Micro-mechanical non-classicality

Mauro Paternostro

Colloquium at the Institute for Quantum Computing

May 2011

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

EPSRC
Pioneering research and skills

Micro-mechanical non-classicality

(or: of the difference between non-classicality and an orange juice)

Mauro Paternostro

Colloquium at the Institute for Quantum Computing

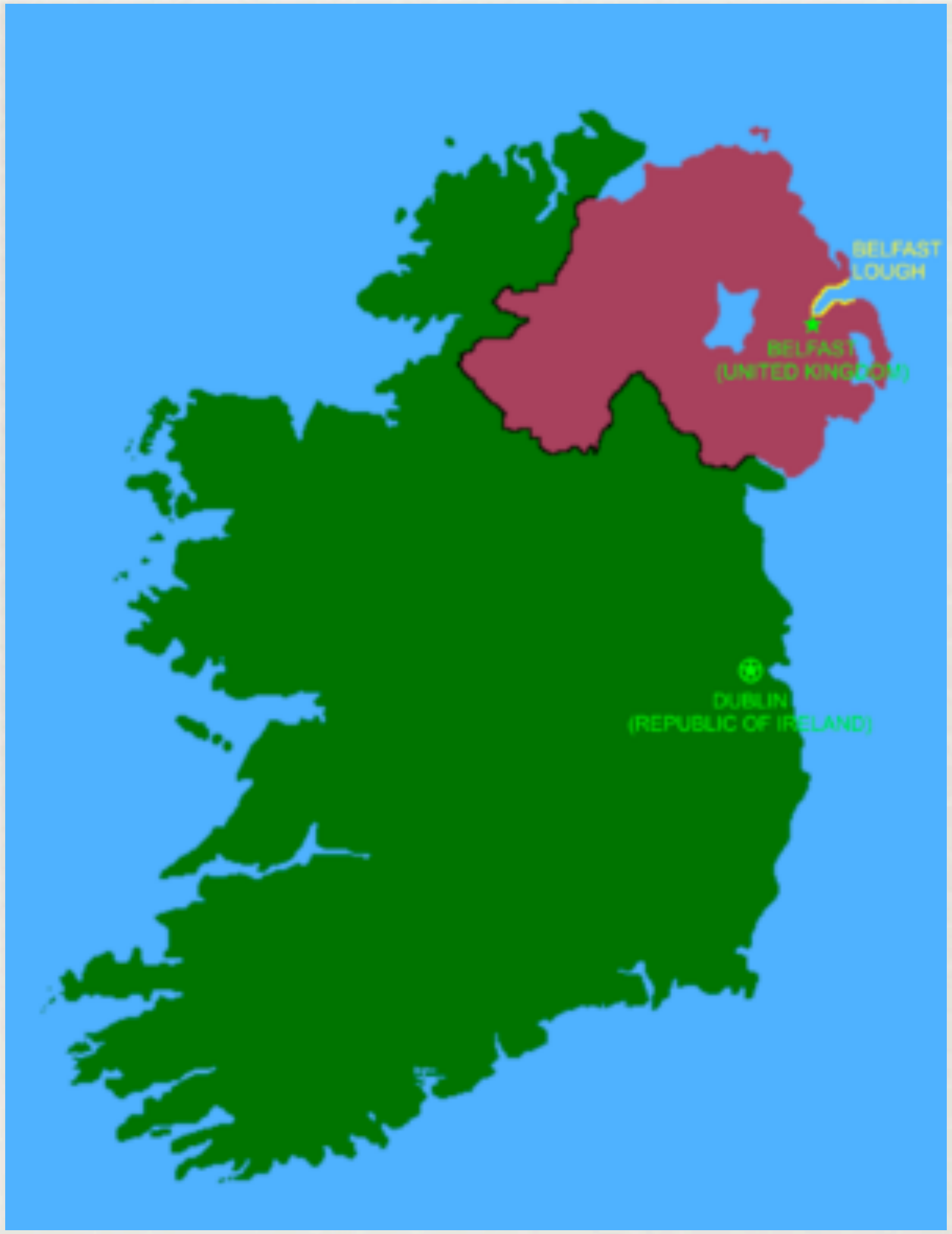
May 2011

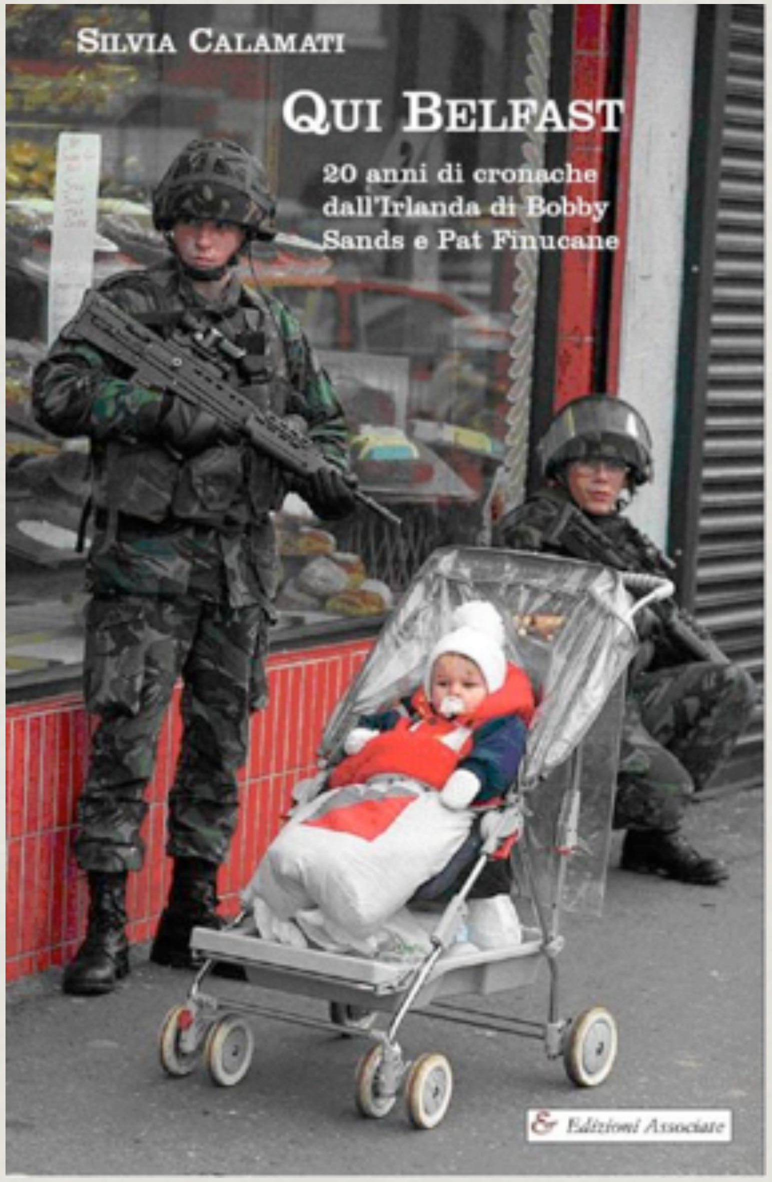
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

EPSRC
Pioneering research and skills

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

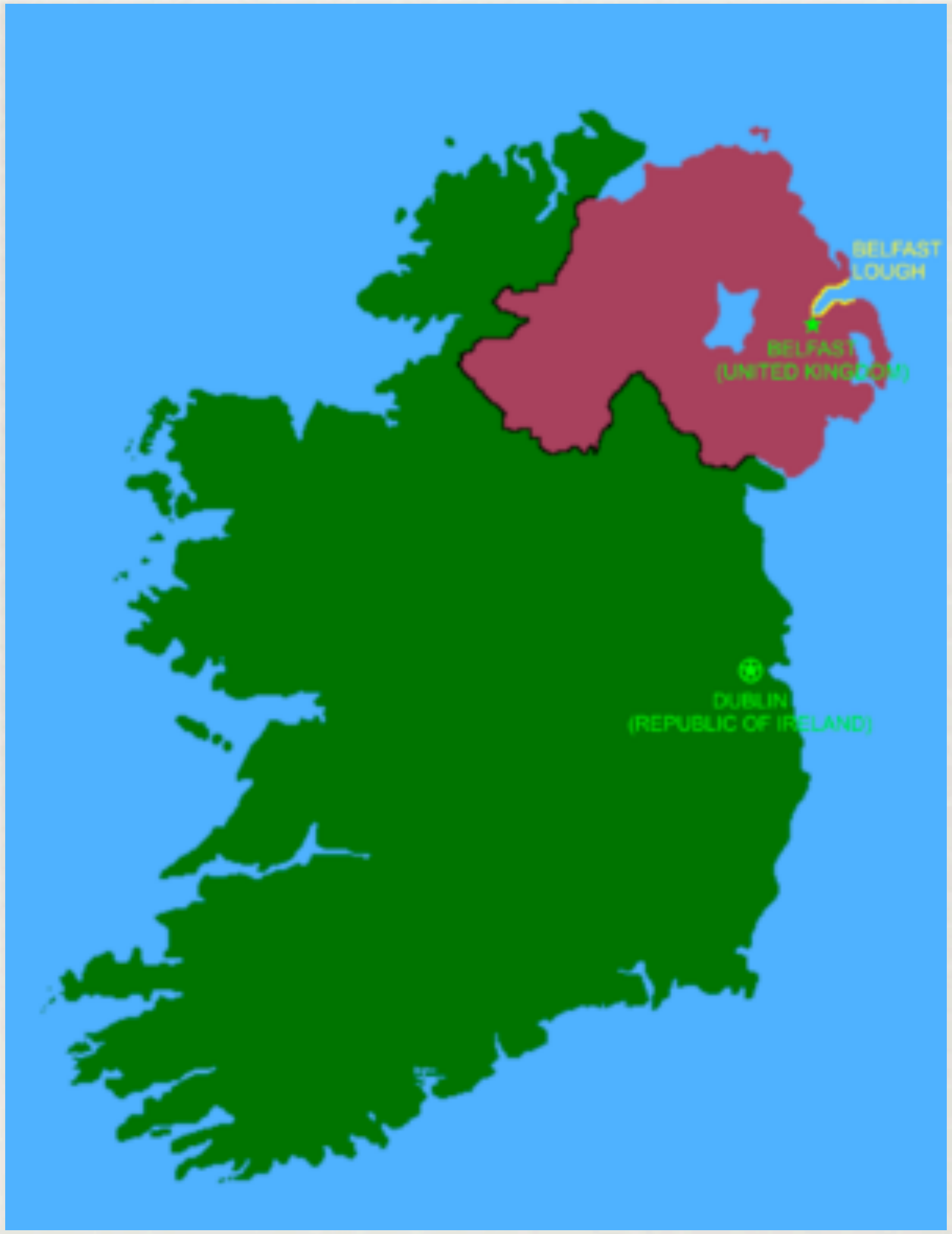
Belfast

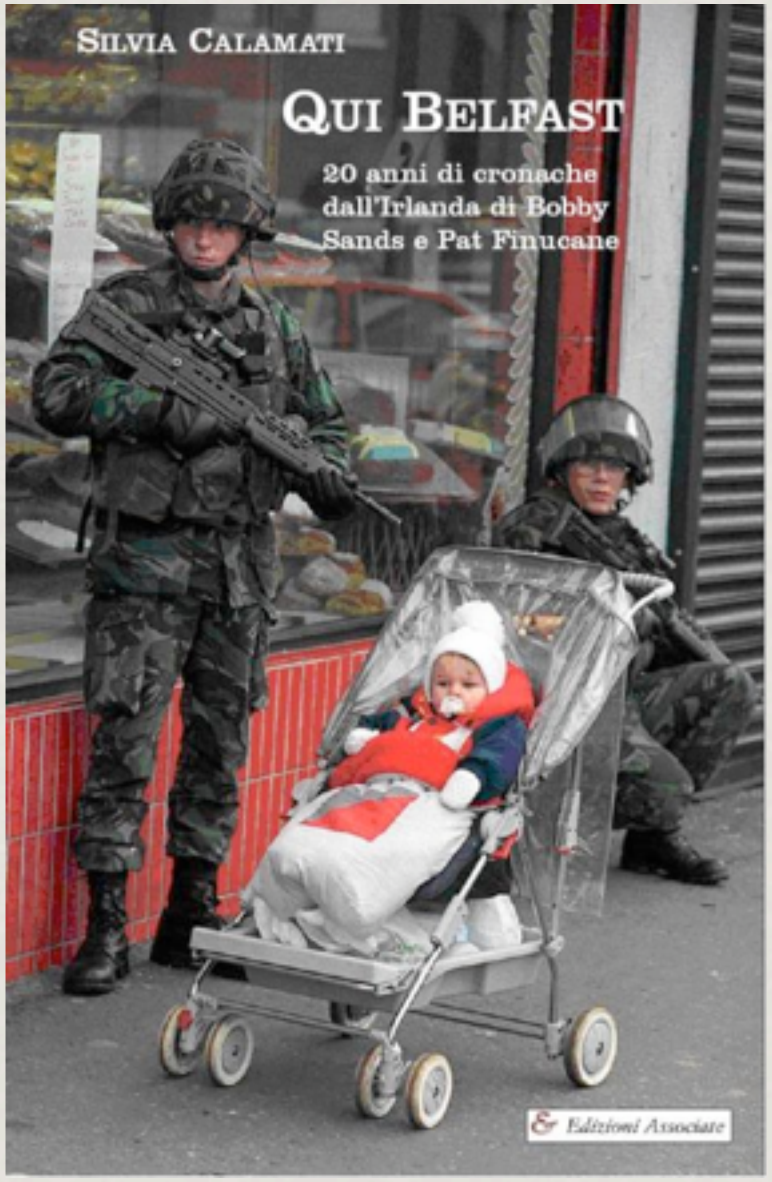




QTEQ
QUANTUM TECHNOLOGY at QUEEN´S
w w w . q t e q . i n f o

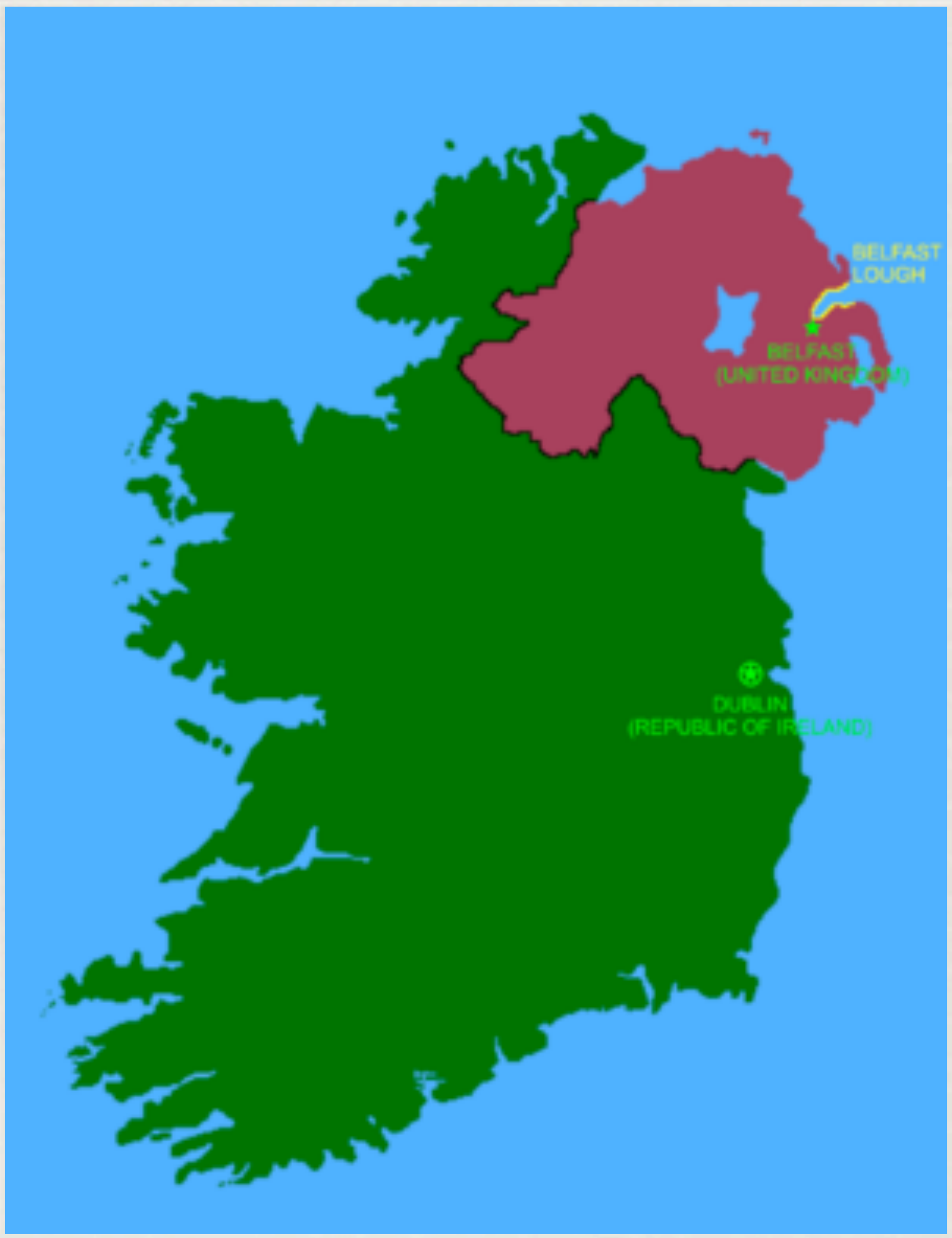
Belfast





QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Belfast



QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Belfast

Belfast's 'best' sons

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Belfast's 'best' sons

Georgie Best!

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Belfast's 'best' sons

Georgie Best!

