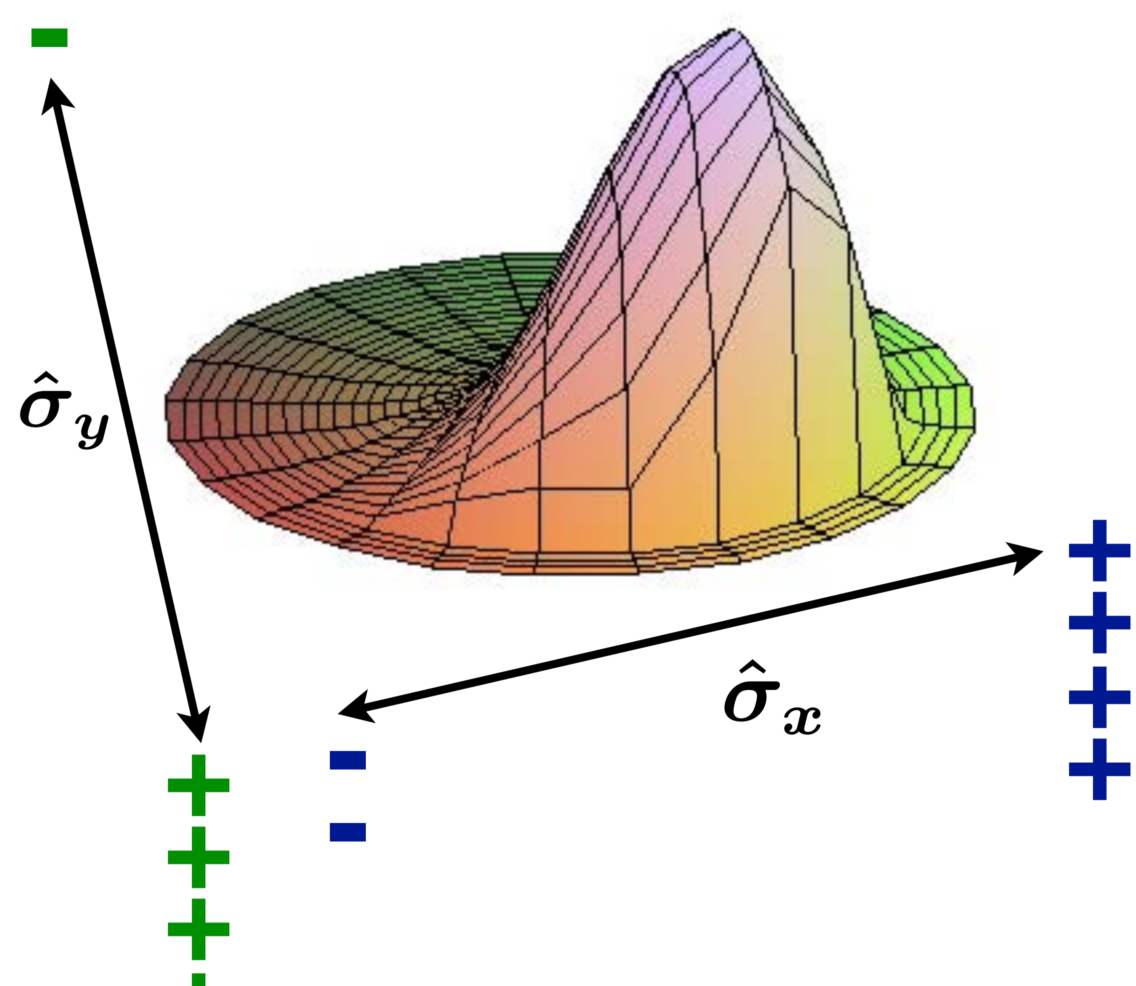

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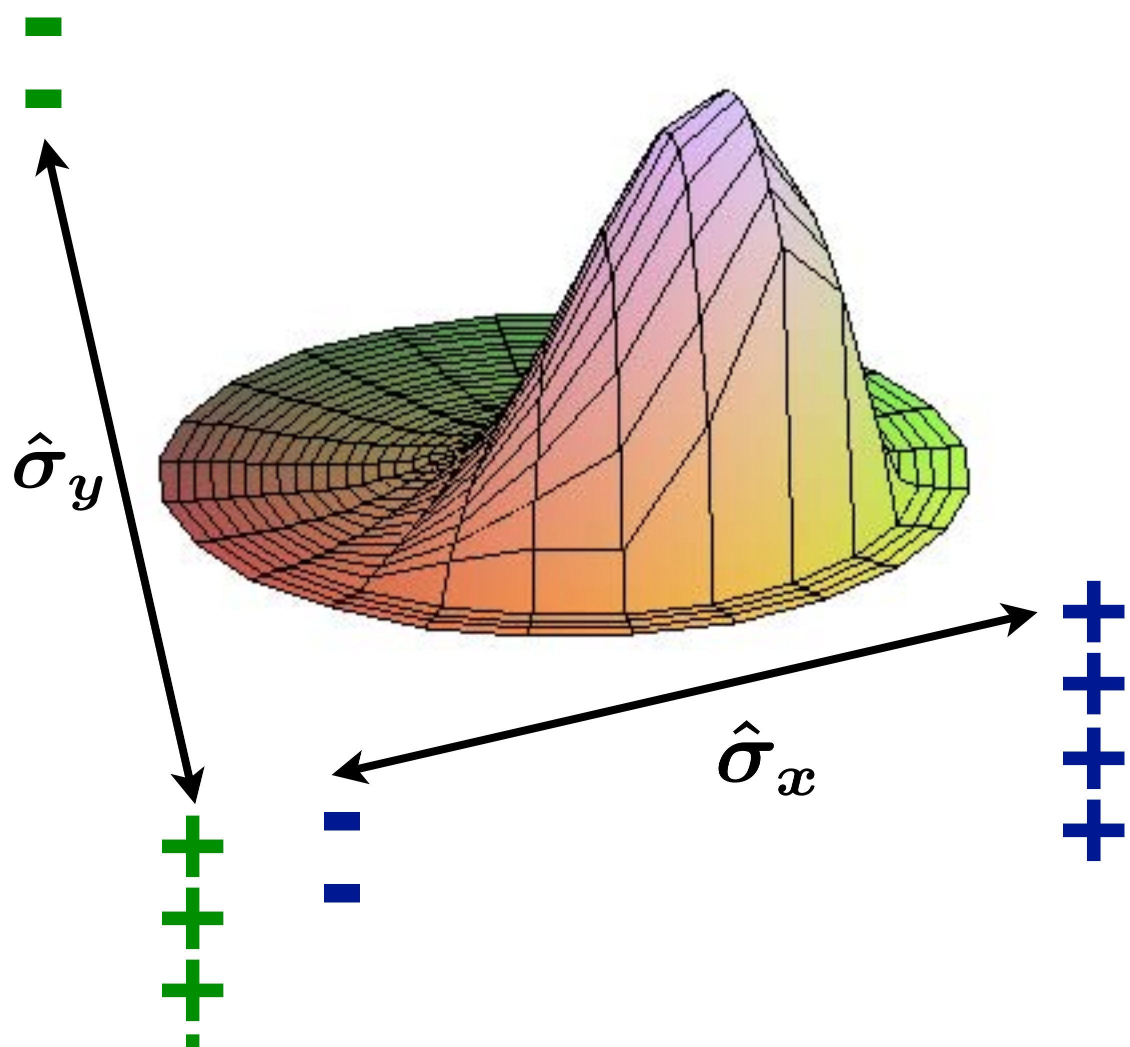

