

High-dimensional Quantum Entanglement and Holographic Ghost Imaging

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EPR correlations in the OAM/angle basis



Violations of generalized Bell inequalities



Full-field quantum measurements



Transmission mask

Bucket detector







Camera









$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B\right)$$



















































$$|\psi\rangle = \sum_{\ell=-\infty}^{\ell=\infty} c_{\ell} |-\ell\rangle |\ell\rangle$$









Severin Fürhapter, Alexander Jesacher, Stefan Bernet, and Monika Ritsch-Marte

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Object





















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PHYSICAL REVIEW LETTERS

week ending 21 AUGUST 2009

Holographic Ghost Imaging and the Violation of a Bell Inequality

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EPR correlations in the OAM/angle basis







PHYSICAL REVIEW A

ATOMIC, MOLECULAR, AND OPTICAL PHYSICS

Quantum theory of rotation angles

D. T. Pegg

Stephen M. Ba

(Received 25 Septem

THIRD SERIES, VOLUME 41, NUMBER 7

Physics

Uncertainty principle for angular position and angular momentum

f Physics 6 (2004) 103

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Angular momentum X

Angular momentum ✓

Angular position \checkmark





MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, Institute for Advanced Study, Princeton, New Jersey (Received March 25, 1935)



If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.







If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.





Can we measure the momentum of photon A and infer the momentum of photon B?

If yes, photon B has reality of momentum Can we measure the position of photon A and infer the position of photon B? If yes, photon B has reality of position

Photon B has simultaneous reality of position and momentum!!!



Photon B has simultaneous reality of position and momentum!!!



What is the resolution to EPR's thoughts?







Journal of Modern Optics Vol. 53, Nos. 5-6, 20 March-15 April 2006, 627-645



Angular EPR paradox

J. B. GÖTTE*, S. FRANKE-ARNOLD and STEPHEN M. BARNETT Department of Physics, University of Strathclyde, Glasgow G4 0NG, UK

Angular position correlations

Angular momentum correlations















 $[\Delta \phi_i | \phi_s]^2 [\Delta \ell_i | \ell_s]^2 \ge \frac{1}{4} \qquad [\Delta \phi_i | \phi_s]^2 [\Delta \ell_i | \ell_s]^2 = 0.024$





Entropy is a measure of the uncertainty associated with a random variable.

Entanglement requires a degree of certainty about the **inferred** outcome of a measurement on B, given a measurement on A, and vice versa

Entropy = 0

Entanglement requires uncertainty about the outcome of the measurements at A and B

Entropy is > 0



 $\ell = \infty$ $|\psi\rangle = \sum c_{\ell} |-\ell\rangle |\ell\rangle$ $\ell = -\infty$



Entropy is low, number of modes is low

Entropy is high, number of modes is high


nature physics

The uncertainty principle in quantum memory

Mario Berta^{1,2}, Matthias Christandl^{1,2}, Roger Colbeck^{1,3}



nature physics

Experimental investigation of the uncertainty principle in the presence of quantum memory and its application to witnessing entanglement

Robert Prevedel¹*, Deny R. Hamel¹, Roger Colbeck², Kent Fisher¹ and Kevin J. Resch¹













$$h(\ell_i | \ell_s) + h(\phi_i | \phi_s) \ge \log_2(2\pi) = 2.65$$

 $h(\ell_i | \ell_s) + h(\phi_i | \phi_s) = 0.89$



Quantum Correlations in Optical Angle–Orbital Angular Momentum Variables

Jonathan Leach,¹ Barry Jack,¹ Jacqui Romero,¹ Anand K. Jha,² Alison M. Yao,³ Sonja Franke-Arnold,¹ David G. Ireland,¹ Robert W. Boyd,² Stephen M. Barnett,³ Miles J. Padgett¹*

Entanglement of the properties of two separated particles constitutes a fundamental signature of quantum mechanics and is a key resource for quantum information science. We demonstrate strong Einstein, Podolsky, and Rosen correlations between the angular position and orbital angular momentum of two photons created by the nonlinear optical process of spontaneous parametric down-conversion. The discrete nature of orbital angular momentum and the continuous but periodic nature of angular position give rise to a special sort of entanglement between these two variables. The resulting correlations are found to be an order of magnitude stronger than those allowed by the uncertainty principle for independent (nonentangled) particles. Our results suggest that angular position and orbital angular momentum may find important applications in quantum information science.

6 AUGUST 2010 VOL 329 SCIENCE www.sciencemag.org

Work provides a clear picture for angles/angular momentum

Both entropy calculation and inferred variances show EPR correlations

Entropy calculation is not effected by cyclic nature of angles

Can use for quantum cryptography, quantum imaging etc



Classical communication

Quantum measurements





















EPR correlations in the OAM/angle basis



Violations of generalized Bell inequalities



Einstein, **Podolsky** and **Rosen** argued that Quantum Mechanics (QM) was an incomplete theory because of its description of entangled systems [EPR, Phys. Rev. 47, 777 (1935)].

А

В





Bell proposed a method of testing local realism.

Bell's theorem: no classical theory based on local-hidden variables can ever fully reproduce quantum mechanical predictions [Bell, Physics 1, 195 (1964)].



Experimental result indicating two photon superpositions in OAM [A. Mair *et al.*, Nature 412, 313 (2001)]

letters to nature

NATURE VOL 412 19 JULY 2001 www.nature.com

Entanglement of the orbital angular momentum states of photons

Alois Mair*, Alipasha Vaziri, Gregor Weihs & Anton Zeilinger

Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria in a coincidence pattern not containing any intensity zeroes. Such a coincidence pattern would also be observed if a shifted hologram together with a mono-mode detector were not able to analyse for superposition states.