Van Morrison

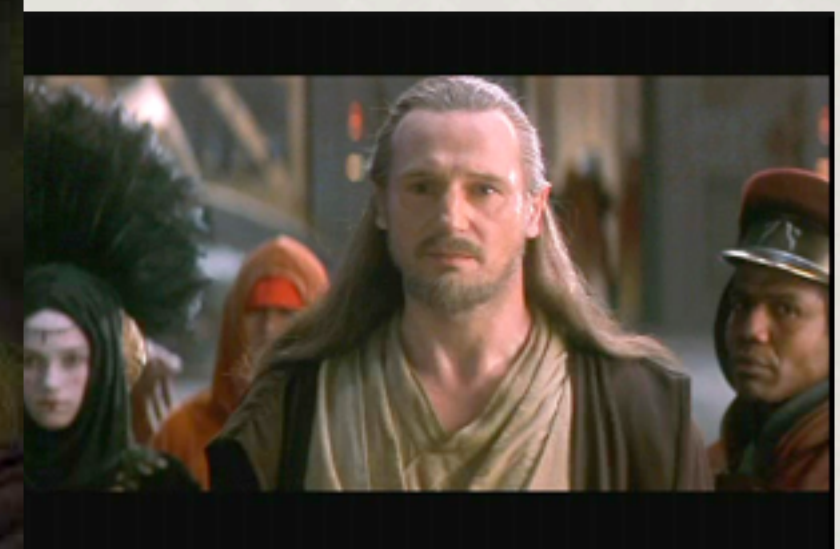
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

Belfast's 'best' sons

Wednesday, 8 June 2011

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Belfast's 'best' sons



Liam Neeson!

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

Queen's heritage

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

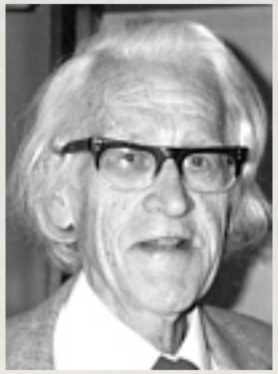
Queen's heritage

Joseph Larmor

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

Queen's heritage

Joseph Larmor

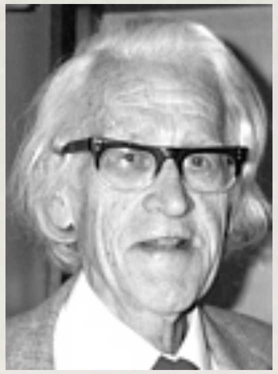


David Bates, FRS

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

Queen's heritage

Joseph Larmor



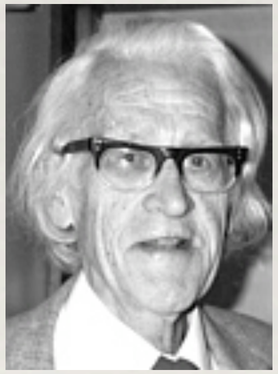
David Bates, FRS

Seamus Heaney

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Queen's heritage

Joseph Larmor



David Bates, FRS

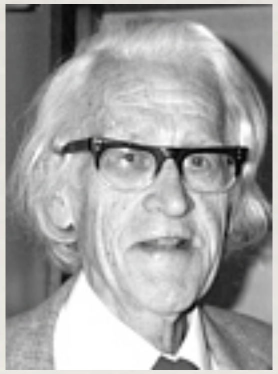
Seamus Heaney

..and John E. Campbell..
(one leg of the Campbell-Baker-Haussdorff trio)

QTEQ QUANTUM TECHNOLOGY at QUEEN'S www.qteq.info

Queen's heritage

Joseph Larmor



David Bates, FRS

John Stuart Bell

Seamus Heaney

..and John E. Campbell.. (one leg of the Campbell-Baker-Haussdorff trio)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Gabriele De Chiara

Jim McCann

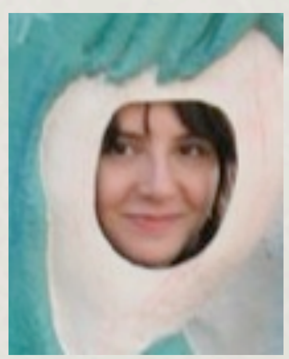
Mauro Paternostro

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

Gabriele De Chiara

Carlo Di Franco

Jim McCann



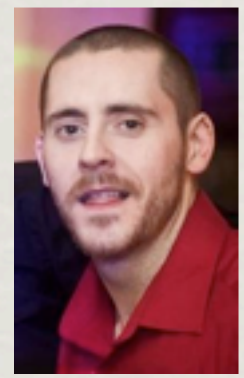
Laura Mazzola

Mauro Paternostro

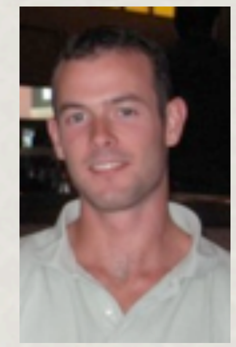
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

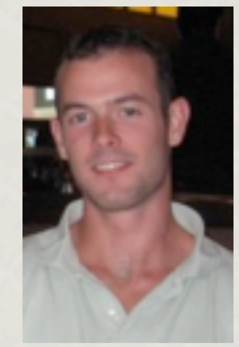
Gabriele De Chiara

Carlo Di Franco



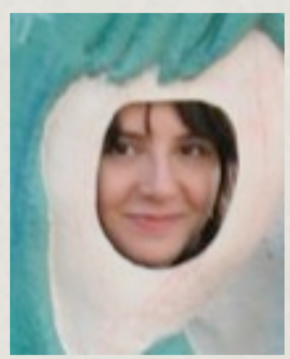
Steve Campbell





Gerard & Ciaran McKeown

Jim McCann



Laura Mazzola

Jie Li

Nicola Lo Gullo

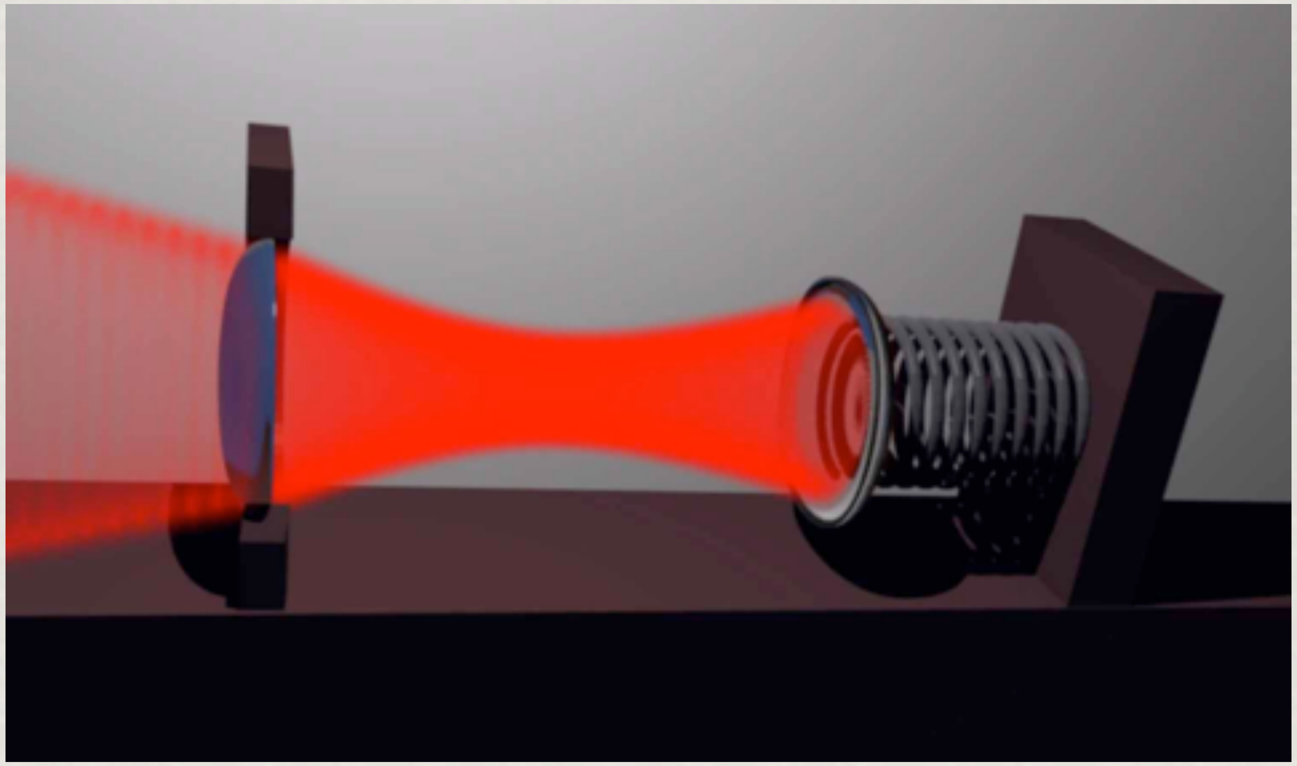
Gareth Cochrane

Mauro Paternostro

Niall Quinn

+ Hugo Lamb & Dave Salford

Let there be light.. and a mirror...vibrating...



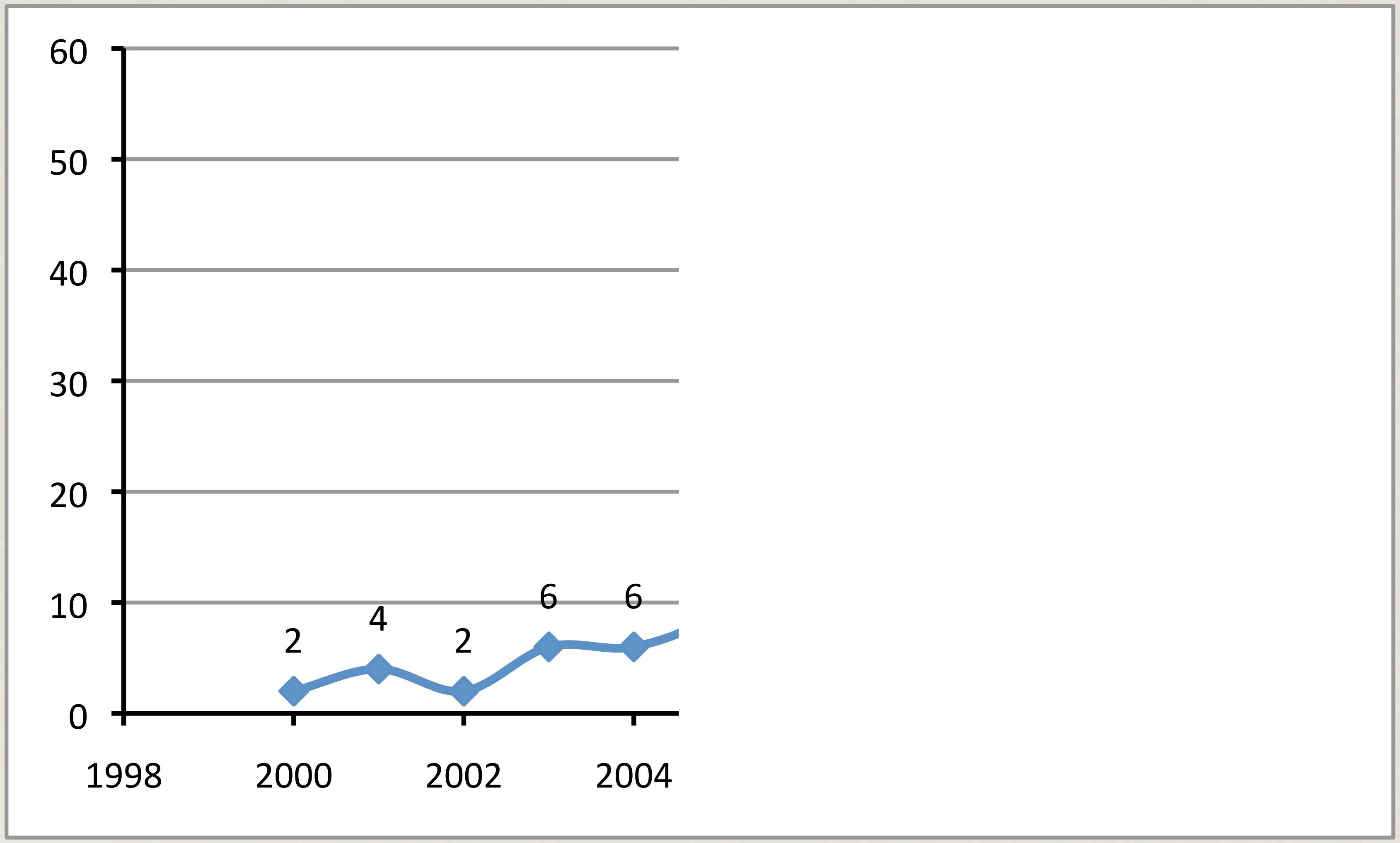
...So, what's the fuss with all this opto-mechanical thingy....

Marco Piani (2011)

An explosion of interest

QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

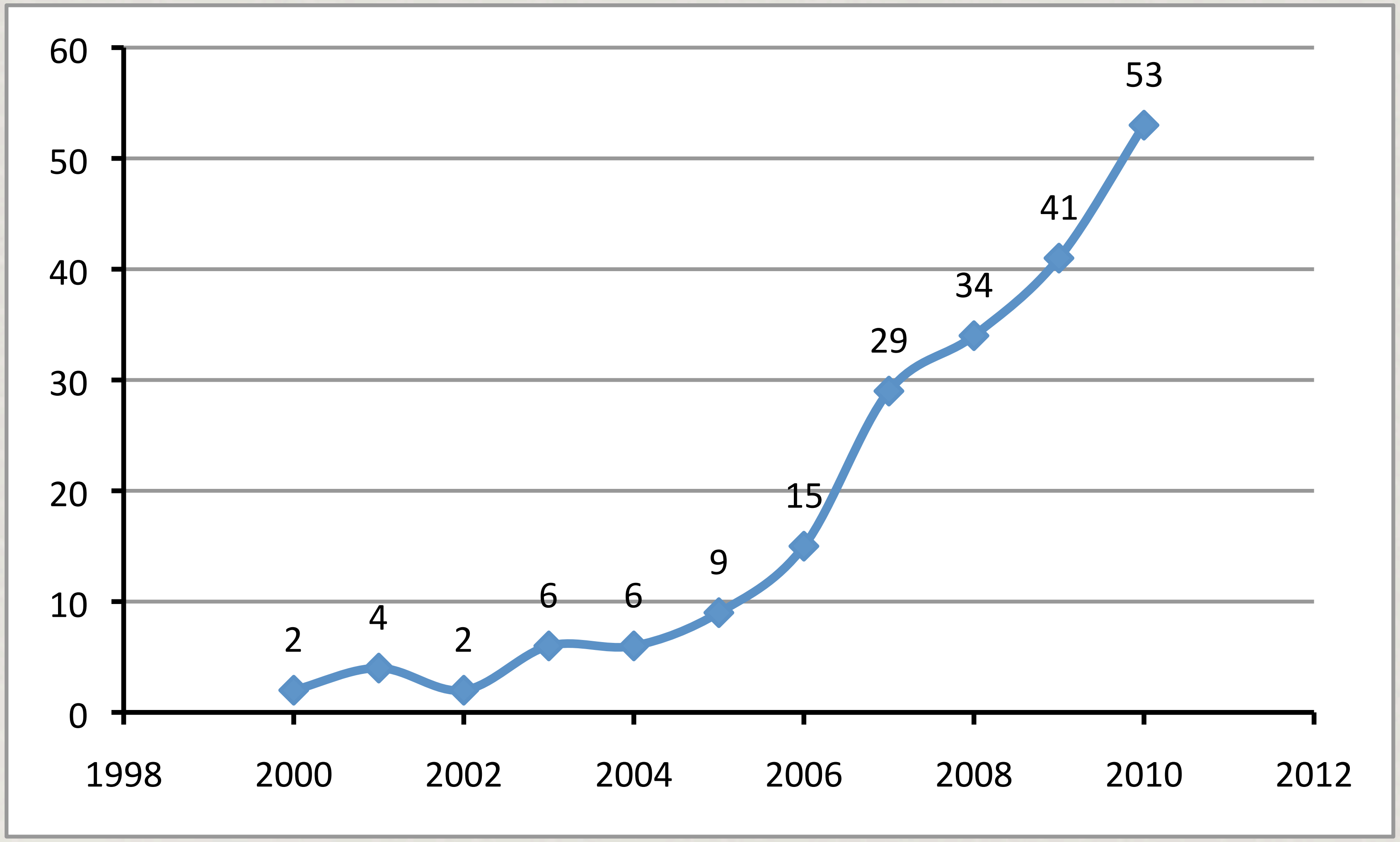
Number of papers on the arXiv containing the word "optomechanics"



QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

An explosion of interest

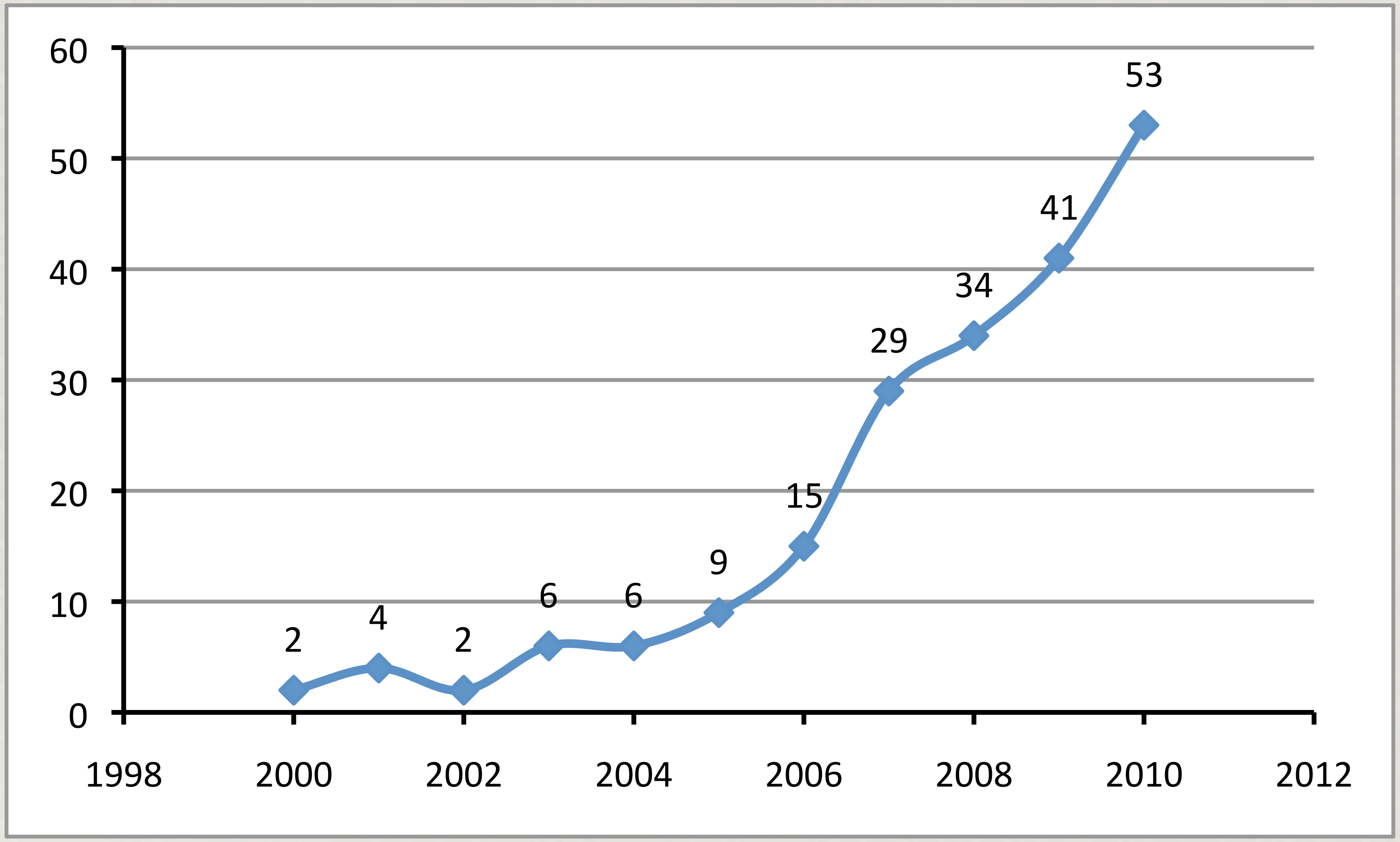
Number of papers on the arXiv containing the word "optomechanics"



QTEQ QUANTUM TECHNOLOGY at QUEEN'S www.qteq.info

An explosion of interest

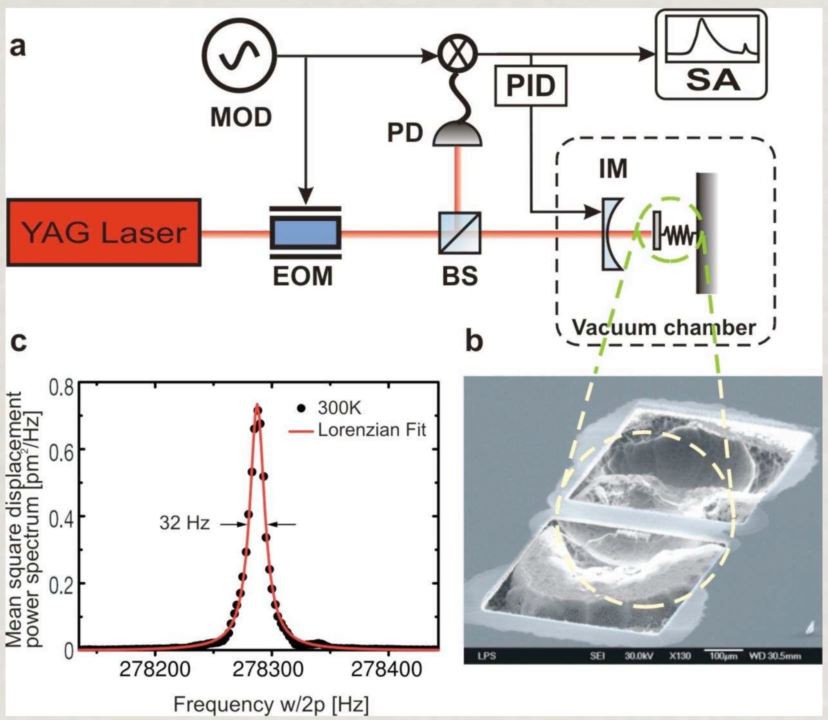
Number of papers on the arXiv containing the word "optomechanics"



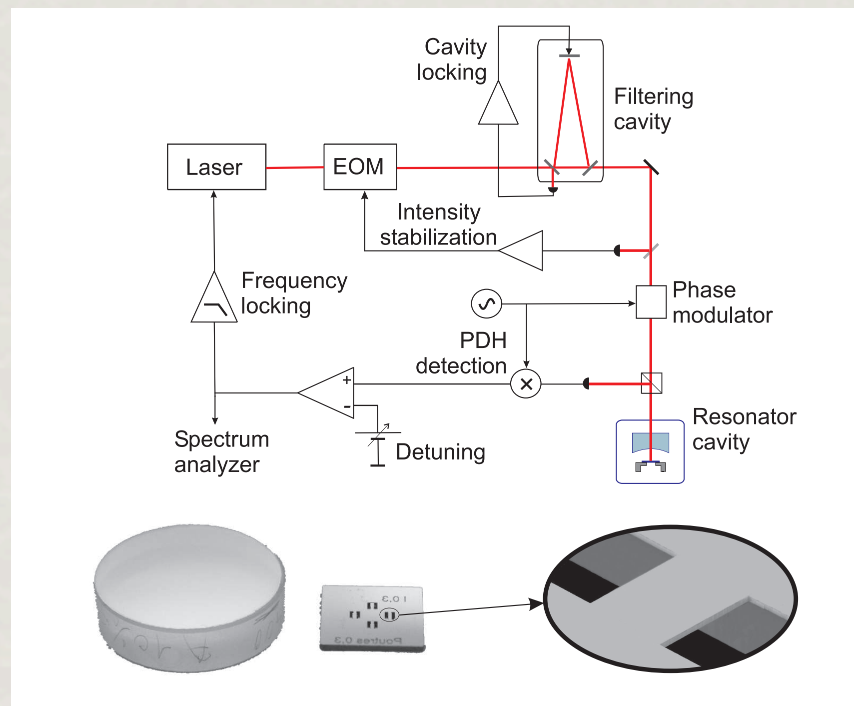
Saturation/Decay? When Nature/Science/Phys. Rev. Lett. will stop publishing papers in the area..

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

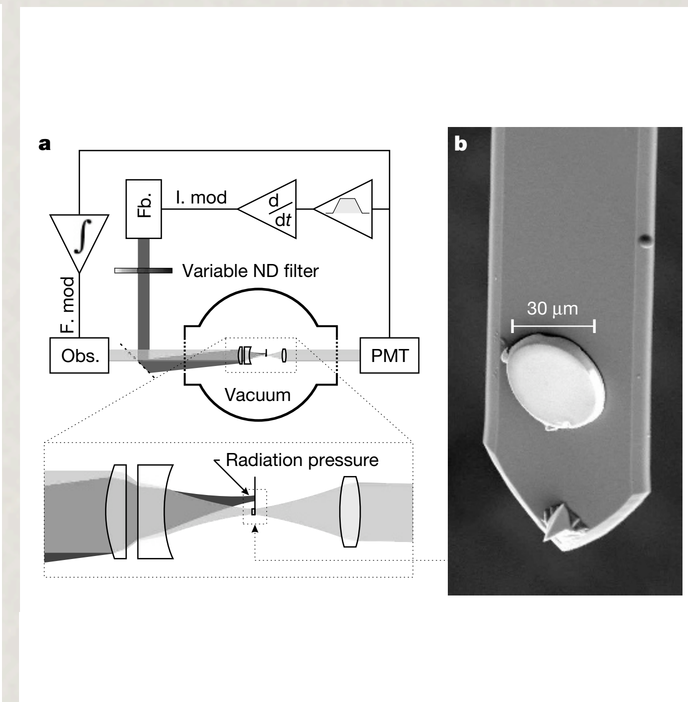
The right experiment at the right time



S. Gigan, et al. Nature **444**, 67 (2006)



O. Arcizet, et al. Nature **444**, 71 (2006)



D. Kleckner and D. Bouwmeester, Nature **444**, 75 (2006)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Somebody thinks it's hot.. while it's actually cold..

ORIGINAL RESEARCH PAPERS Gigan, S. *et al.* Self-cooling of a micromirror by radiation pressure. *Nature* **444**, 67–70 (2006) | Arcizet, O. *et al.* Radiation pressure cooling and optomechanical instability of a micromirror. *Nature* **444**, 71–74 (2006) | Schliesser, A. *et al.* Radiation pressure cooling of a micromechanical oscillator using dynamical backaction. *Phys. Rev. Lett.* **97**, 243905 (2006) | O'Connell, A. D. *et al.* Quantum ground state and single-phonon control of a mechanical resonator. *Nature* **464**, 697–703 (2010)
FURTHER READING Kippenberg, T. J. & Vahala, K. J. Cavity optomechanics: Back-action at the mesoscale. *Science* **321**, 1172–1176 (2008) | Marquardt, F. & Girvin, S. M. Optomechanics. *Physics* **2**, 40 (2009)

nature MILESTONES | PHOTONS

<u>*http://www.nature.com/milestones/*</u>

MILESTONES TIMELINE

1600s–1800s	Debate on the character of light (Milestone 1)
1861	Maxwell's equations (Milestone 2)
1900	Planck's theory of black-body radiation (Milestone 3)
1905	Special relativity (Milestone 4)
1923	Compton effect (Milestone 5)
1947	Quantum electrodynamics (Milestone 6)
1948	Holograms (Milestone 7)
1954	Solar cells (Milestone 8)
1960	The laser (Milestone 9)
1961	Nonlinear optics (Milestone 10)
1963	Quantum optics (Milestone 11)
1964	Bell inequality (Milestone 12)
1966	Optical fibres (Milestone 13)
1970	CCD cameras (Milestone 14)
	Semiconductor lasers (Milestone 15)
1981	High-resolution laser spectroscopy and frequency metrology (Milestone 16)
1982–1985	Quantum information (Milestone 17)
1987	Photonic crystals (Milestone 18)
1993	Blue light-emitting diodes (Milestone 19)
1998	Plasmonics (Milestone 20)
2000	Metamaterials (Milestone 21)
2001	Attosecond science (Milestone 22)
2006	Cavity optomechanics (Milestone 23)

QTEQ

QUANTUM TECHNOLOGY at QUEEN'S

www.qteq.info

Somebody thinks it's hot.. while it's actually cold..

ORIGINAL RESEARCH PAPERS Gigan, S. *et al.* Self-cooling of a micromirror by radiation pressure. *Nature* **444**, 67–70 (2006) | Arcizet, O. *et al.* Radiation pressure cooling and optomechanical instability of a micromirror. *Nature* **444**, 71–74 (2006) | Schliesser, A. *et al.* Radiation pressure cooling of a micromechanical oscillator using dynamical backaction. *Phys. Rev. Lett.* **97**, 243905 (2006) | O'Connell, A. D. *et al.* Quantum ground state and single-phonon control of a mechanical resonator. *Nature* **464**, 697–703 (2010)

FURTHER READING Kippenberg, T. J. & Vahala, K. J. Cavity optomechanics: Back-action at the mesoscale. *Science* **321**, 1172–1176 (2008) | Marquardt, F. & Girvin, S. M. Optomechanics. *Physics* **2**, 40 (2009)

nature MILESTONES | PHOTONS

http://www.nature.com/milestones/

MILESTONES TIMELINE

1600s–1800s	Debate on the character of light (Milestone 1)
1861	Maxwell's equations (Milestone 2)
1900	Planck's theory of black-body radiation (Milestone 3)
1905	Special relativity (Milestone 4)
1923	Compton effect (Milestone 5)
1947	Quantum electrodynamics (Milestone 6)
1948	Holograms (Milestone 7)
1954	Solar cells (Milestone 8)
1960	The laser (Milestone 9)
1961	Nonlinear optics (Milestone 10)
1963	Quantum optics (Milestone 11)
1964	Bell inequality (Milestone 12)
1966	Optical fibres (Milestone 13)
1970	CCD cameras (Milestone 14)
	Semiconductor lasers (Milestone 15)
1981	High-resolution laser spectroscopy and frequency metrology (Milestone 16)
1982–1985	Quantum information (Milestone 17)
1987	Photonic crystals (Milestone 18)
1993	Blue light-emitting diodes (Milestone 19)
1998	Plasmonics (Milestone 20)
2000	Metamaterials (Milestone 21)
2001	Attosecond science (Milestone 22)
2006	Cavity optomechanics (Milestone 23)

QTEQ
QUANTUM TECHNOLOGY at QUEEN´S
www.qteq.info

Answering Marco's question

Answering Marco's question

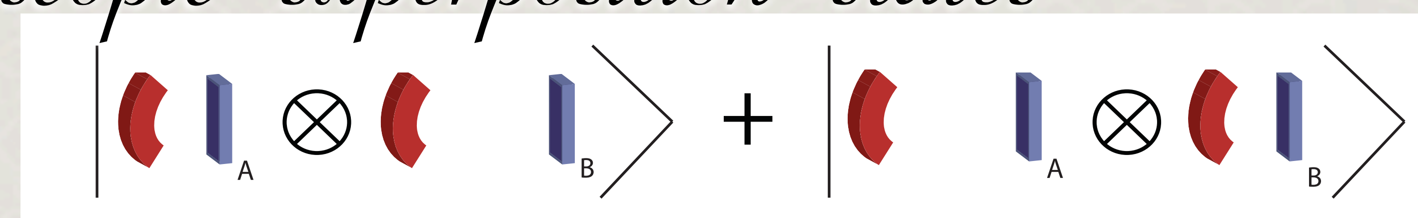
Research at the boundary of the quantum world

QTEQ
QUANTUM TECHNOLOGY at QUEEN´S
www.qteq.info

Answering Marco's question

Research at the boundary of the quantum world

Preparation of mesoscopic superposition states

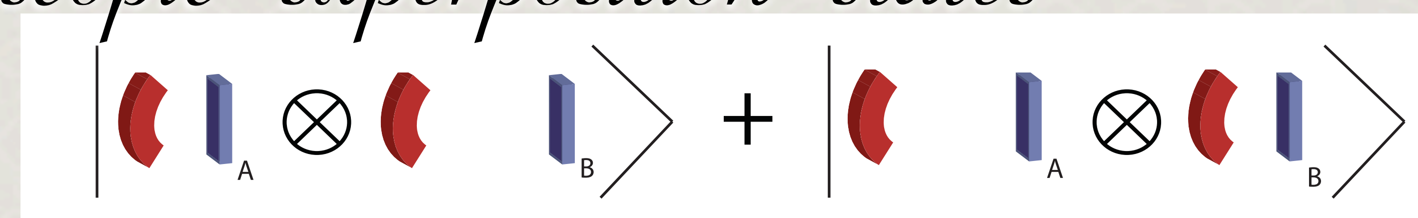


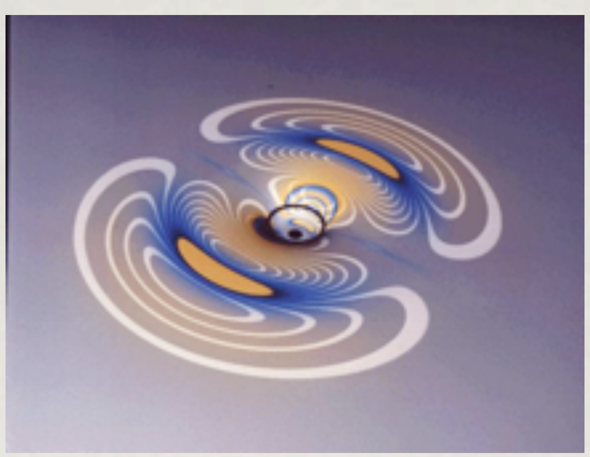
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Answering Marco's question

Research at the boundary of the quantum world

Preparation of mesoscopic superposition states



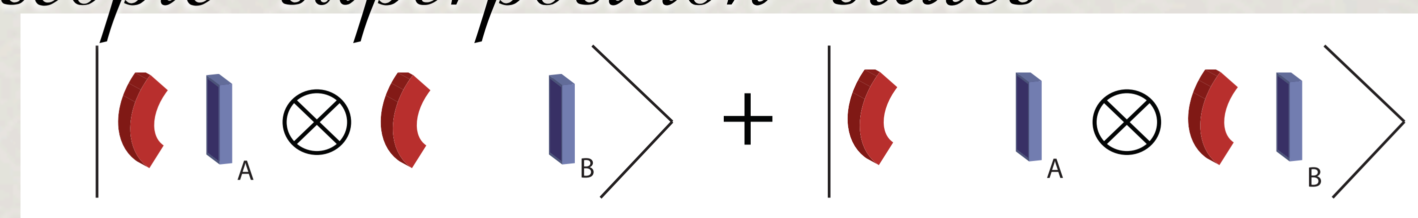


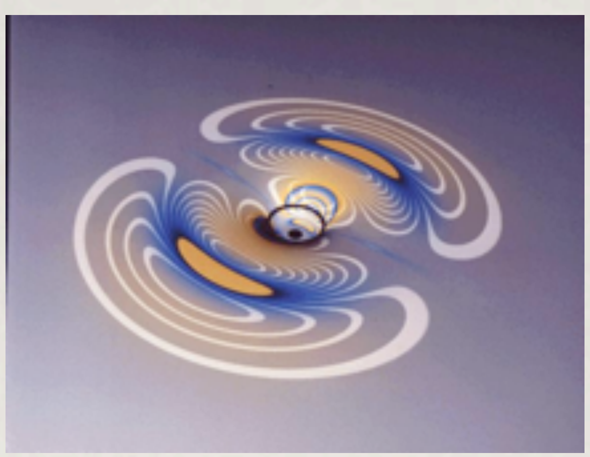
Gravitational waves and collapse models

Answering Marco's question

Research at the boundary of the quantum world

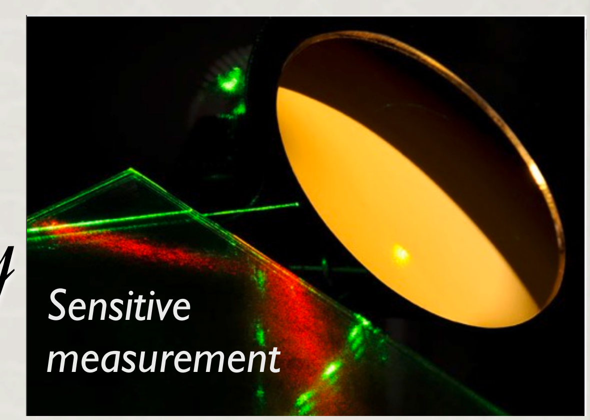
Preparation of mesoscopic superposition states