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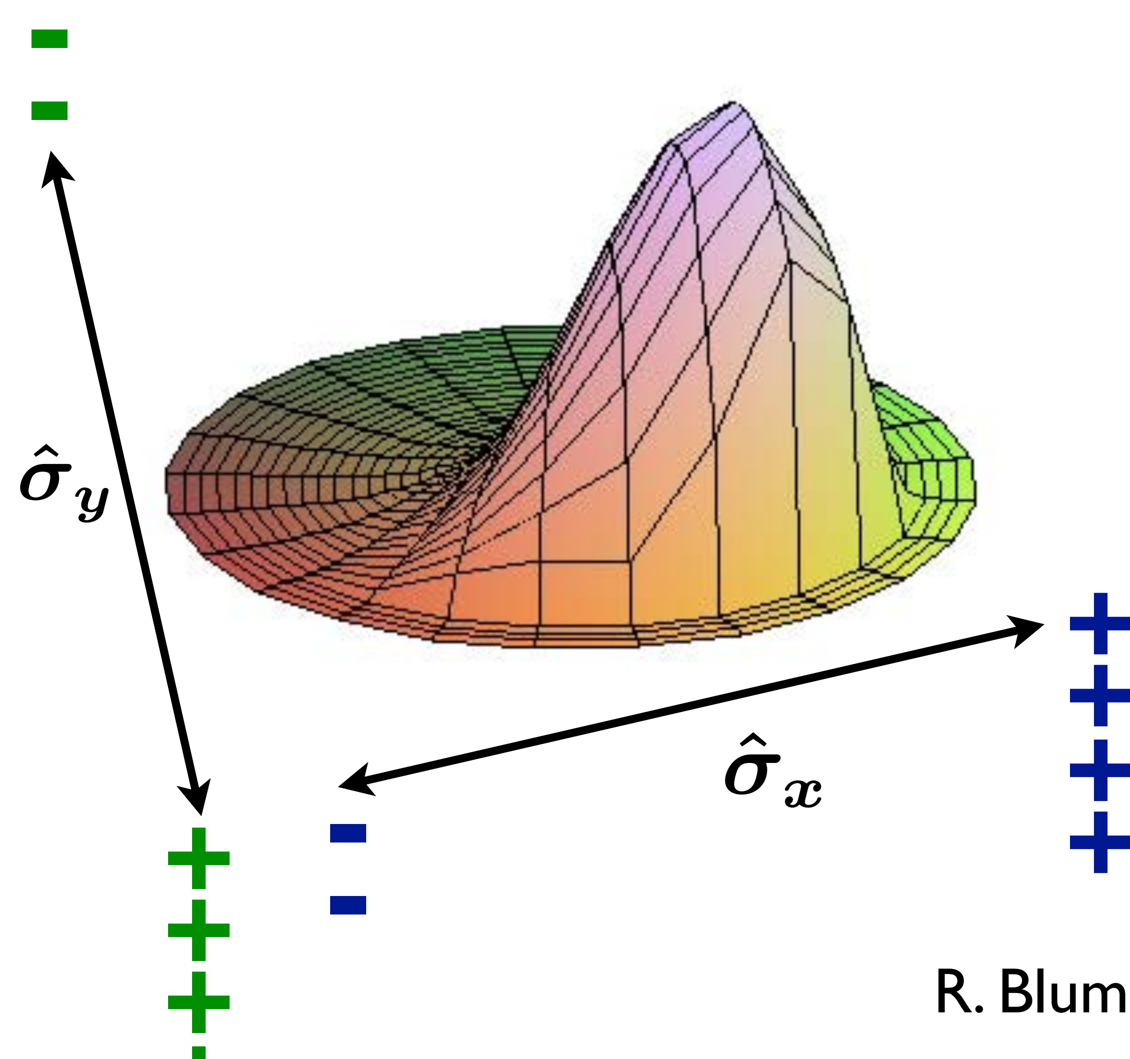


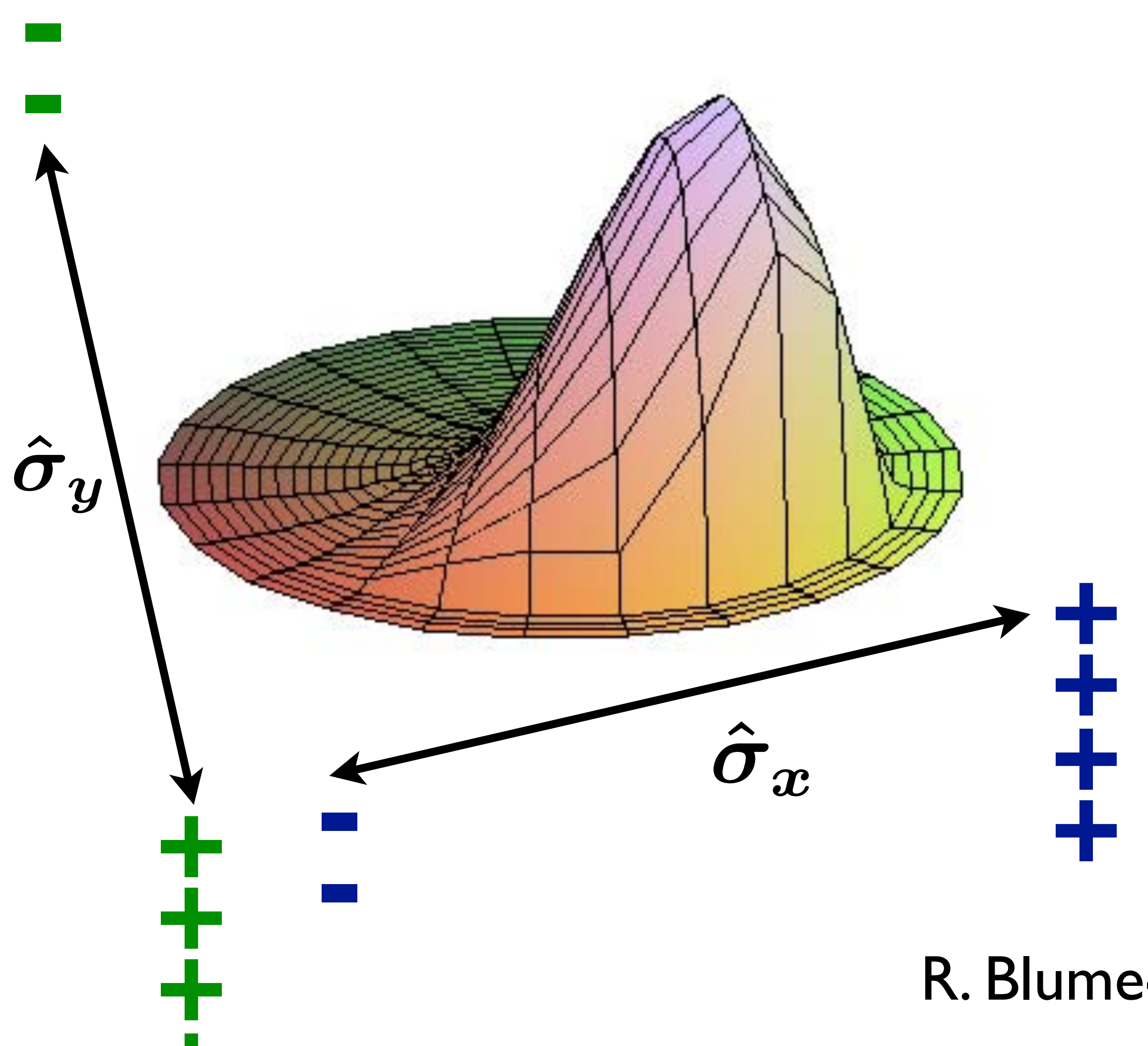
R. Blume-Kohout

Existing methods

Bayesian Update $P_{\mathrm{posterior}}(\rho)d\rho \;\alpha\; \mathrm{Prob}[f|\rho]P_{\mathrm{prior}}(\rho)d\rho$