An entangled state represents correctly both the correlation of the eigenmodes and the correlations of their superpositions. Having experimentally confirmed the quantum superposition for l = 0 and $l = \pm 2$, it is reasonable to expect the quantum superposition will also occur for the other states. Nevertheless, ultimate confirmation of entanglement will be a Bell inequality experiment generalized to more states²⁵. Such an experiment will be a major experimental challenge, and we are preparing to perform it.





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| | 2 | ndu Popescu ^{1,2} | Serge Massar,5 and S | ² Nicolas Gisin, ³ Noah Linden, ⁴ | Daniel Collins,1,2 |
| | ngdom | TL, United King | dall Avenue, Bristol BS8 | boratory, University of Bristol, Tyn | ¹ H.H. Wills Physics Lal |
| | Switzerland | 211 Geneva 4, Sv | cole-de-Médecine, CH- | T-Packara Laboratories, Stoke Giff University of Geneva, 20, rue de l'I | ³ Group of Applied Physics, U |
| | dom | V, United Kingdo | ity Walk, Bristol BS8 1 | nematics, Bristol University, Univer | ⁴ Department of Mathe |
| | ixelles, Belgium | ohe, B1050 Bruxe | 225, Boulevard du Trion ed 10 January 2002) | Université Libre de Bruxelles, CP (Received 23 July 2001: publish | Service de Physique Théorique, |
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Figure 3 | Experimental Bell-type parameter S_d versus number of dimensions d. $S_d > 2$ violates local realism for any $d \ge 2$. The plot compares the theoretically predicted violations by a maximally entangled state and the LHV limit with the experiments. Violations are observed for up to d = 12. Errors were estimated assuming Poisson statistics.



Figure 4 | Experimental coincidence rates proportional to the probability of measuring the state $|\ell_s\rangle \otimes |\ell_i\rangle$ with $\ell_s, \ell_i = -5, ..., +5$. The coloured and greyed-out bars depict the measurement results with and without the application of Procrustean filtering respectively. The measurement time was 20 s for each combination of ℓ_s and ℓ_i .



Measurement of qubits

Daniel F. V. James,^{1,*} Paul G. Kwiat,^{2,3} William J. Munro,^{4,5} and Andrew G. White^{2,4}
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⁵Hewlett-Packard Laboratories, Filton Road, Stoke Gifford, Bristol BS34 8QZ, United Kingdom (Received 20 March 2001; published 16 October 2001)

We describe in detail the theory underpinning the measurement of density matrices of a pair of quantum two-level systems ("qubits"). Our particular emphasis is on qubits realized by the two polarization degrees of freedom of a pair of entangled photons generated in a down-conversion experiment; however, the discussion applies in general, regardless of the actual physical realization. Two techniques are discussed, namely, a tomographic reconstruction (in which the density matrix is linearly related to a set of measured quantities) and a maximum likelihood technique which requires numerical optimization (but has the advantage of producing density matrices that are always non-negative definite). In addition, a detailed error analysis is presented, allowing errors in quantities derived from the density matrix, such as the entropy or entanglement of formation, to be estimated. Examples based on down-q





the produced from a given input into the device, ormation quantities such as the degree of entangleneasurements have been discussed for this tomoghic reconstruction technique to two new regimes, reconstruction of the state of the system provided v quantum-state tomography can be performed for eve this in one- and two-qutrit systems.

Particle Dimension (d) and Number of Operations (d²ⁿ), $(j_1, \dots, j_n \in \{1, 2, \dots, d\})$

FIG. 2. The measurement scaling for tomography on n qudits results from the necessity to measure every basis state on every subsystem in every permutation. The measurements scale as $d^{2n} - 1$, where d is the *particle dimension*, e.g., d = 2 for a qubit and n is the number of particles.

PHYSICAL REVIEW A 66, 012303 (2002)

Qudit quantum-state tomography

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Recently quantum tomography has been proposed as a fundamental tool for prototyping a few qubit quan-



















EPR correlations in the OAM/angle basis



Violations of generalized Bell inequalities



Full-field quantum measurements









 SLMs in signal/idler arms
Multi-pixel detector array (not shown in picture)
Converts position to time)

We can detect coincidences for near-field/far-field/ intermediate-fields

We can simultaneously measure 8 quantum states with no scanning































 $(\Delta_{\inf} x_s)^2 (\Delta_{\inf} p_{x_s})^2 = 0.023\hbar$






































Our multi-pixel linear array, single photon detector allows to make.....

- a demonstration of spatial EPR in which the strength of the position or momentum is measured without scanning any of the optical or electronic system
- measure the correlations in the near-field, far-field and intermediate planes

What is the significance of how the correlations change as we move from the near-field to the far-field?

- the polarisation analogy has implications for locality and hidden variable theories









Holographic ghost imaging

Violations of Bell's inequality Can a single image violate this?



EPR correlations in the OAM/angle basis

QKD with spatial states How best to maximise bit/photon? Optimal basis for measurements?



Violations of generalized Bell inequalities

Differences between EPR/Bell? How best to characterise states?



Full-field quantum measurements

Power of SLM for quantum optics Implications for Bell's inequalities?



Questions

What is ghost imaging? Is this a quantum effect?

What is required to show a non-local quantum effect?

Is our image the result of a quantum effect?

What is the angular EPR paradox? What is required to demonstrate this?

What is the difference between

- violating a Bell-type inequality
- demonstrating EPR correlations?