Gravitational waves and collapse models

Sensors, transducers, metrology

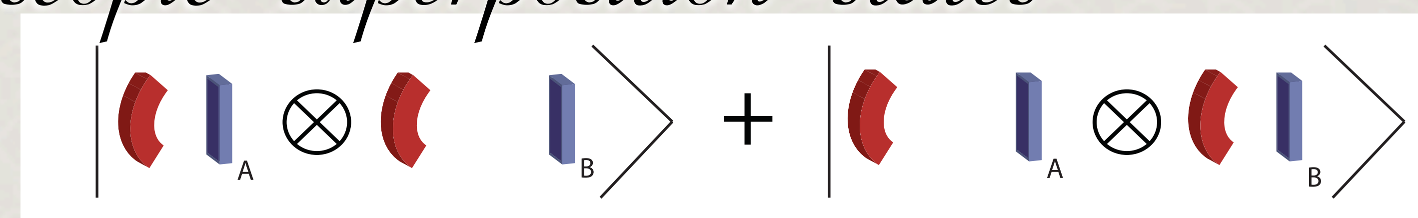


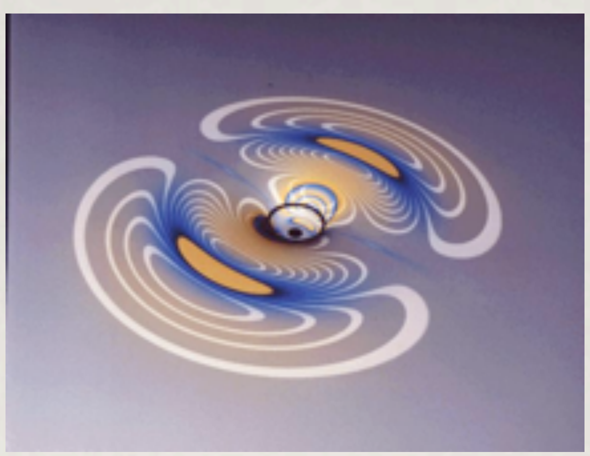
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Answering Marco's question

Research at the boundary of the quantum world

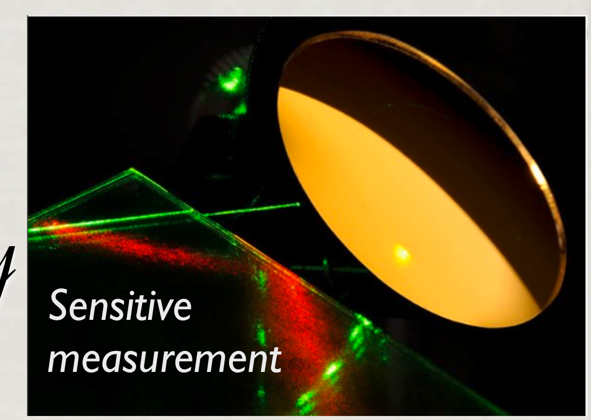
Preparation of mesoscopic superposition states





Gravitational waves and collapse models

Sensors, transducers, metrology



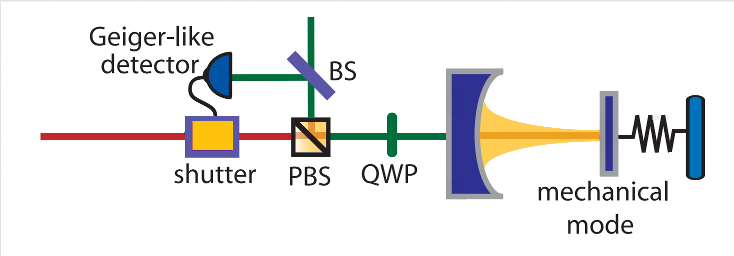
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Menu of the day

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Menu of the day

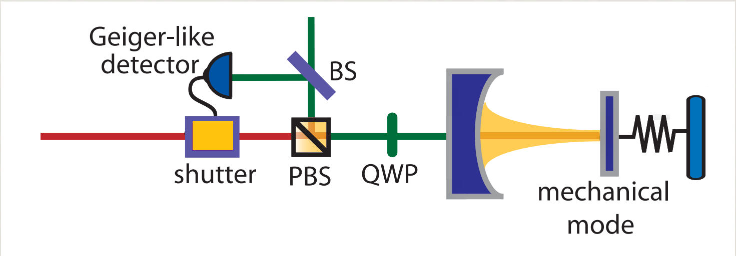
1. Non-classicality by photon-subtraction



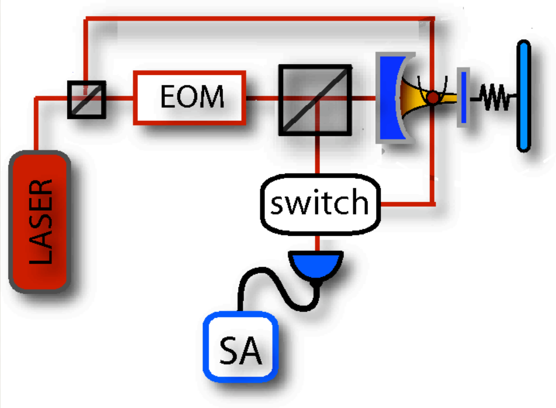
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Menu of the day

1. Non-classicality by photon-subtraction



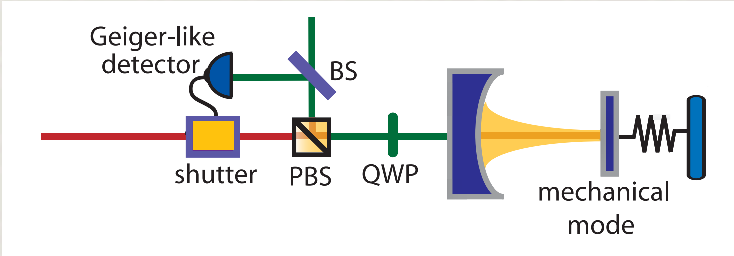
2. Entanglement and diagnostics through cold-atom induced dynamics



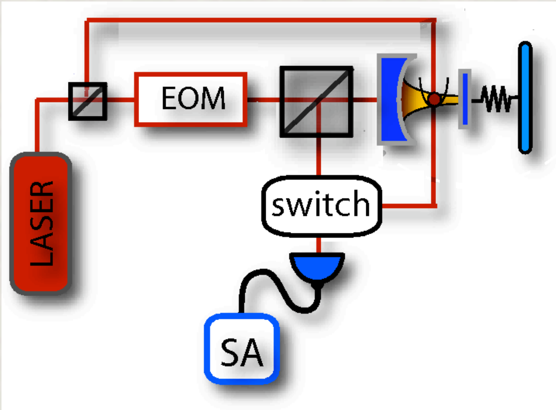
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Menu of the day

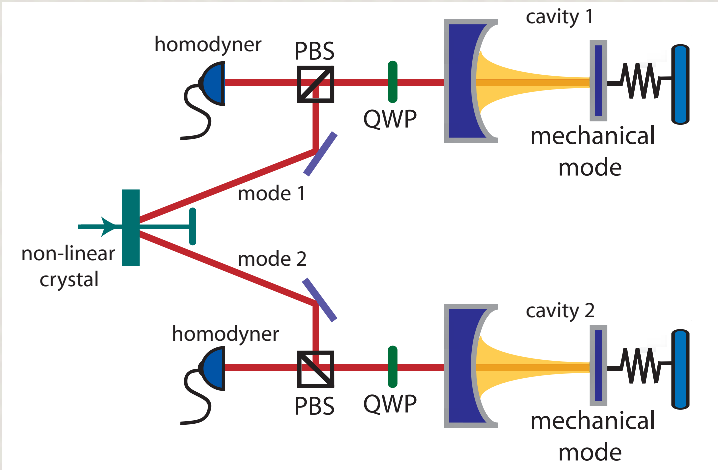
1. Non-classicality by photon-subtraction



2. Entanglement and diagnostics through cold-atom induced dynamics



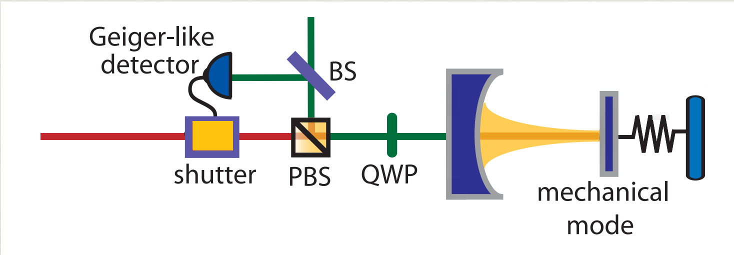
3. Quantum opto-mechanical networking



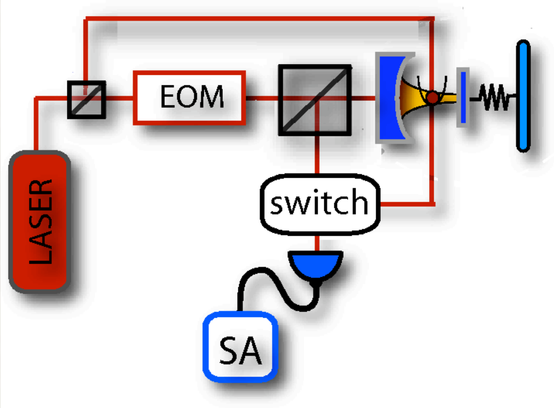
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

First course

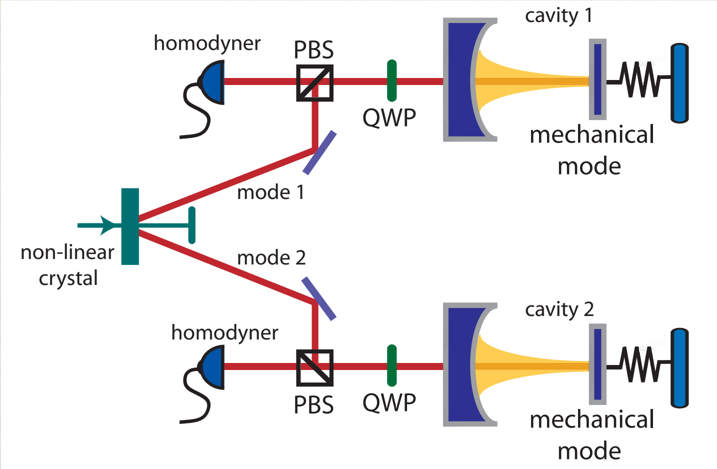
1. Non-classicality by photon-subtraction



2. Entanglement and diagnostics through cold-atom induced dynamics

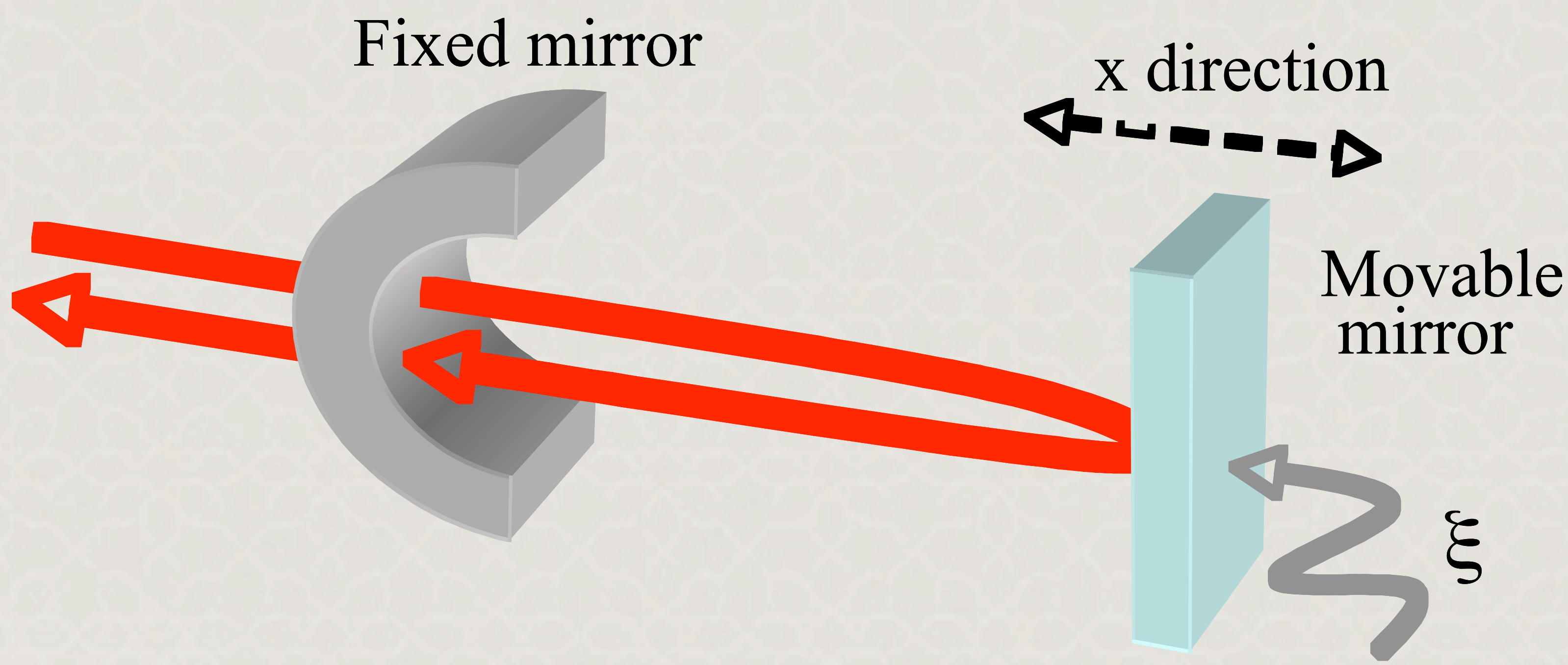


3. Quantum opto-mechanical networking

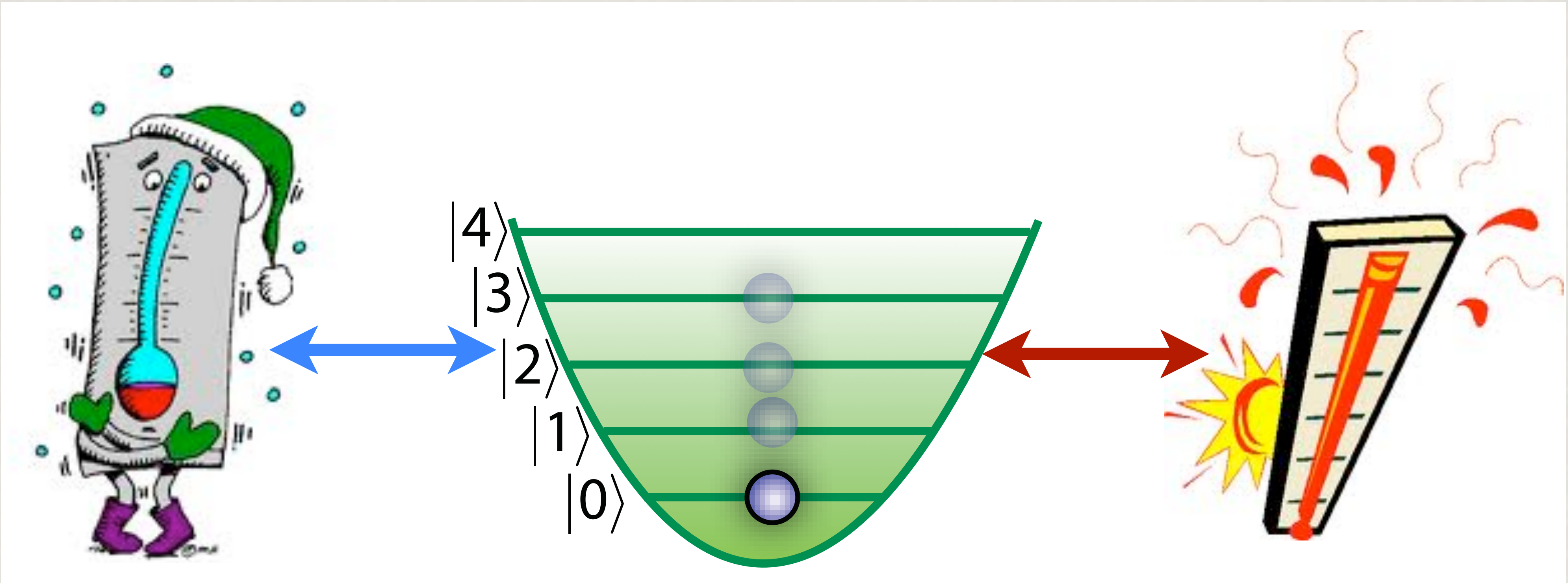


QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Define the context



Define the context



Opto-mechanical coupling

Brownian motion at temperature T

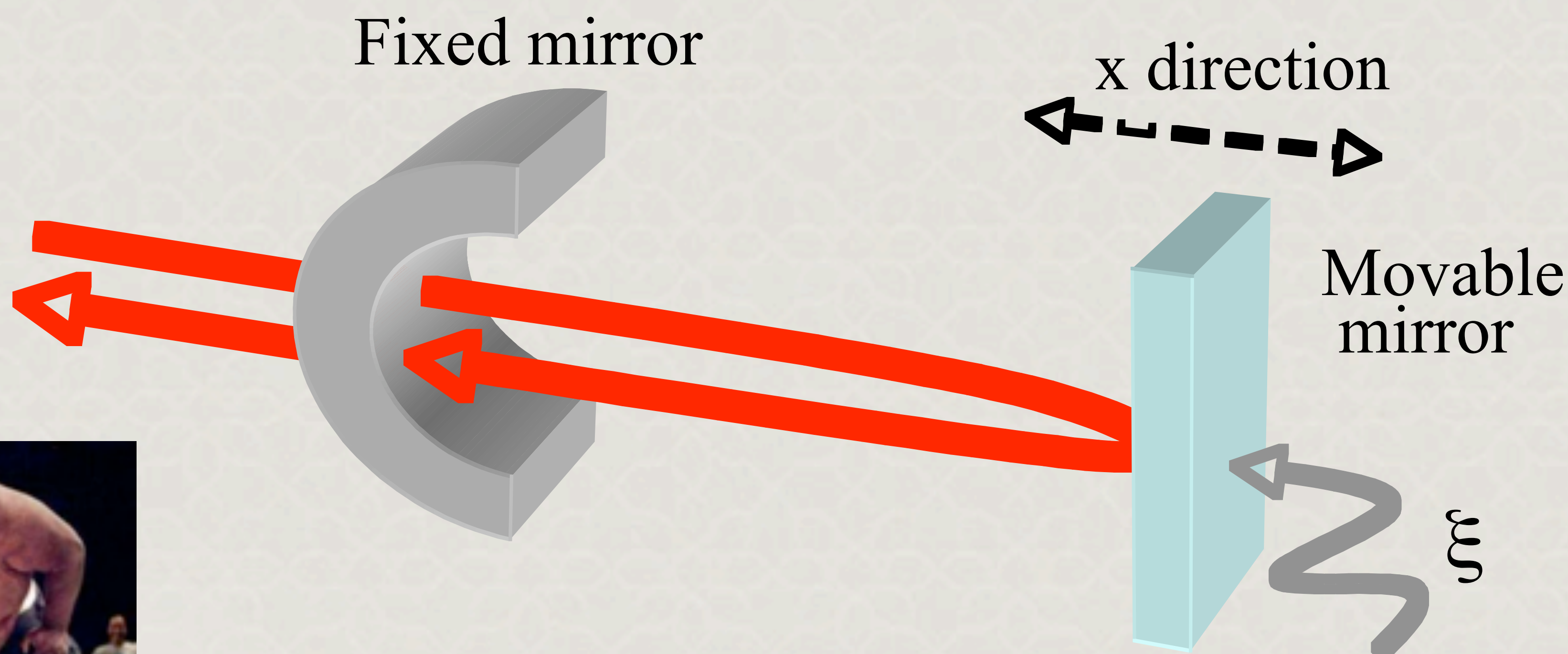
QTEQ QUANTUM TECHNOLOGY at QUEEN´S
www.qteq.info