From posterior compute estimate and error bar

depends on prior

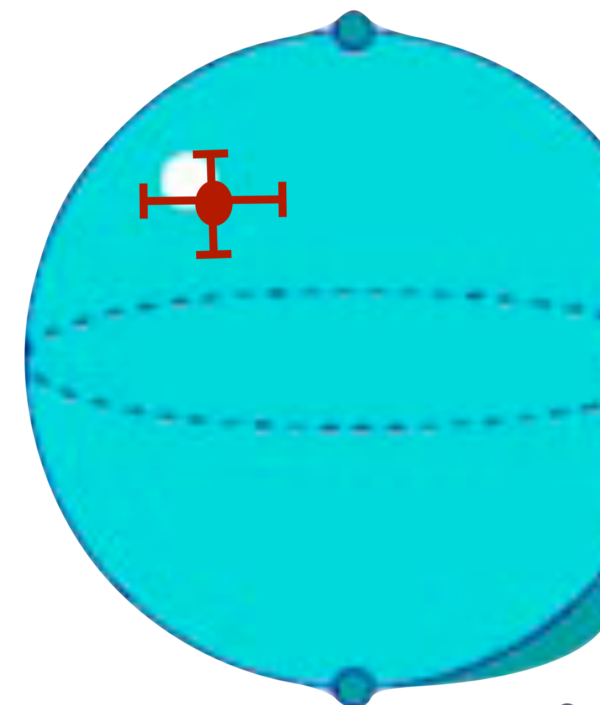


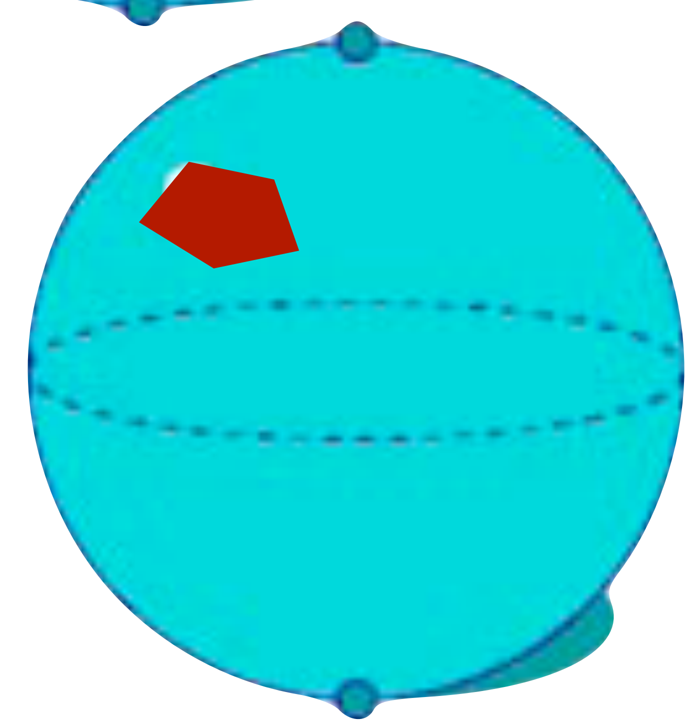
R. Blume-Kohout

Experimenter's Goal

- Given experimental data
- Compute estimate & error bar (region)
- Requirement:
  True state lies within region with high probability (over data)

  Region is small



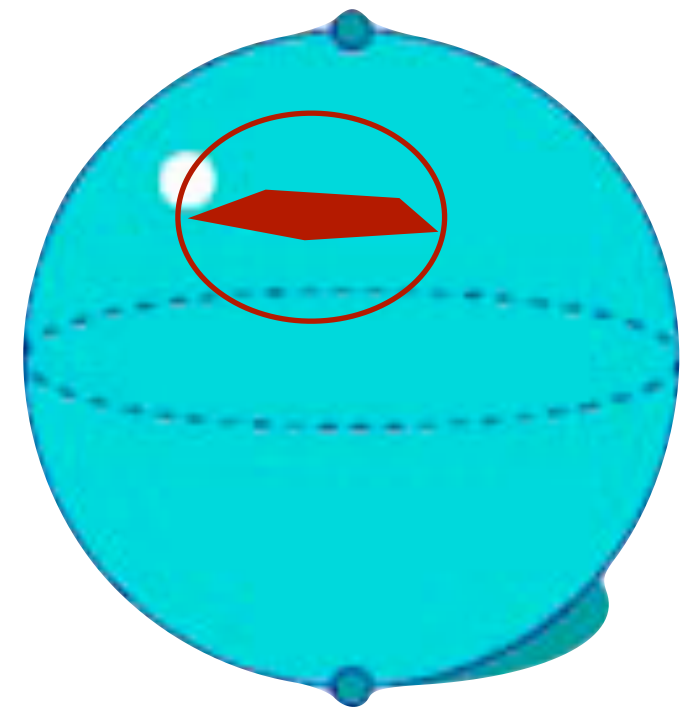


Strict necessity

- Scientific Community
(otherwise reported experimental data may be falsified)
- Quantum Cryptography
(has tomography as subroutine)
- Fault-tolerant Quantum Computing
(needs certified components)

Existing methods

- Large deviations bounds
- Specific measurements
- Focus on efficiency



Compressed sensing

Gross, Liu, Flammia, Becker & Eisert, PRL 2010
Gross, IEEE Trans. Inf. Th. 2011
Liu, NIPS 2011

Matrix-Product-State tomography

Cramer, Plenio, Flammia, Gross, Bartlett, Somma, London-Cardinal, Liu & Poulin
Nat. Comm. 2011

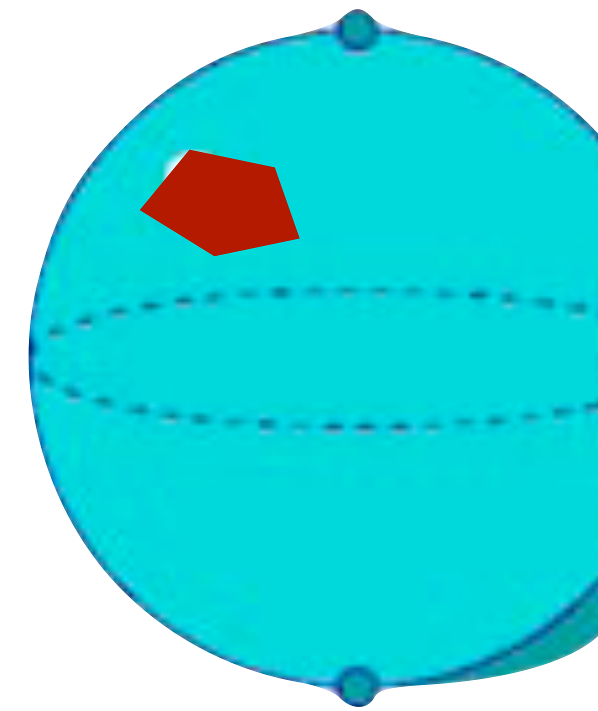
Analysis of experiments

Sugiyama, Turner, &Murao
PRA 2011

Main Result

Our Work

- Construct small regions
- For *any* given measurement



independent & identical measurement

$\{E_i\}$ outcome

$$B^n = E_3 \otimes E_1 \otimes \cdots \otimes E_3 \otimes E_2$$

adaptive measurements

setting dependent on previous outcome

$\{E_i\}$

$$B^n = E_3^1 \otimes E_1^2 \otimes \cdots \otimes E_3^{n-1} \otimes E_2^n$$

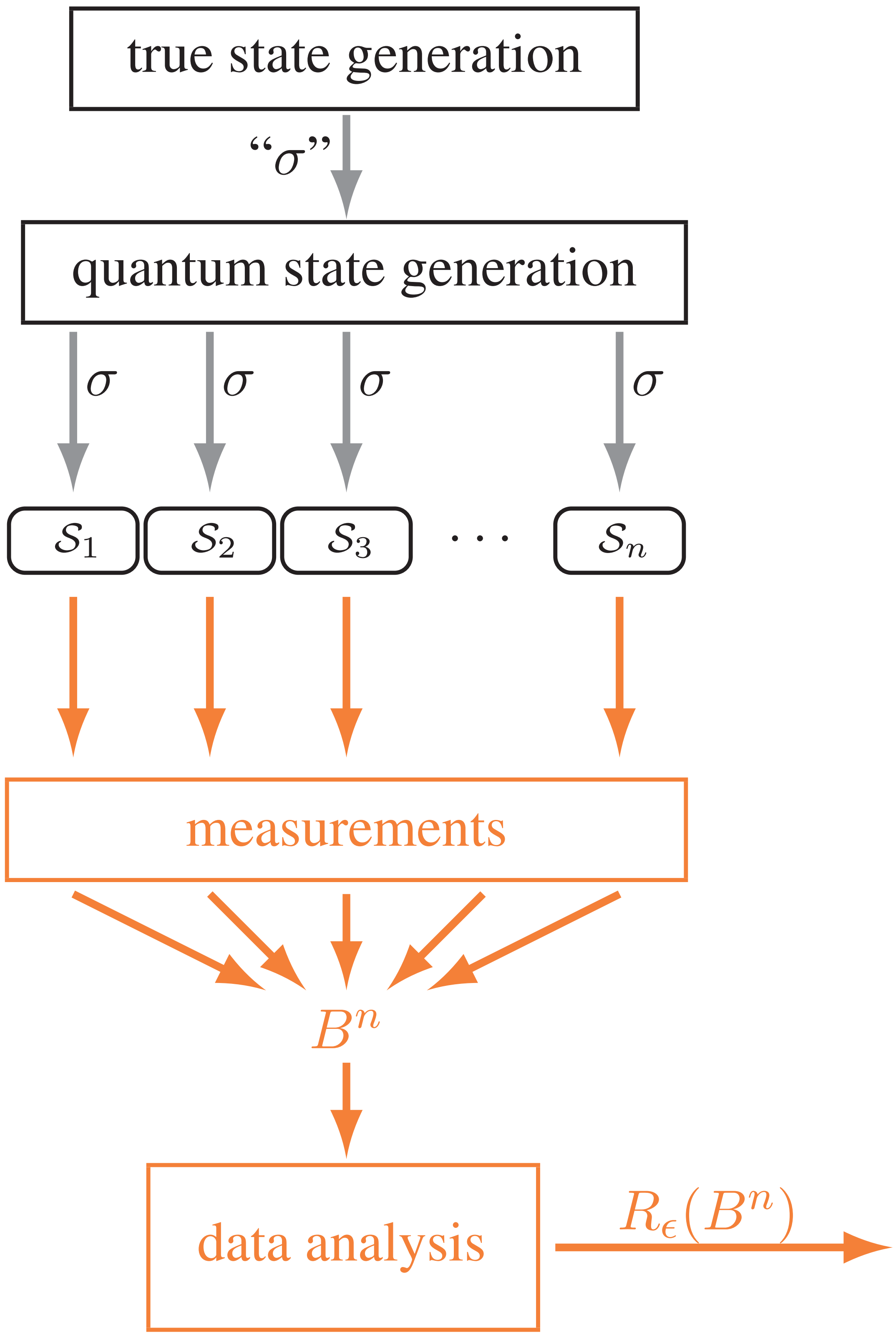
coherent measurements

$\{B^n\}$

B^n

closely related work R. Blume-Kohout QIP 2012 (talk M.Ch.)

Main Result



We derive region estimators such that the true state is contained in the region with high probability (over the data):

$$\mathrm{Prob}_{B^n}[\sigma \in R_\epsilon(B^n)] \geq 1 - \epsilon$$

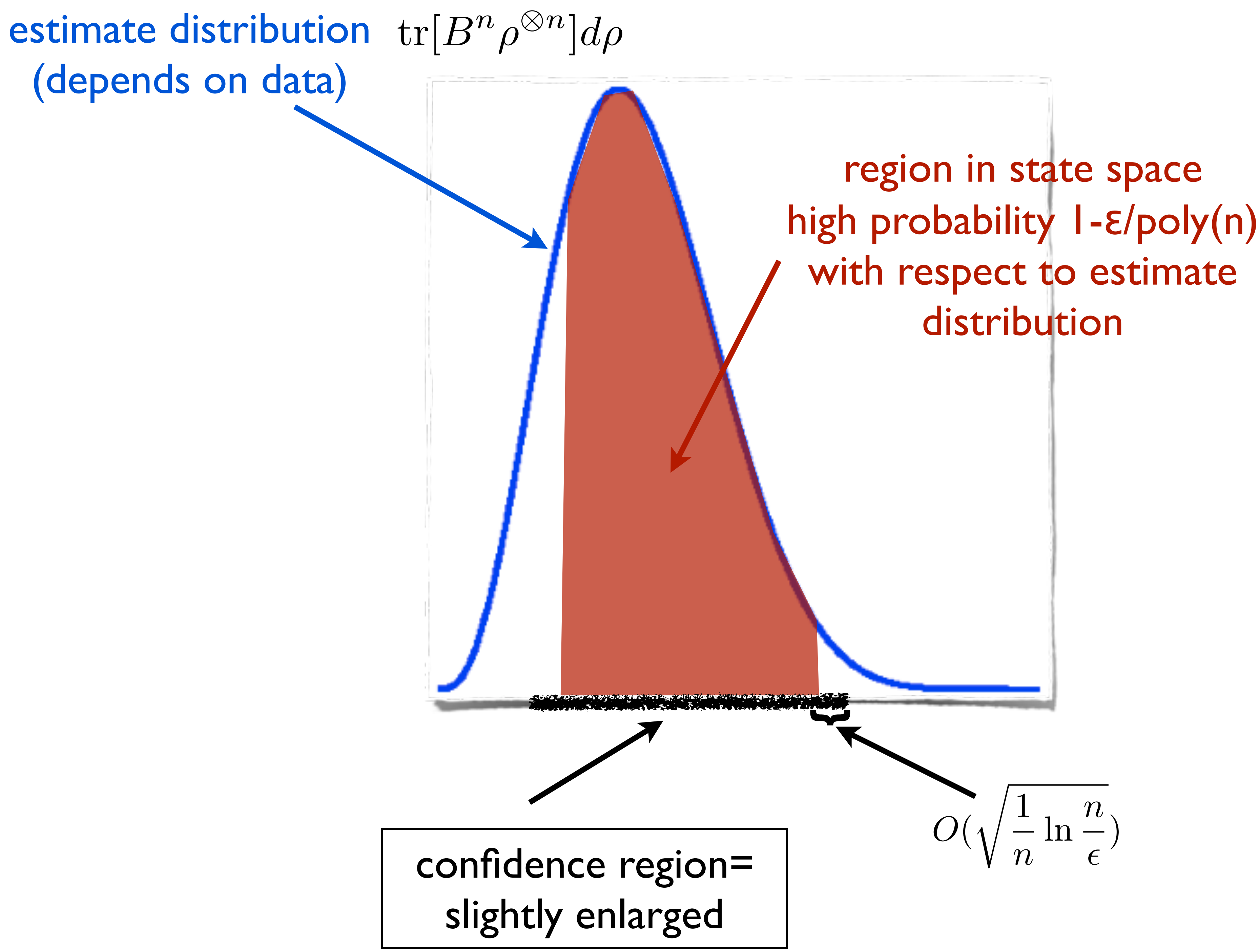
true state

region

data

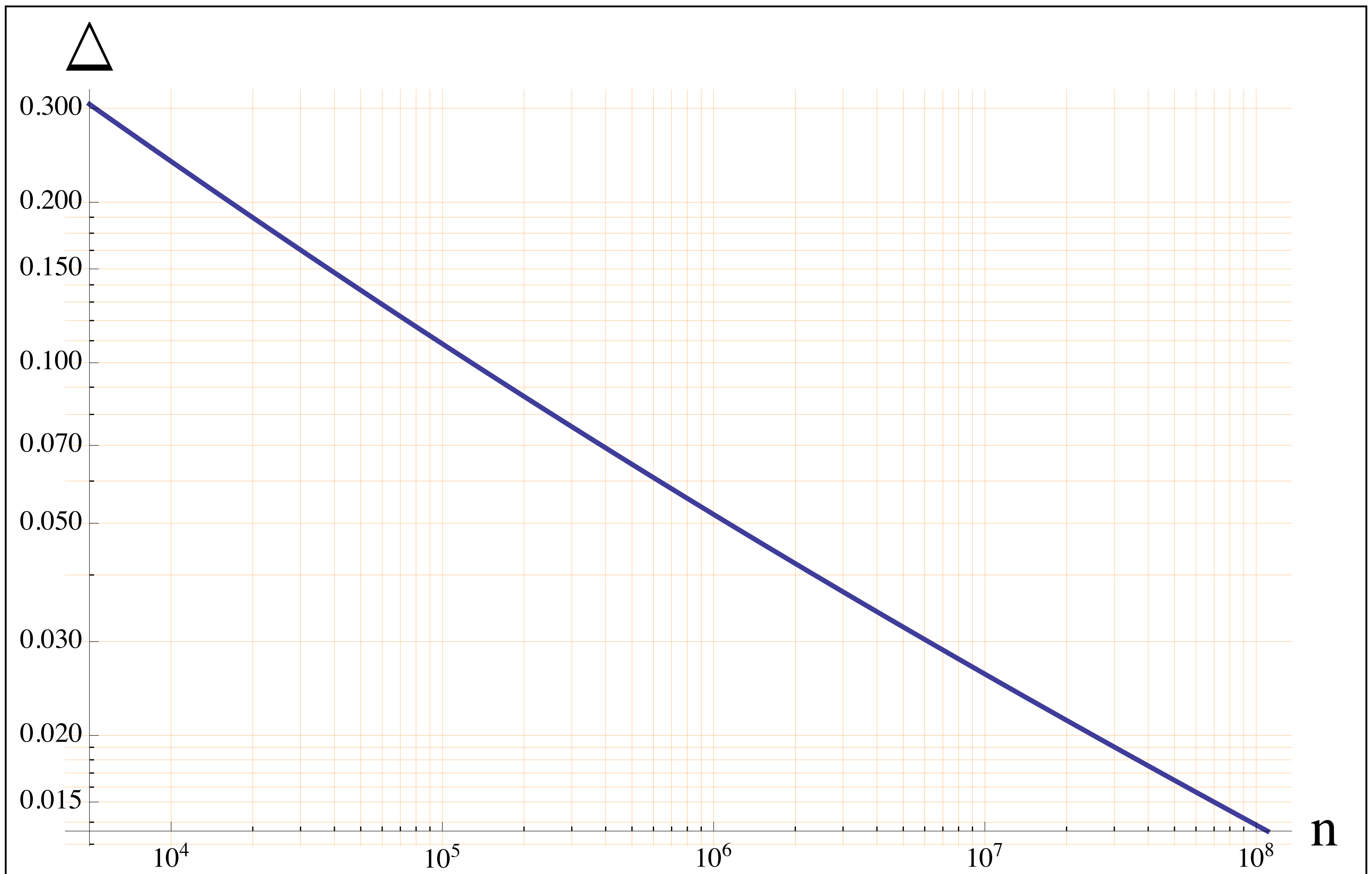
The size of the regions is minimal (in certain ways)