An unfair game

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

An unfair game

$$\hat{\mathcal{H}}_{mc} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_m}{2}(\hat{P}_m^2 + \hat{Q}_m^2) - \hbar\chi\hat{n}_f\hat{Q}_m + i\hbar E(e^{-i\omega_0 t}\hat{a}^\dagger - e^{i\omega_0 t}\hat{a})$$

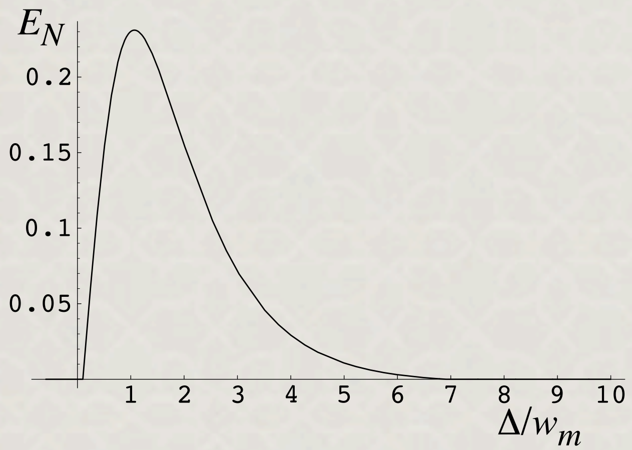


You against 8 blood-thirsty parameters..

Using a stick and a carrot..

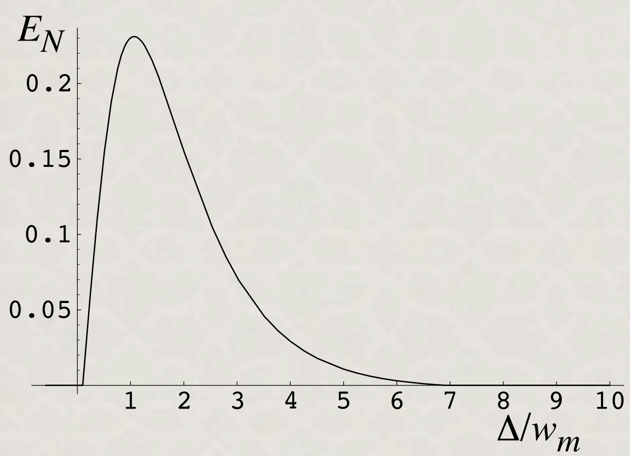
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Using a stick and a carrot..

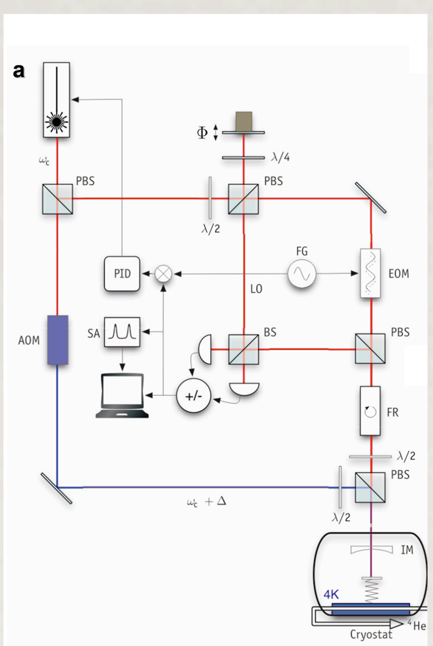


D. Vitali,et al. PRL **104**, 243602 (2007)

Using a stick and a carrot..



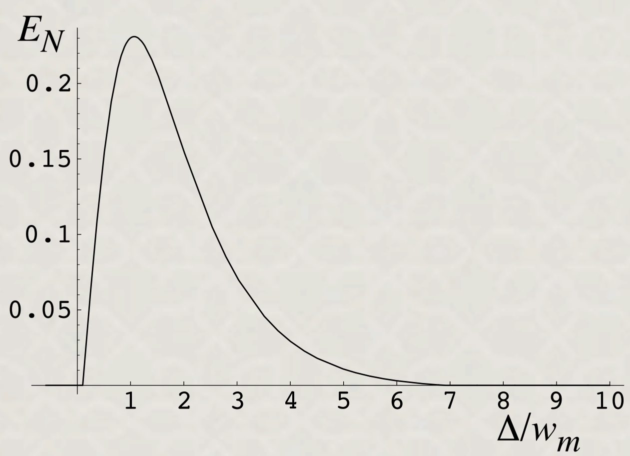
D. Vitali, et al. PRL ***104****, 243602 (2007)*



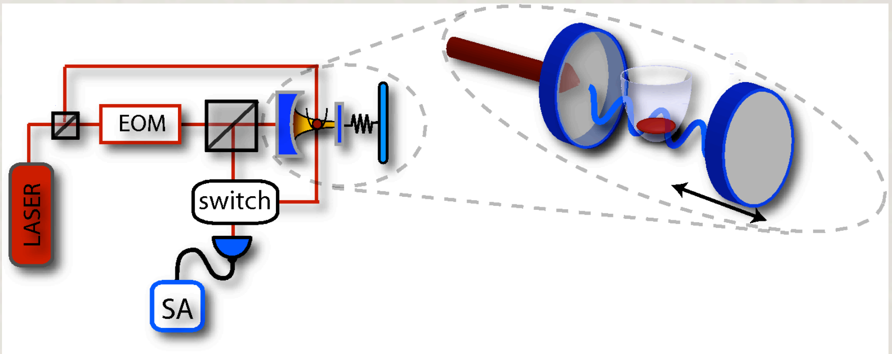
S. Groeblacher, et al., Nature 460, 724 (2009)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

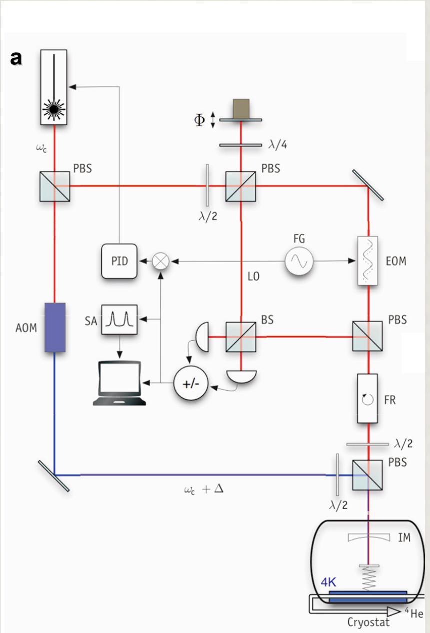
Using a stick and a carrot..



D. Vitali,et al. PRL **104**, 243602 (2007)



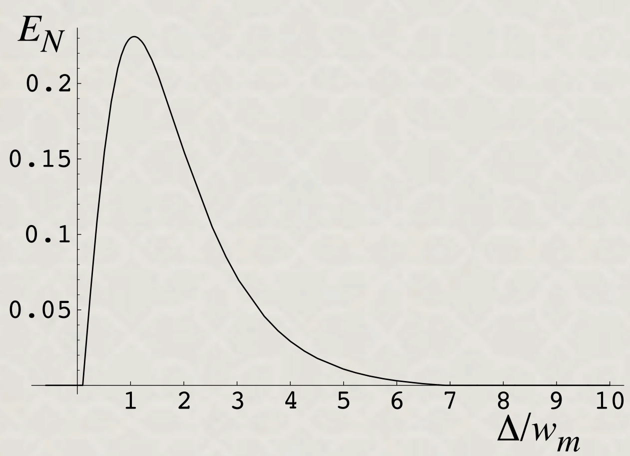
M.Paternostro, G. De Chiara,
and G. M. Palma, PRL **104**, 243602 (2010)



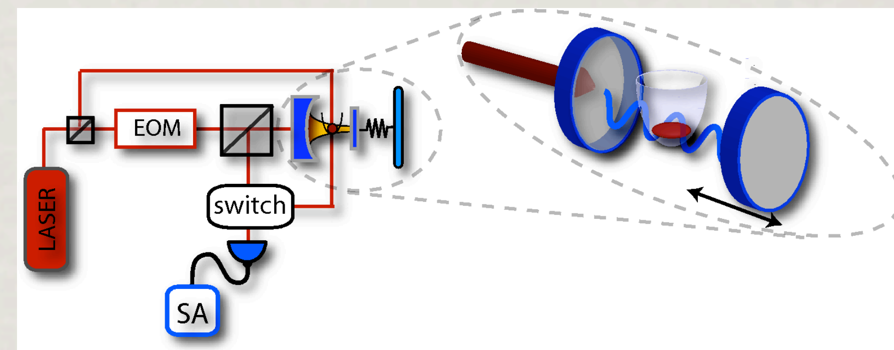
S. Groeblacher, et al., Nature 460, 724 (2009)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

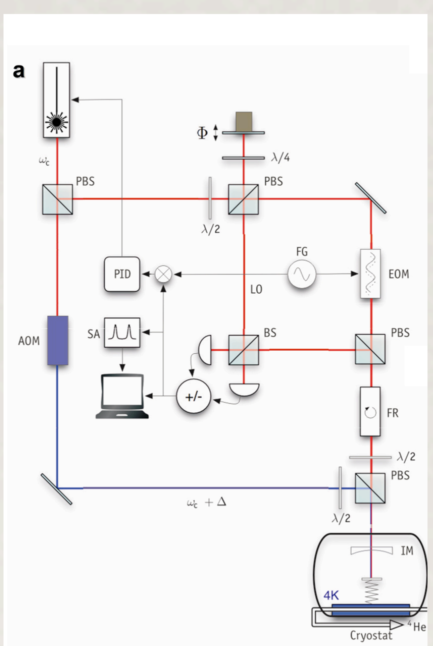
Using a stick and a carrot..



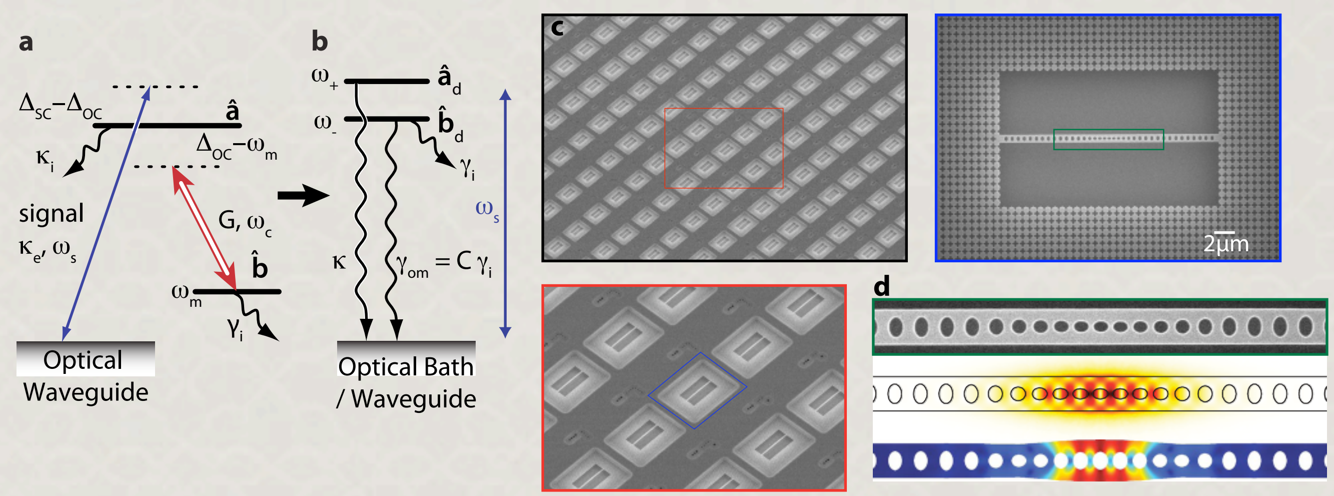
D. Vitali,et al. PRL **104**, 243602 (2007)



M.Paternostro, G. De Chiara, and G. M. Palma, PRL **104**, 243602 (2010)



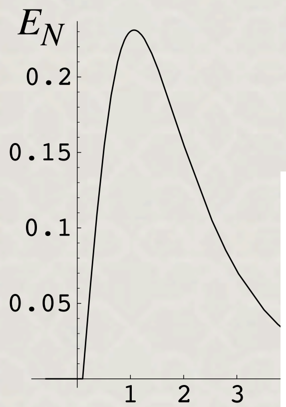
S. Groeblacher, et al., Nature 460, 724 (2009)



A. H. Safavi-Naeini, et al., Nature **472**, 69 (2011).

QTEQ QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Using a stick and a carrot..

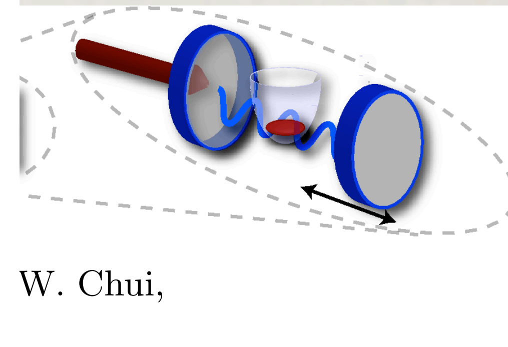


D. Vitali,

M. Aspelmeyer and K. C.
095001 (2008).
I. Favero and K. Karrai, Nat
F. Marquardt and S. M. Gir
C. Genes, A. Mari, D. Vitali
Mol. Opt. Phys. 57, 33 (200
K. W.
Stamper
F. Bren
Science
I. Wilso
penberg,
F. Marq
Phys. Re
C. Gene
pelmeye

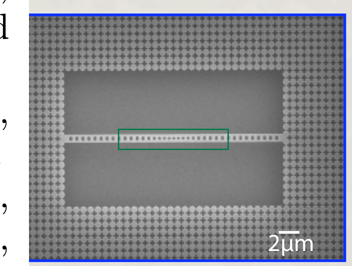
P. Verlot, A. Tav
and A. Heidmann
A. Schliesser, P. D
T. J. Kippenberg,
T. Corbitt, Y. Ch
D. Ottaway, H.
C. Wipf, and N. M
(2007).
J. D. Teufel, J.
Lehnert, Phys. Re
D. J. Wilson, C. A
Phys. Rev. Lett.
A. Schliesser, R.
and T. J. Kippenberg, Nat

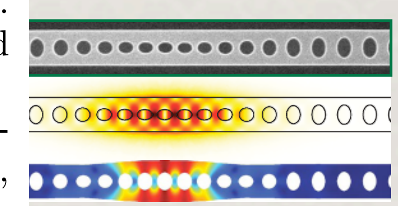
D. E. Chang, C. A. Regal, S. B. Papp, D. J. Wilson, J. Ye, O. Painter, H. J. Kimble, and P. Zoller, PNAS **107**, 1005 (2010).
P. F. Barker and M. N. Schneider, Phys. Rev. A **81**, 023826 (2010).
I. Wilson-Rae, P. Zoller, and A. Imamoglu, Phys. Rev. Lett. **92**, 075507 (02 2004).
M. Poggio, C. L. Degen, H. J. Mamin, and D. Rugar, Phys. Rev. Lett. **99**, 017201 (2007).
D. Kleckner and D. Bouwmeester, Nature **444**, 75 (2006).
T. Corbitt, C. Wipf, T. Bodiya, D. Ottaway, D. Sigg, N. Smith, S. Whitcomb, and N. Mavalvala, Phys. Rev. Lett. **99**, 160801 (2007).
A. A. Clerk, F. Marquardt, and K. Jacobs, New J. Phys. **10**, 095010 (2008).
K. Hammerer, M. Aspelmeyer, E. Polzik, and P. Zoller, Phys. Rev. Lett. **102**, 020501 (2009).
W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003).
M. Paternostro, D. Vitali, S. Gigan, M. S. Kim, C. Brukner, J. Eisert, and M. Aspelmeyer, Phys. Rev. Lett. **99**, 250401 (2007).

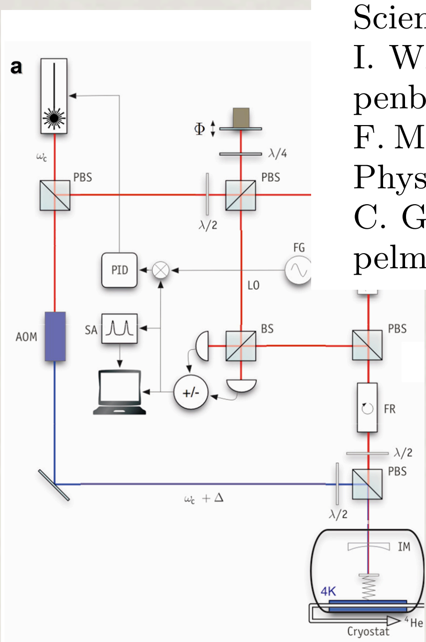


W. Chui,
. Schwab,
. Schwab,
. C. Bial-
D. Sank,
inis, and
Scherer,
07 2005).
. Blaser,
Bäuerle,
57 (2006).
ard, and

G. De Chiara, **104**, 243602 (2010)







J. D. Thompson, B. M. Zwickl, A. M. Jayich, F. Marquardt, S. M. Girvin, and J. G. E. Harris, Nature **452**, 72 (2008).
C. A. Regal, J. D. Teufel, and K. W. Lehnert, Nature Phys. **4**, 555 (2008).

S. Groeblacher, et al., Nature **460**, *724 (2009)*

re **472**, *69 (2011).*

Fair enough..

QTEQ
QUANTUM TECHNOLOGY at QUEEN´S
w w w . q t ϵ q . i n f o

Fair enough..

$$\boldsymbol{\sigma} = \begin{pmatrix} \boldsymbol{m} & \boldsymbol{c} \\ \boldsymbol{c}^T & \boldsymbol{f} \end{pmatrix}$$

QTEQ QUANTUM TECHNOLOGY at QUEEN'S www.qteq.info

Fair enough..

$$\boldsymbol{\sigma} = \begin{pmatrix} \boldsymbol{m} & \boldsymbol{c} \\ \boldsymbol{c}^T & \boldsymbol{f} \end{pmatrix}$$

$$\boldsymbol{K\sigma} + \boldsymbol{\sigma K}^t = -\boldsymbol{D}$$

QTEQ QUANTUM TECHNOLOGY at QUEEN´S www.qteq.info

Fair enough..

$$\sigma = \begin{pmatrix} m & c \\ c^T & f \end{pmatrix} \qquad K\sigma + \sigma K^t = -D$$

$$D = \mathrm{diag}[0, \gamma_m(2\bar{n}+1), \kappa, \kappa]$$

QTEQ QUANTUM TECHNOLOGY at QUEEN'S www.qteq.info

Fair enough..

$$\boldsymbol{\sigma} = \begin{pmatrix} \boldsymbol{m} & \boldsymbol{c} \\ \boldsymbol{c}^T & \boldsymbol{f} \end{pmatrix}$$

$$\boldsymbol{K\sigma} + \boldsymbol{\sigma K}^t = -\boldsymbol{D}$$

$$\boldsymbol{D}=\mathrm{diag}[0, \gamma_m(2\bar{n}+1), \kappa, \kappa]$$

$$\chi(\eta, \lambda)=\mathrm{e}^{-\frac{1}{2}\tilde{\mathbf{q}}\boldsymbol{\sigma}\tilde{\mathbf{q}}^t}$$

QTEQ QUANTUM TECHNOLOGY at QUEEN'S www.qteq.info

Fair enough..

$$\boldsymbol{\sigma} = \begin{pmatrix} \boldsymbol{m} & \boldsymbol{c} \\ \boldsymbol{c}^T & \boldsymbol{f} \end{pmatrix} \qquad \boldsymbol{K\sigma} + \boldsymbol{\sigma K}^t = -\boldsymbol{D}$$

$$\boldsymbol{D} = \mathrm{diag}[0, \gamma_m(2\bar{n}+1), \kappa, \kappa]$$

$$\chi(\eta, \lambda) = \mathrm{e}^{-\frac{1}{2}\tilde{\mathbf{q}}\boldsymbol{\sigma}\tilde{\mathbf{q}}^t}$$

$$\varrho_{fm} = \pi^{-2}\int \chi(\eta, \lambda)\hat{D}_m^\dagger(\eta)\otimes\hat{D}_f^\dagger(\lambda)d^2\eta\, d^2\lambda$$

QTEQ

QUANTUM TECHNOLOGY at QUEEN'S

www.qteq.info

Squeezing ?

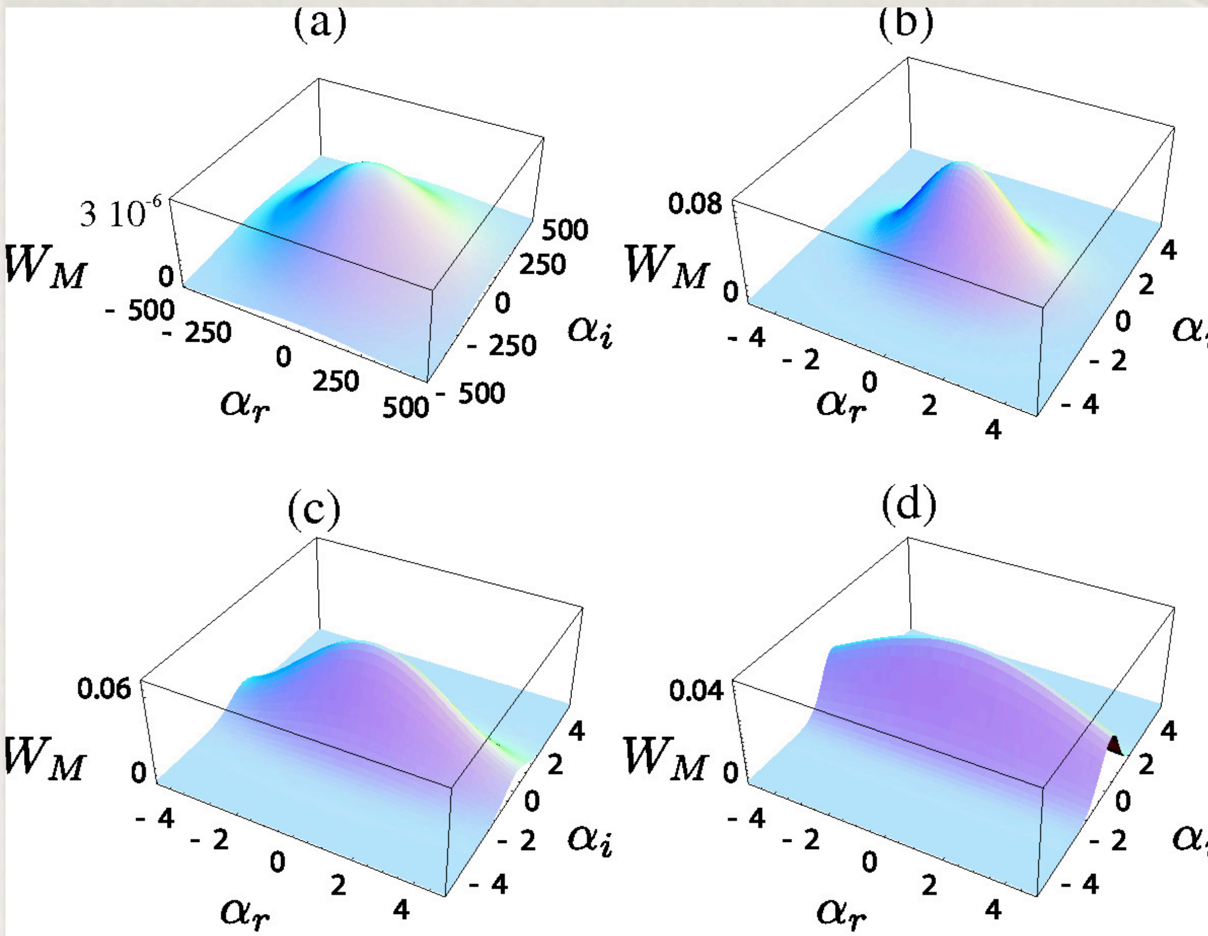
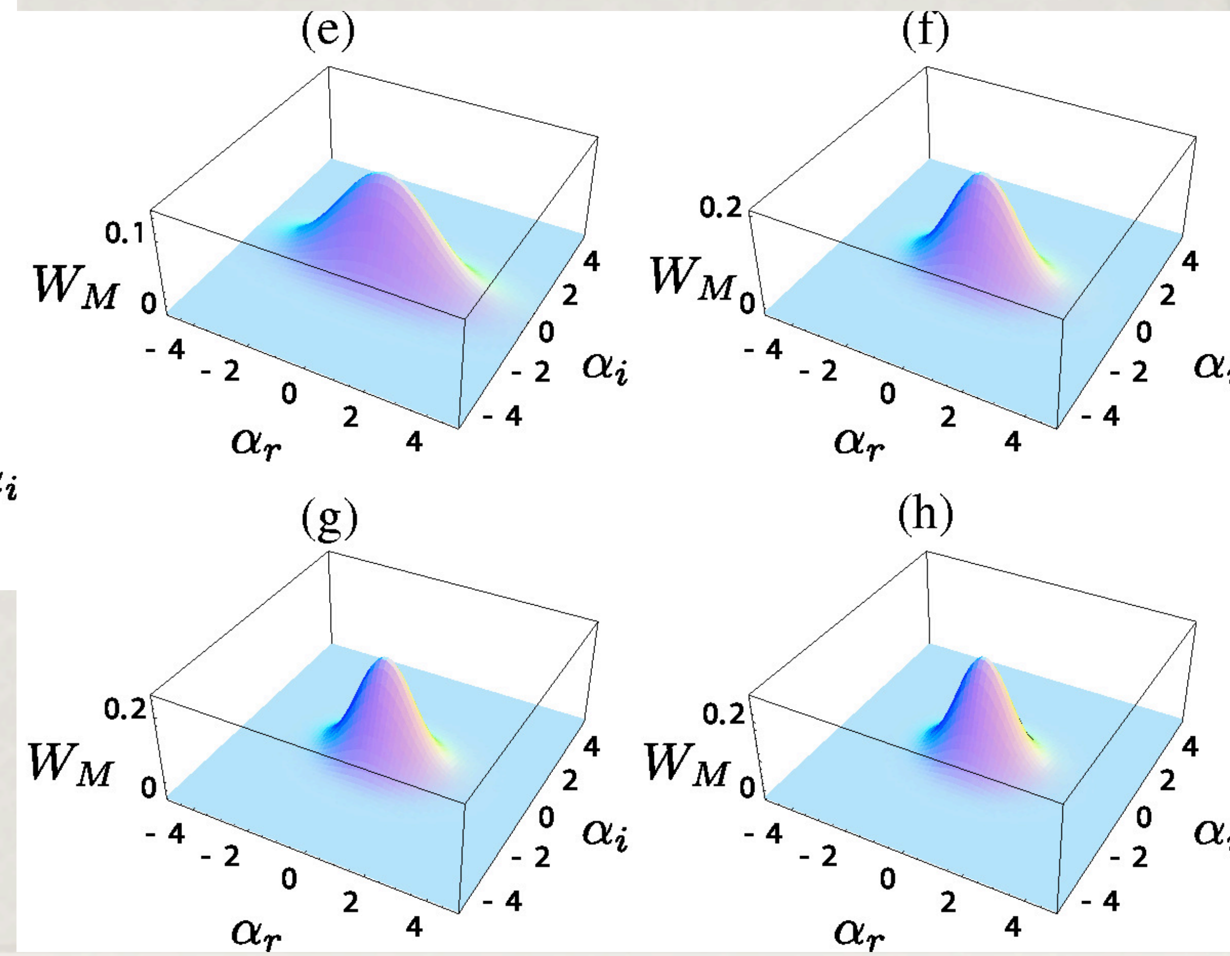


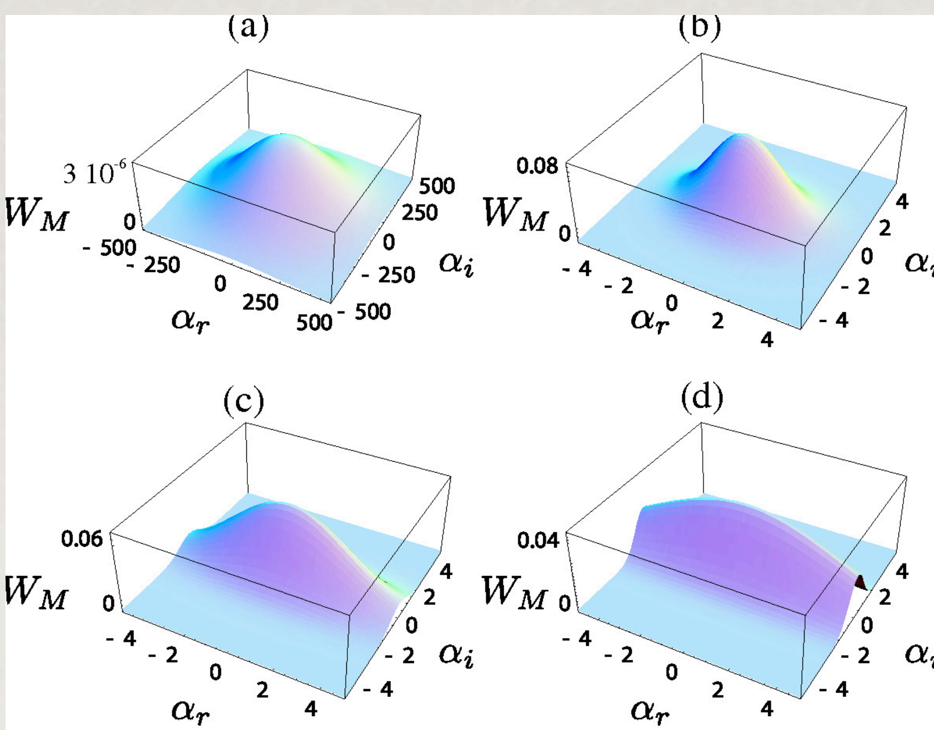
FIG. 1: Wigner function associated with the mirror state plotted against the rescaled detuning $\Delta = j\omega_m/10$ $(j \in \mathbb{Z})$. Each panel corresponds to a set value of j. We have taken $j = 0$ (**a**), 4 (**b**), 7 (**c**), 10 (**d**), 13 (**e**), 16 (**f**), 19 (**g**) and 20 (**h**).



M. Paternostro (unpublished)

QTEQ
QUANTUM TECHNOLOGY at QUEEN´S
www.qteq.info

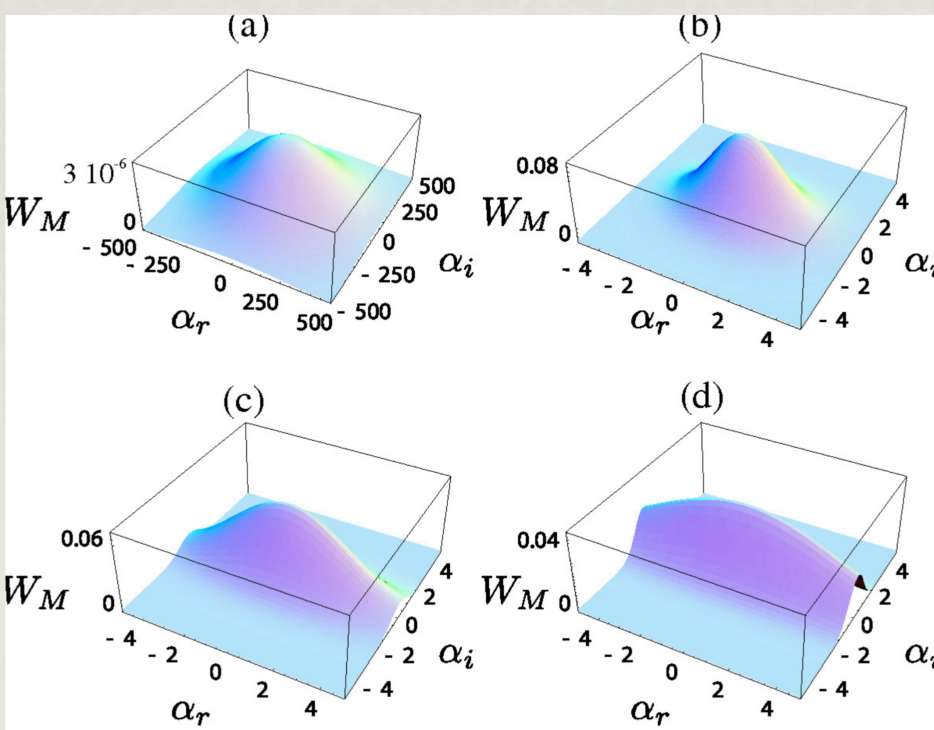
Squeezing ?



QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

Squeezing ?