Definition of Regions $R_\epsilon(B^n)$



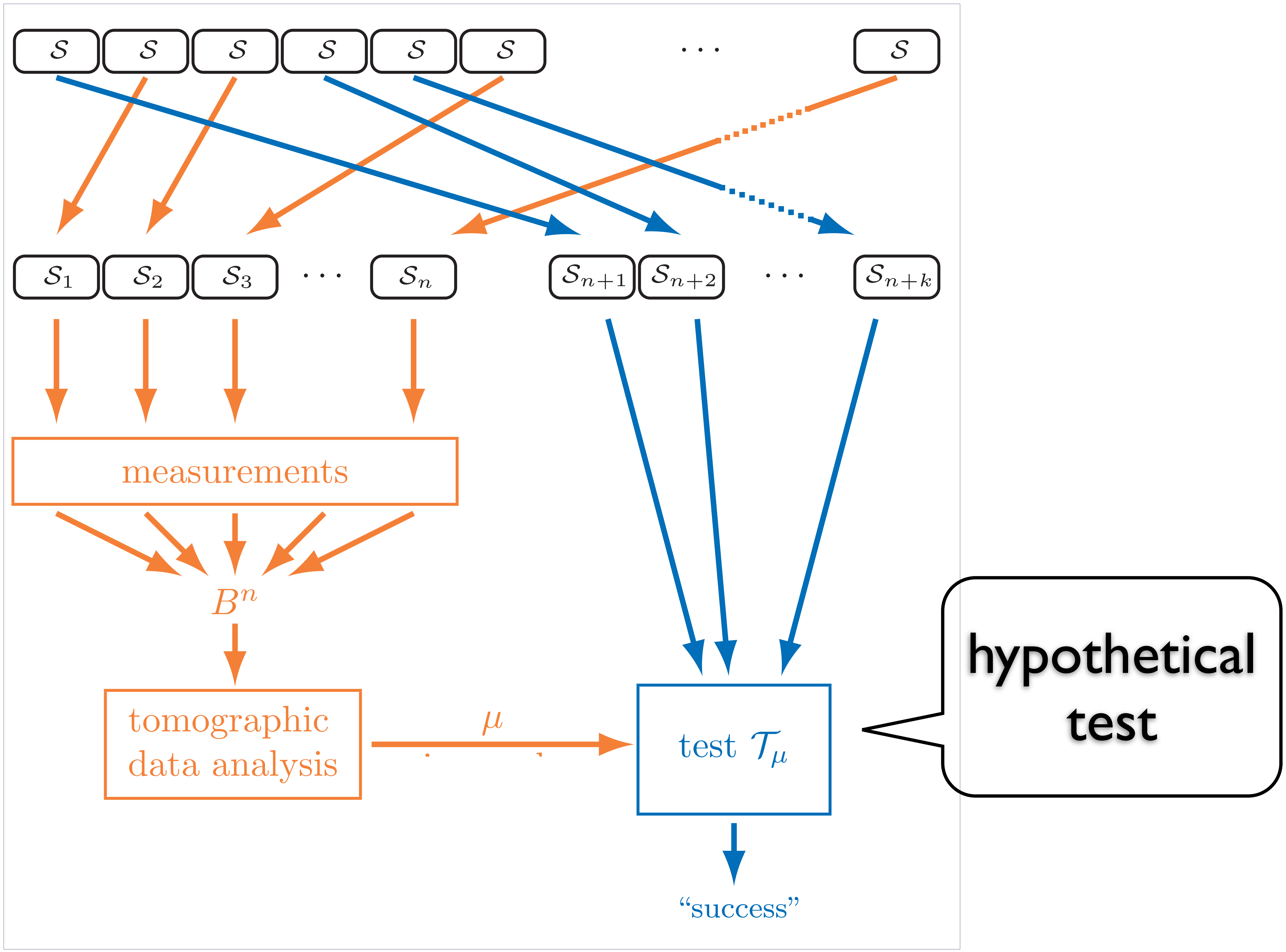
Error Bar

- fixed error probability $\varepsilon = 10^{-6}$
- Pauli measurements on qubit
- relative frequencies (0 78, 0.22), (0.66, 0.44) and (0.91, 0.09)



Technique

Measure & Predict



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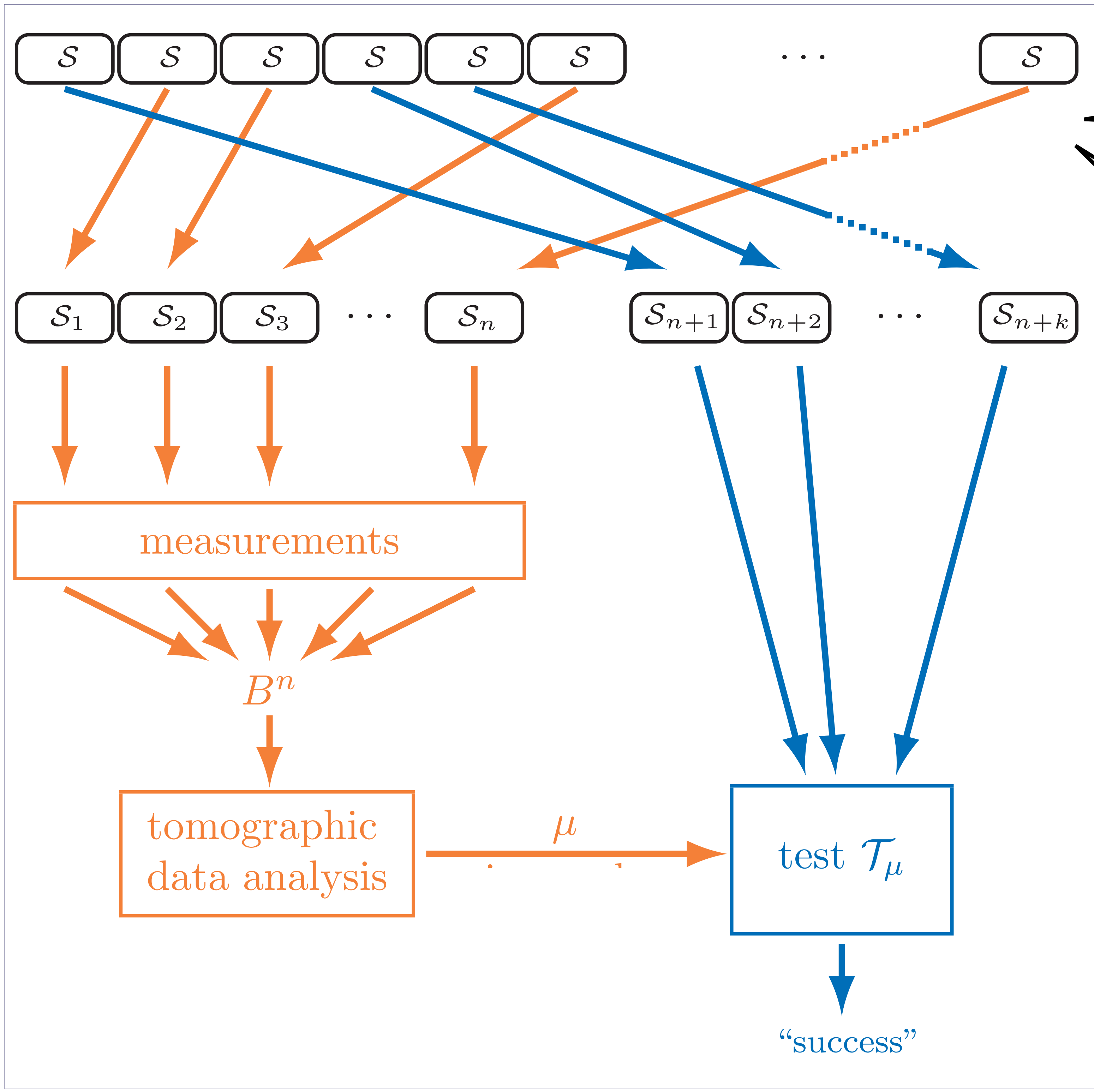