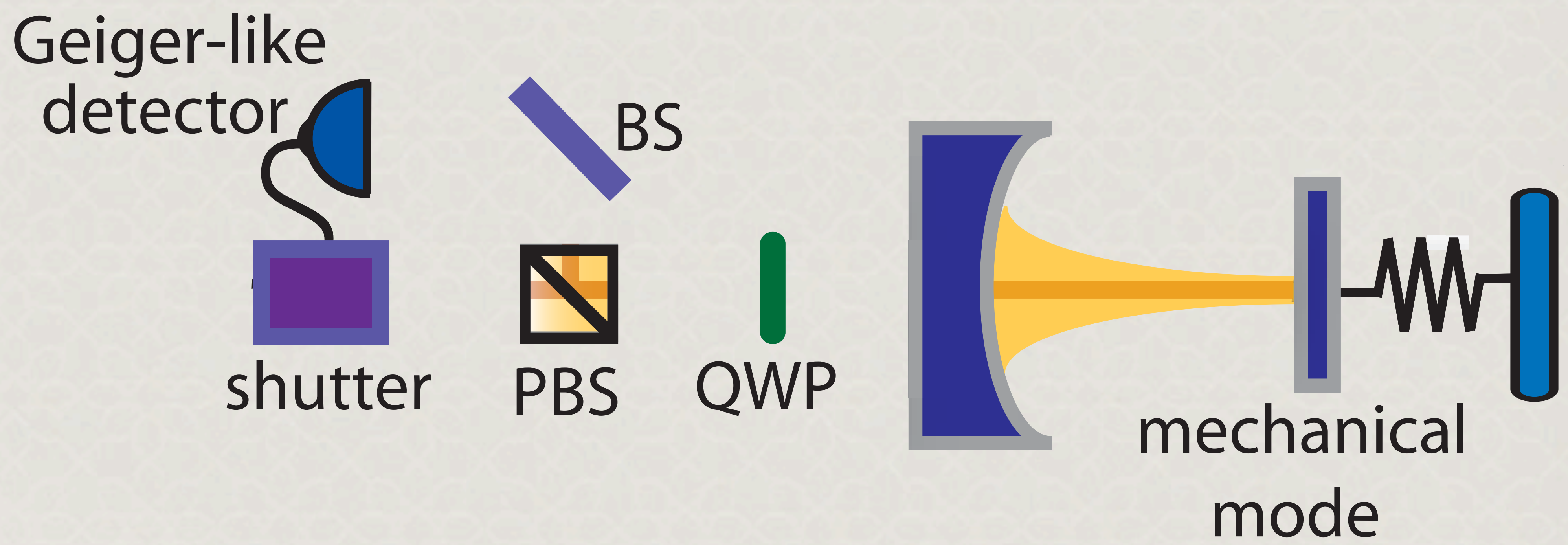
..but with no vitamin C at all..



QUANTUM TECHNOLOGY at QUEEN´S
w w w . q t e q . i n f o

Credit when is due

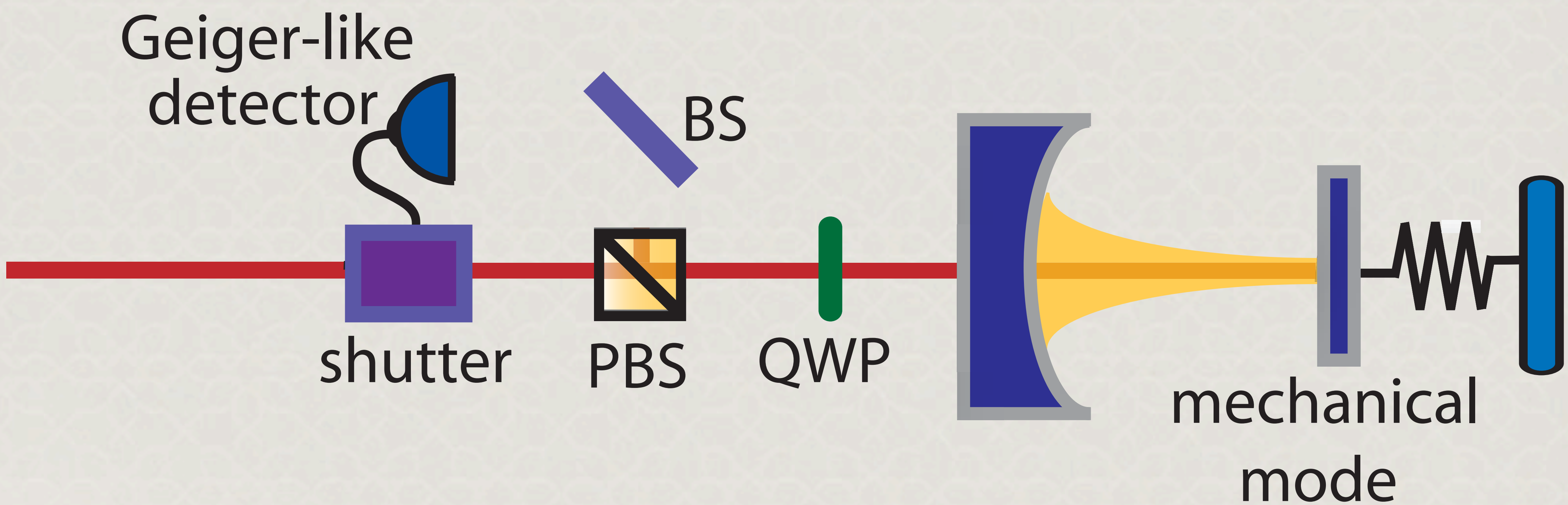
Less is more



M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

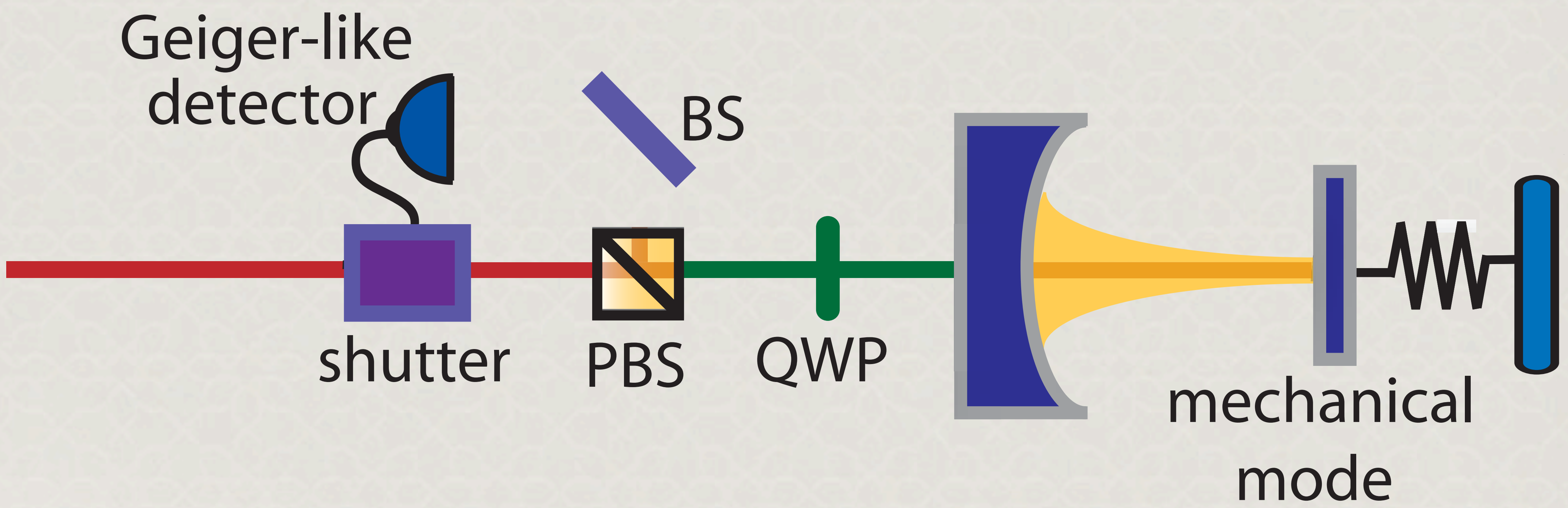
QTEQ QUANTUM TECHNOLOGY at QUEEN'S www.qteq.info

Less is more



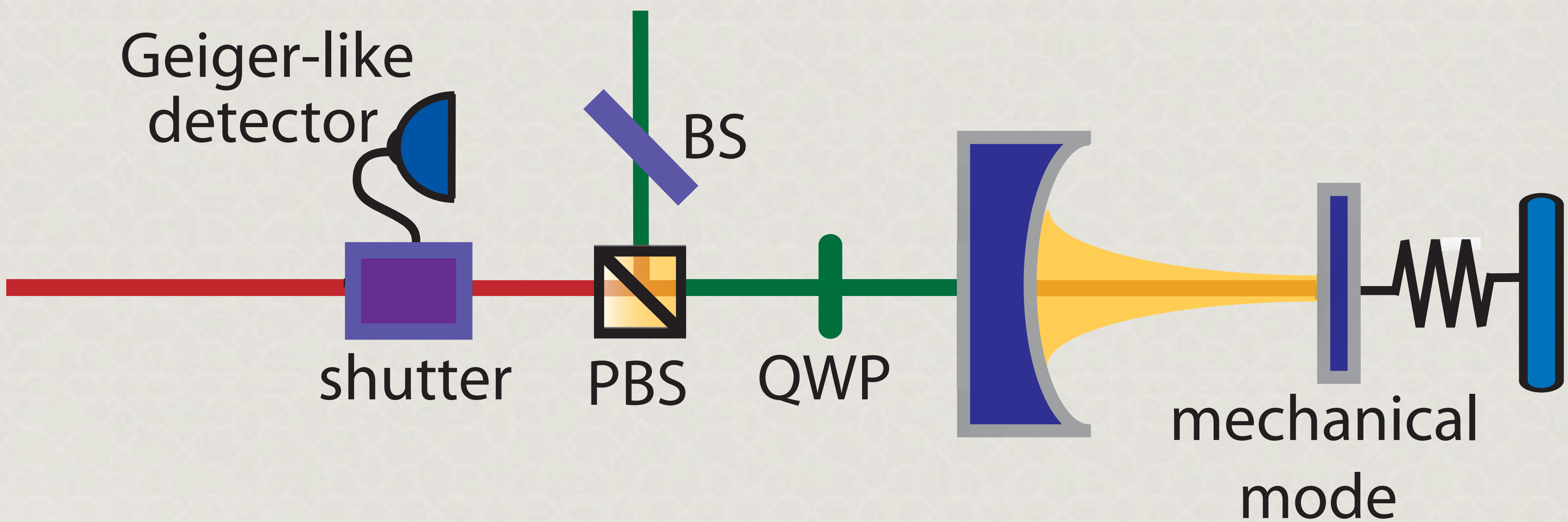
M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

Less is more



M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

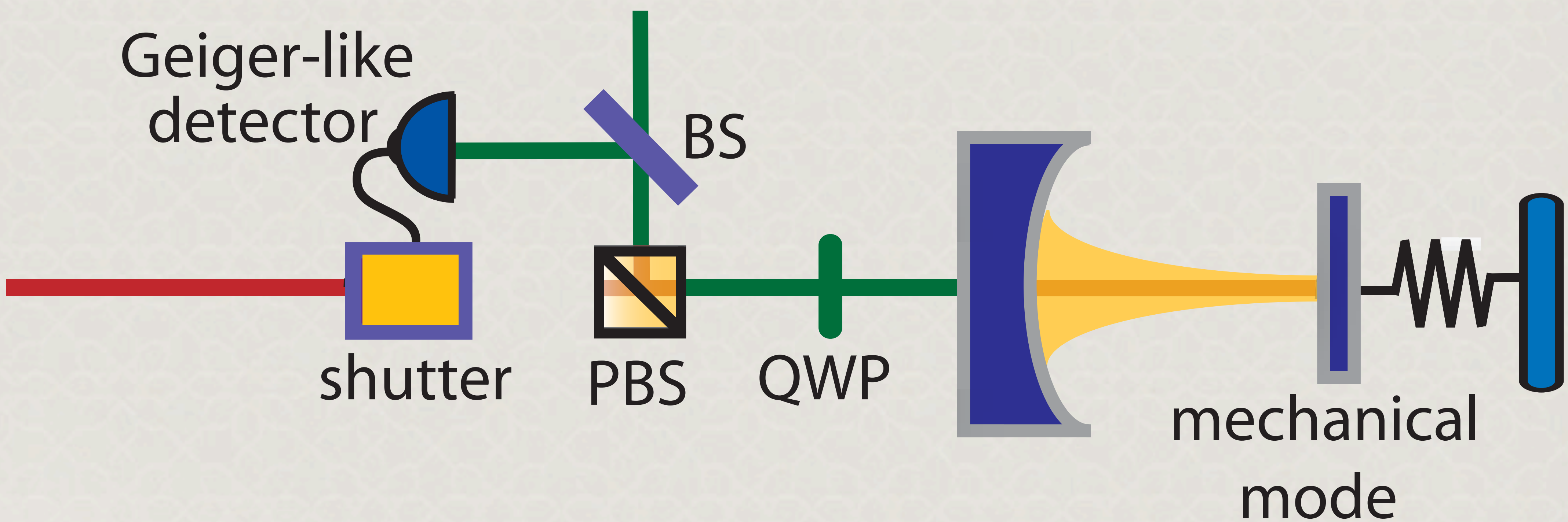
Less is more



M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

QTEQ QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Less is more



M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Less is more

M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

QTEQ QUANTUM TECHNOLOGY at QUEEN'S www.qteq.info

Less is more

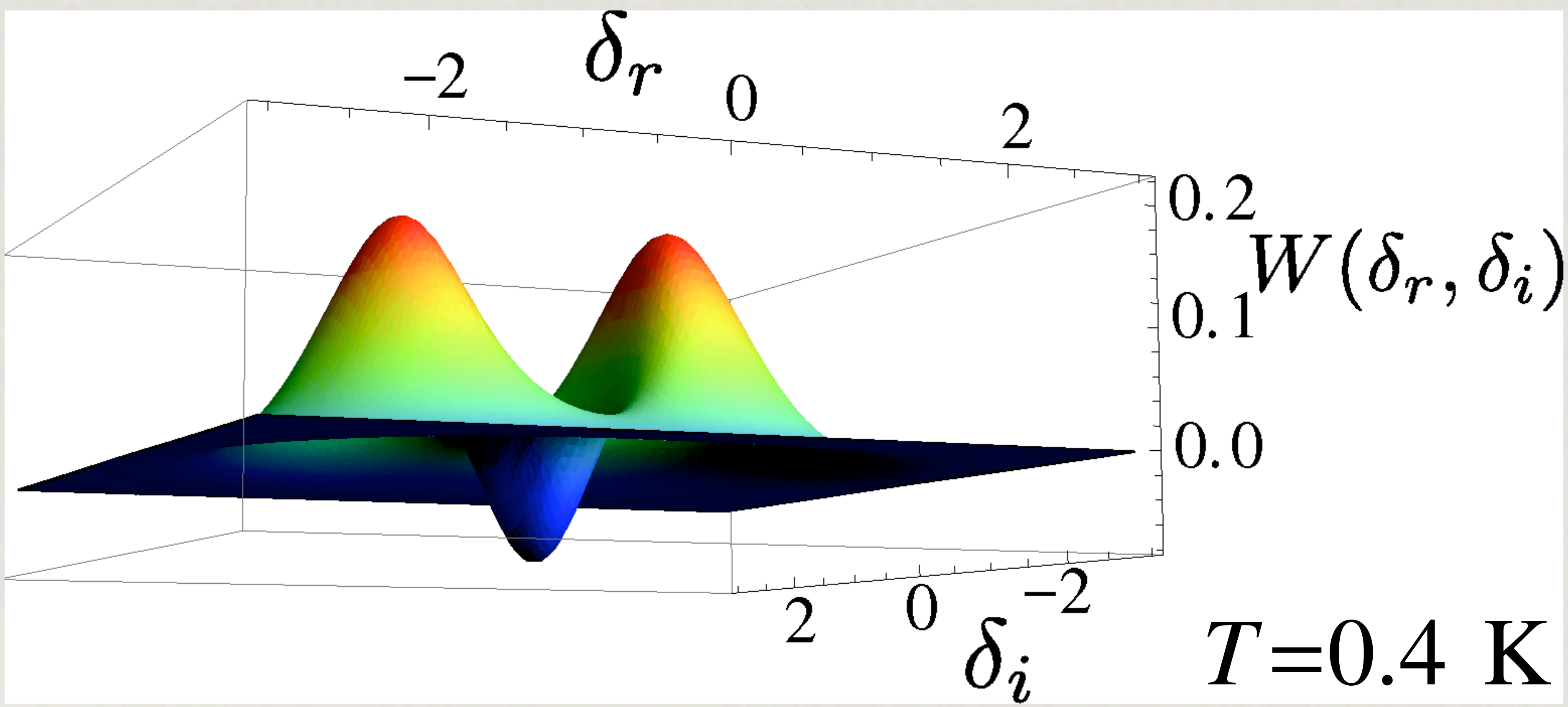
$$W(\delta_r, \delta_i) = \pi \mathcal{N} \mathcal{A}(\boldsymbol{\sigma}) \exp\left[-2\left(\delta_i^2/m_{11} - \delta_r^2/m_{22}\right)\right]$$

M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Less is more

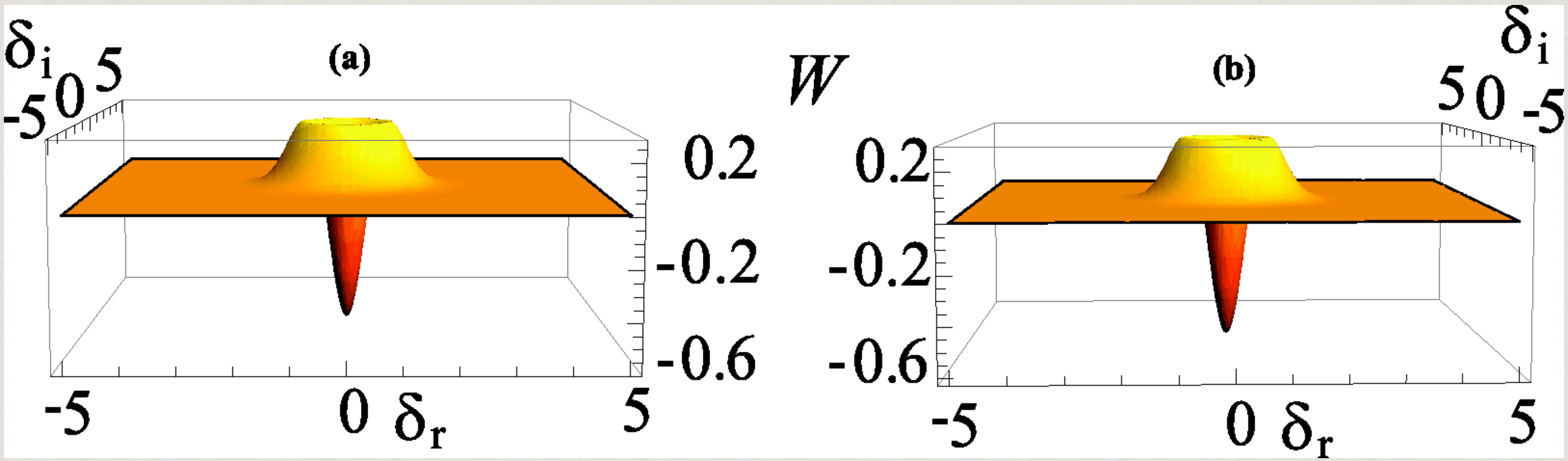
$$W(\delta_r, \delta_i) = \pi \mathcal{N} \mathcal{A}(\boldsymbol{\sigma}) \exp\left[-2\left(\delta_i^2/m_{11} - \delta_r^2/m_{22}\right)\right]$$