sample
n out of n+k

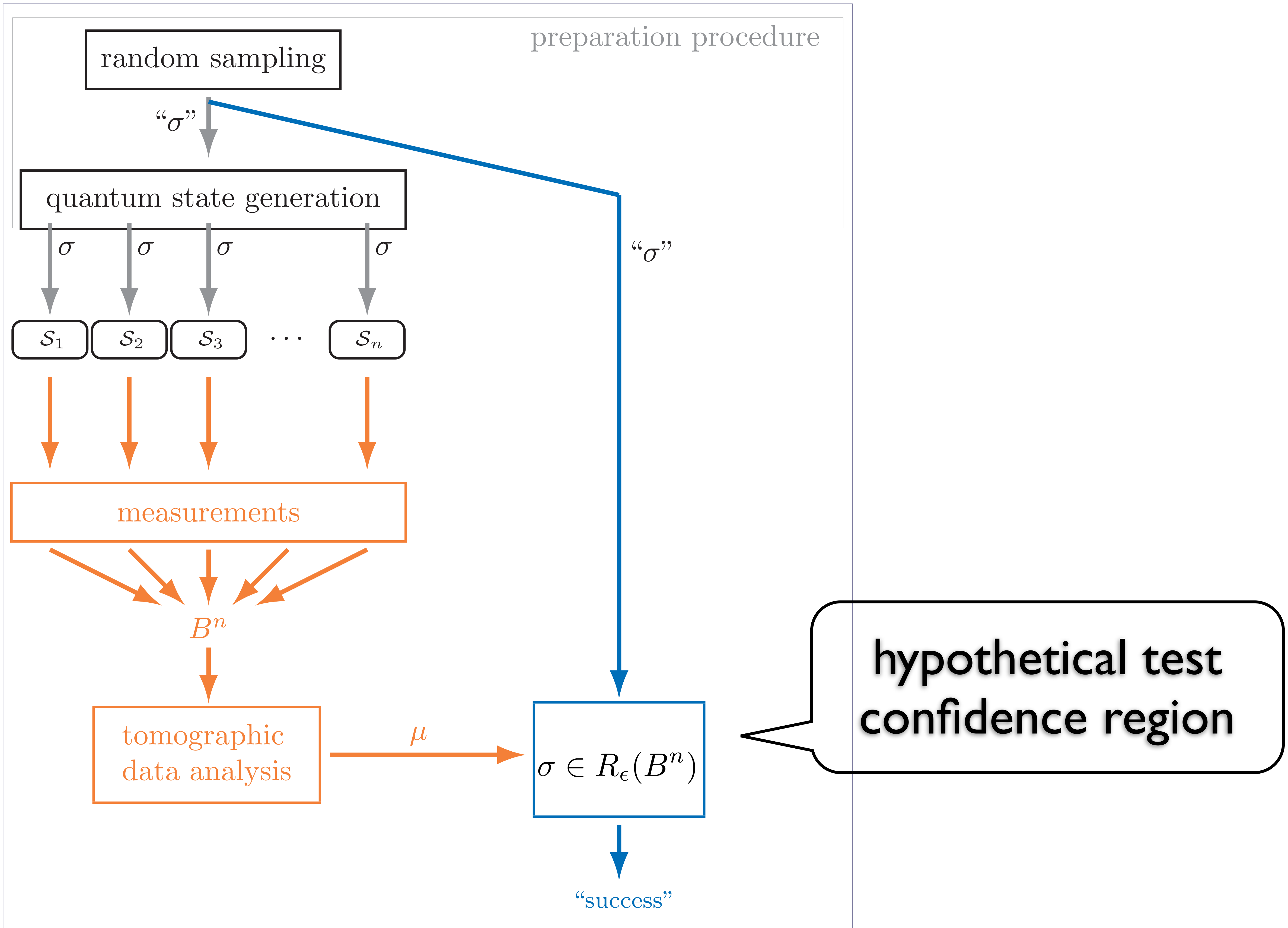
permutation-
invariant
take first n

Quantum de Finetti

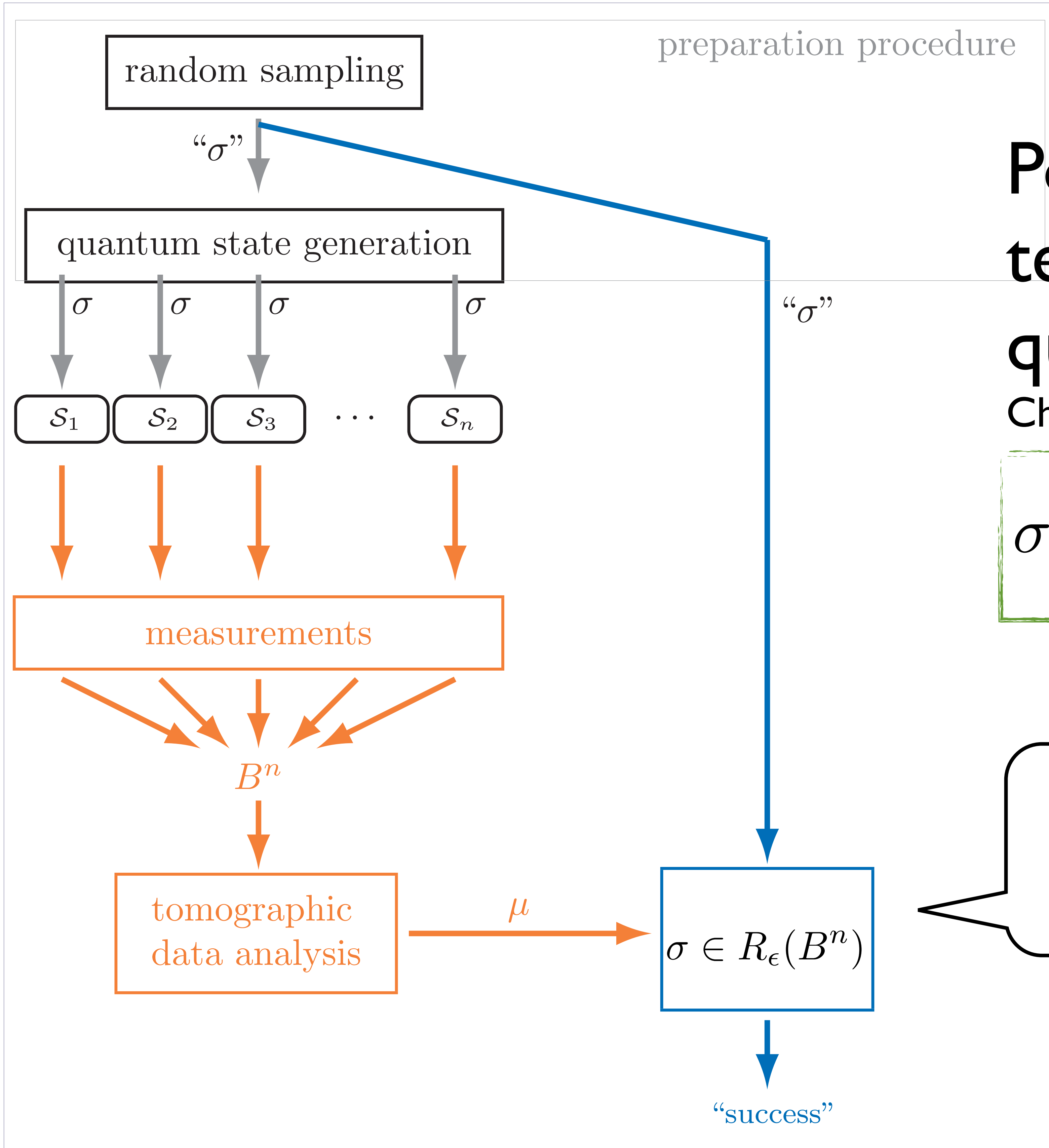
$$\rho^{n+k} \approx \int P(\sigma)\sigma^{\otimes(n+k)} d\sigma$$

large k

Measure & Predict



Measure & Predict



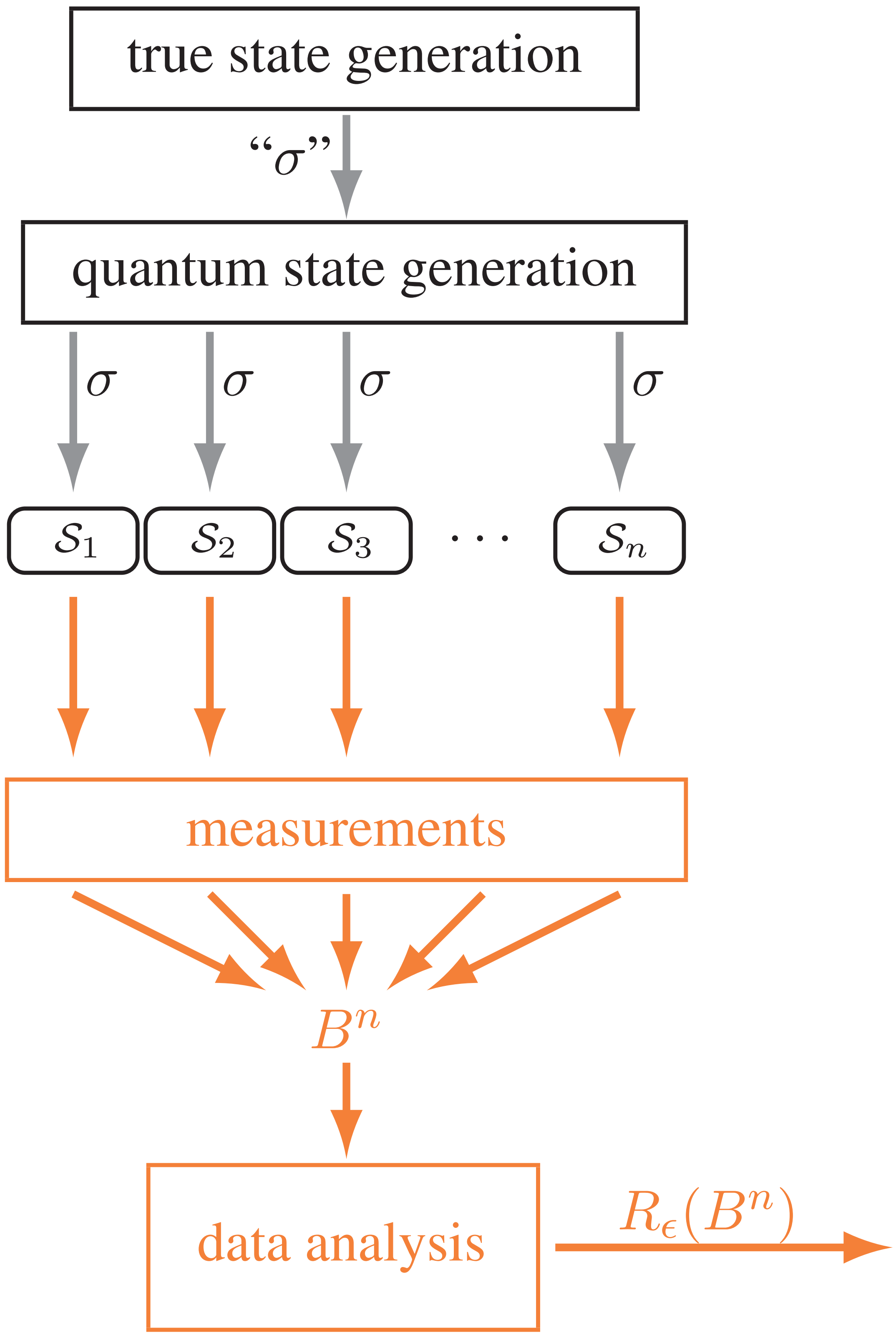
Post-selection technique for quantum channels

Ch., König, Renner, PRL 2009

$$\boxed{\sigma^{\otimes n} \leq \text{poly}(n) \int \rho^{\otimes n} d\rho}$$

hypothetical test
confidence region

Main Result



We derive region estimators such that the true state is contained in the region with high probability (over the data):

$$\mathrm{Prob}_{B^n}[\sigma \in R_\epsilon(B^n)] \geq 1 - \epsilon$$

true state

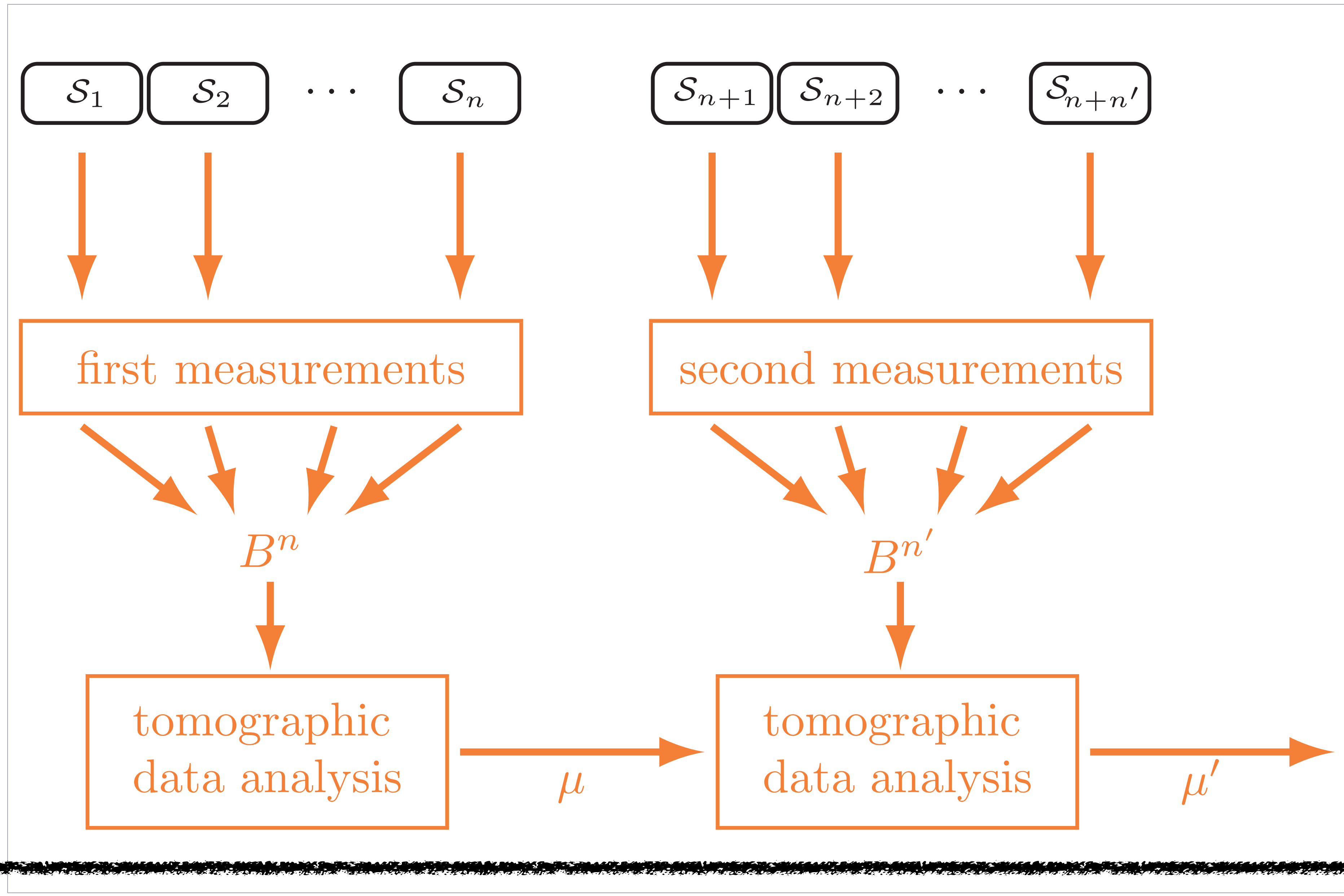
region

data

The size of the regions is minimal (in certain ways)

Features

Update the estimate distribution



$$\mu_{B^n \otimes B^{n'}}(\sigma) := \frac{c_{B^n}}{c_{B^n \otimes B^{n'}}} \mu_{B^n}(\sigma) \, Q_{B^{n'}}(\sigma) \qquad Q_{B^{n'}}(\sigma) := \mathrm{tr}[\sigma^{\otimes n'} B^{n'}]$$

•

Fourier Transform

- Convenient when processing information further

generalised spherical harmonics

$$\nu_{B^n}(x) = \sum_{\ell,m}^{n} \nu_{B^n}(\ell, m) y_{\ell,m}(x)$$

- Update rule

derived from U(d^2) Clebsch-Gordan coefficients

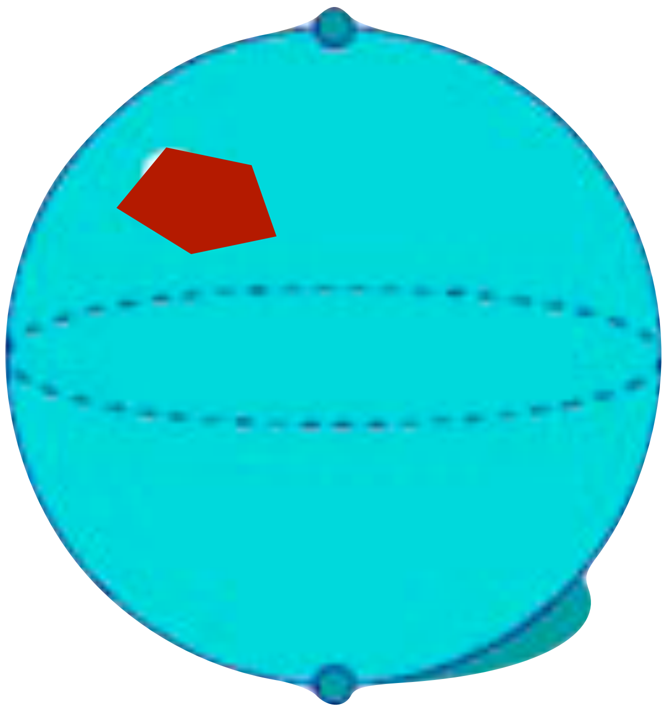
$$\nu_{B^n \otimes B^{n'}}(\ell'', m'') = (\nu_{B^n} * q_{B^{n'}})(\ell'', m'')$$

$$:= \frac{c_{B^n}}{c_{B^n \otimes B^{n'}}} \sum_{\ell,m}^{\ell_{\max}} \sum_{\ell',m}^{\ell'_{\max}} \nu_{B^n}(\ell, m) q_{B^{n'}}(\ell', m') \left\{ \begin{array}{ccc} \ell & \ell' & \ell'' \\ m & m' & m'' \end{array} \right\}$$

- Description grows by degree of outcome

$$\ell''_{\max} \leq \ell_{\max} + \ell'_{\max} \leq n + n'$$

Conclusion



$$\mathrm{Prob}_{B^n}[\sigma \in R_\epsilon(B^n)] \geq 1 - \epsilon$$

true state

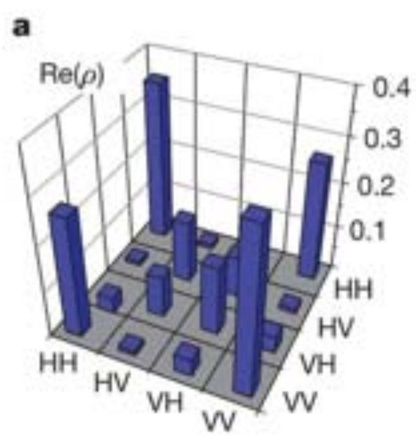
region

data

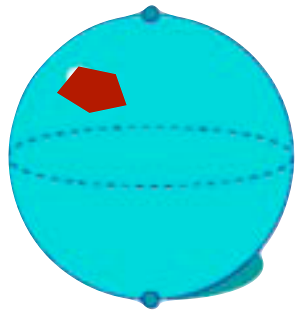
Use it in your next experiment!

Past, Present, Future

Nature
author, author, ...author

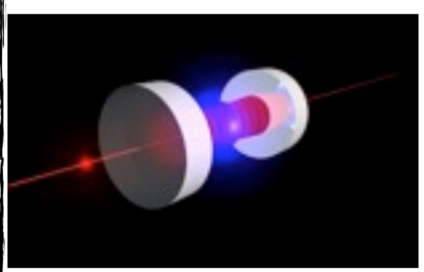


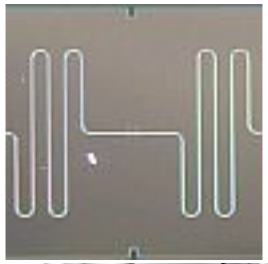
Nature
author, author, ...author



Nature
author, author, ...author

see quantum attachment

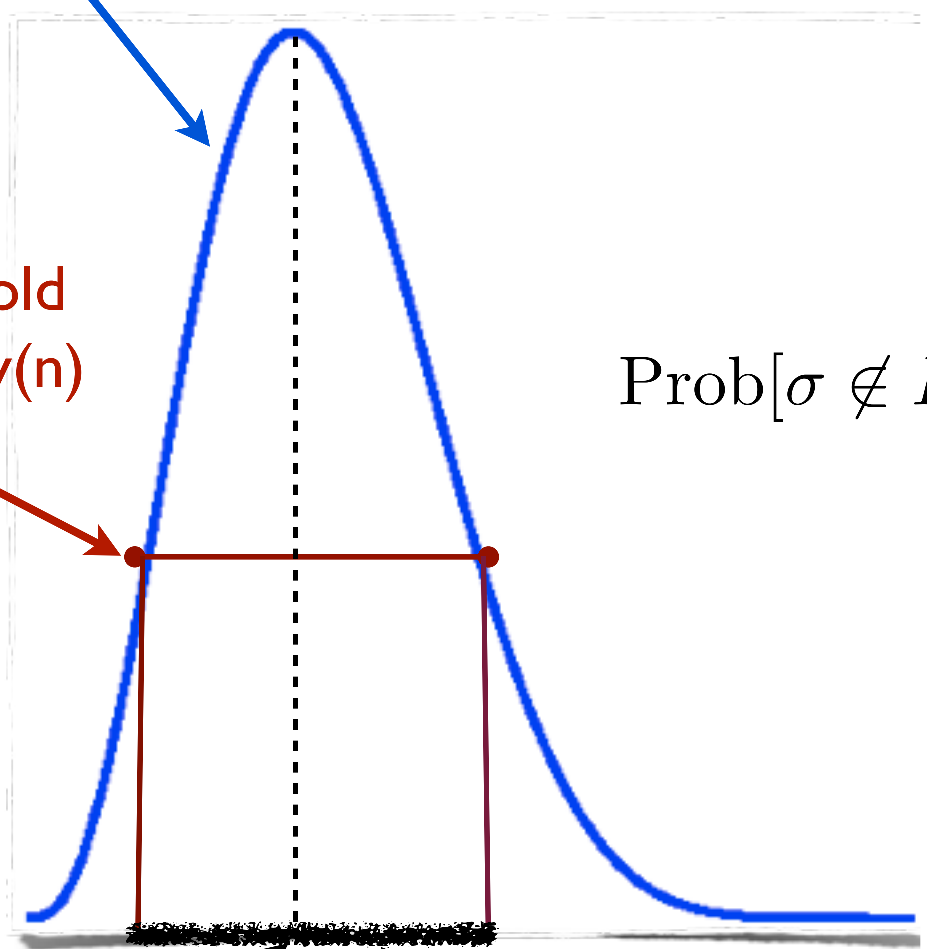




QIP Custom-Made Proof

likelihood ratio function
(depends on data B^n)

threshold
= ε/poly(n)



ρ_{B^n}

confidence region=
states whose
likelihood ratio is
above threshold

$$\left\{\rho : \frac{\mathrm{tr} B^n \rho^{\otimes n}}{\mathrm{tr} B^n \rho_{B^n}^{\otimes n}} > \epsilon/\mathrm{poly}(n)\right\}$$

$$\frac{\mathrm{tr} B^n \sigma^{\otimes n}}{\mathrm{tr} B^n \sigma_{B^n}^{\otimes n}} \leq \epsilon/\mathrm{poly}(n)$$

$$\mathrm{Prob}[\sigma \notin R_\epsilon(B^n)] = \sum_{B^n} \mathrm{tr} B^n \sigma^{\otimes n}$$

$$\leq \epsilon/\mathrm{poly}(n) \sum_{B^n} \mathrm{tr} B^n \sigma_{B^n}^{\otimes n}$$

postselection technique

$$\leq \epsilon/\mathrm{poly}(n) \sum_{B^n} \mathrm{tr} B^n \mathrm{poly}(n) \int \rho^{\otimes n} d\rho$$

$$\leq \epsilon \ \mathrm{tr}(\sum_{B^n} B^n)(\int \rho^{\otimes n} d\rho) = \epsilon$$