M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

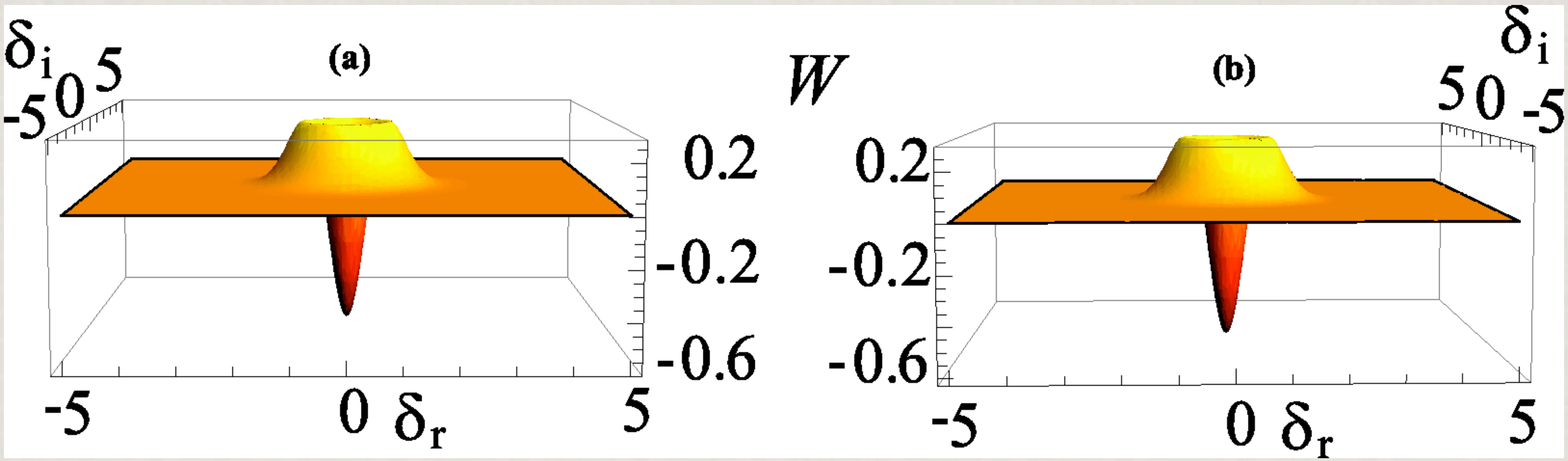
Spot the differences



M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

Spot the differences

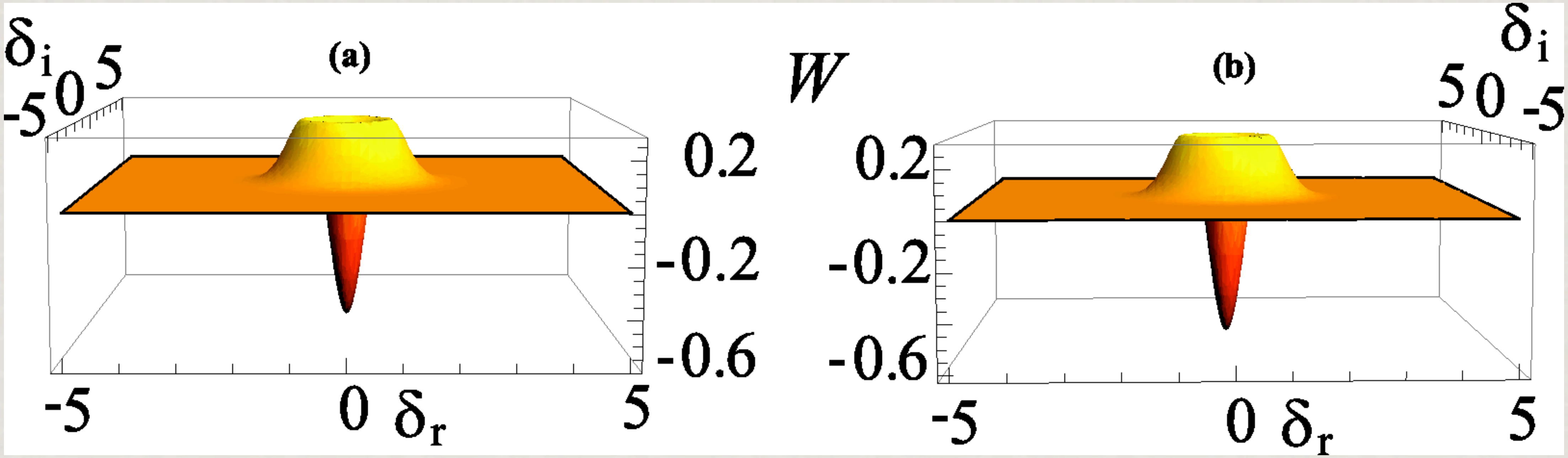


Conditional mechanical state at low temperature

M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Spot the differences



Conditional mechanical state at low temperature

Two-mode squeezed state at low squeezing

M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Spot the differences

Traced out

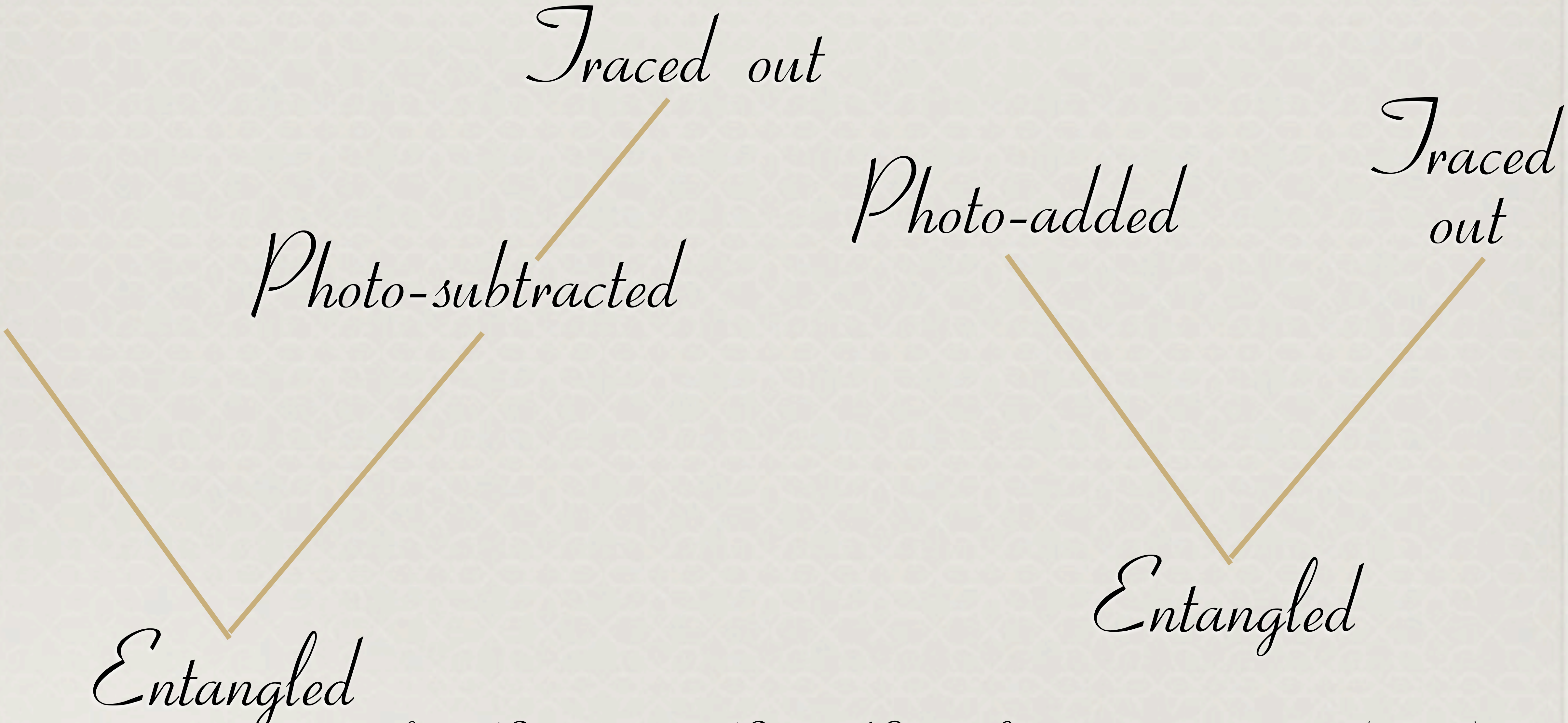
Photo-subtracted

Entangled

M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN´S
w w w . q t e q . i n f o

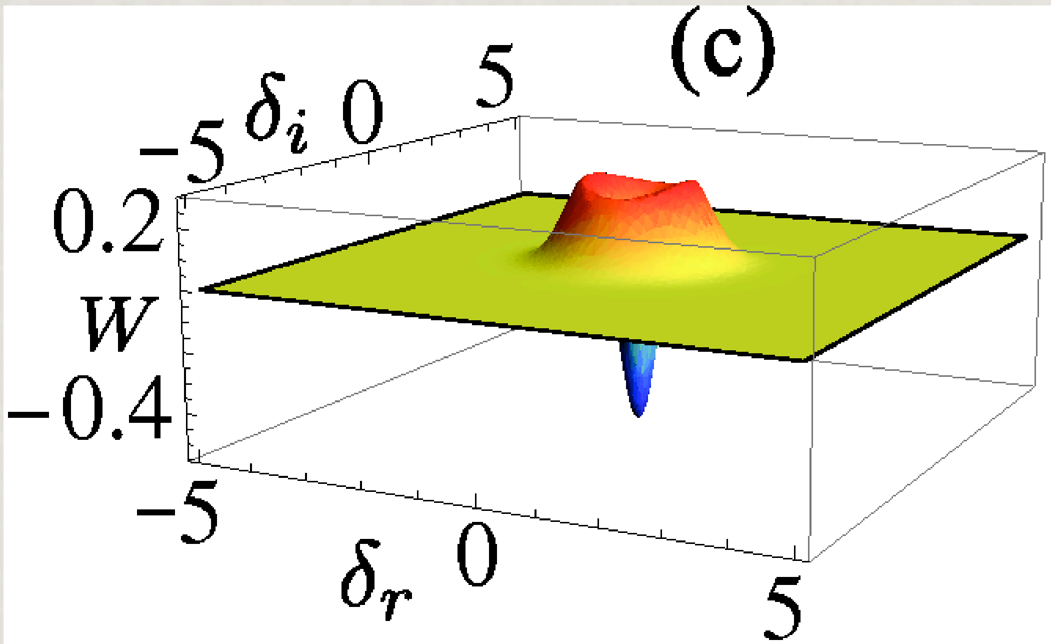
Spot the differences



M. Paternostro, Phys. Rev. Lett. 106, 183601 (2011)

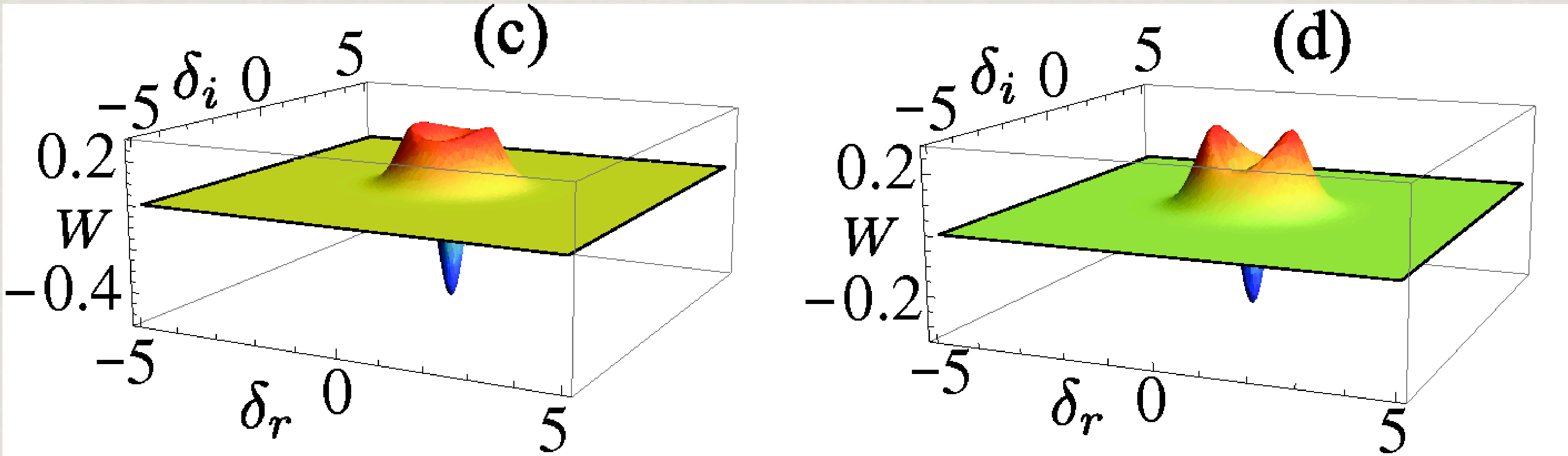
Temperature is the key

Temperature is the key

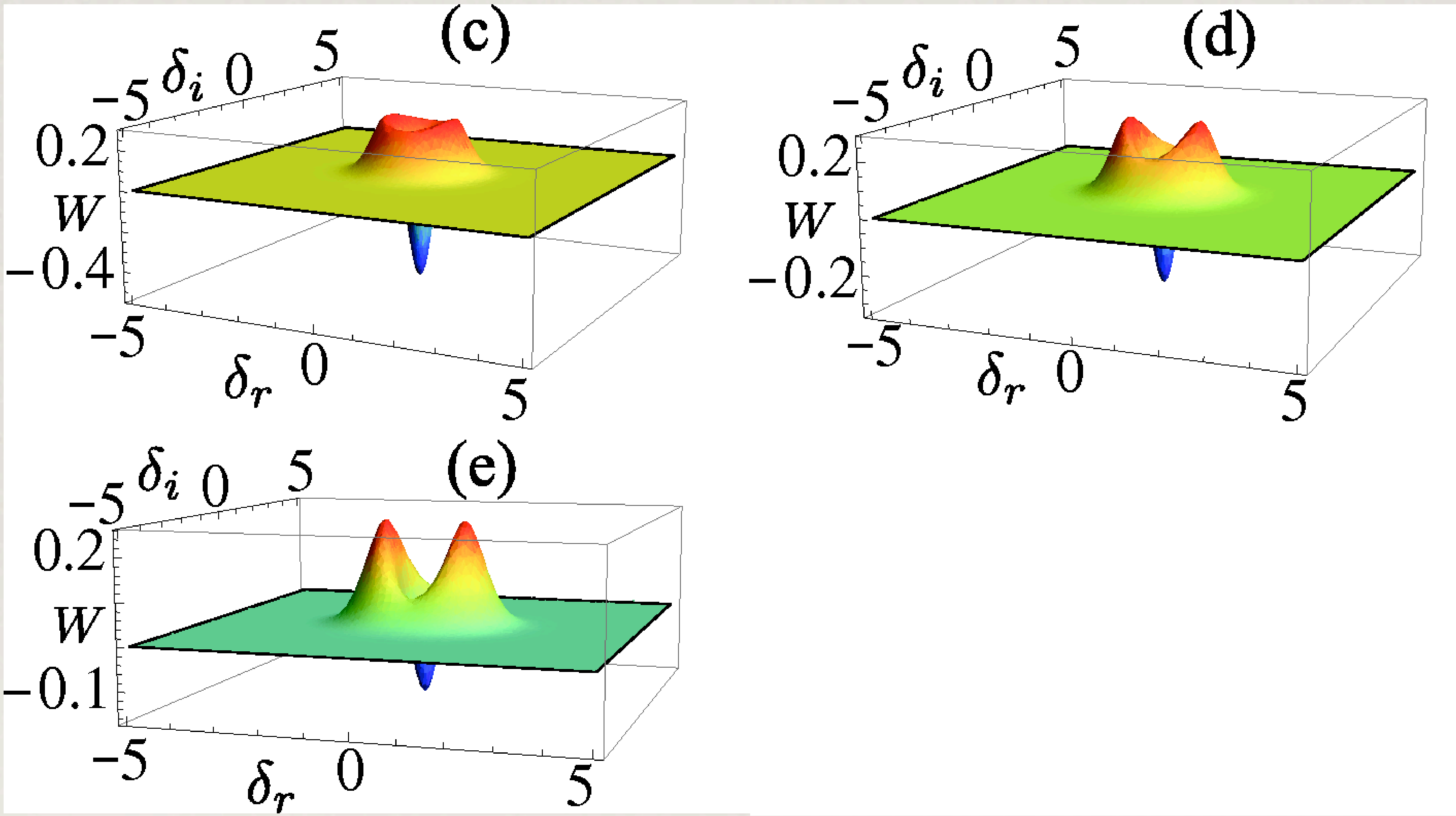


QTEQ
QUANTUM TECHNOLOGY at QUEEN´S
w w w . q t e q . i n f o

Temperature is the key

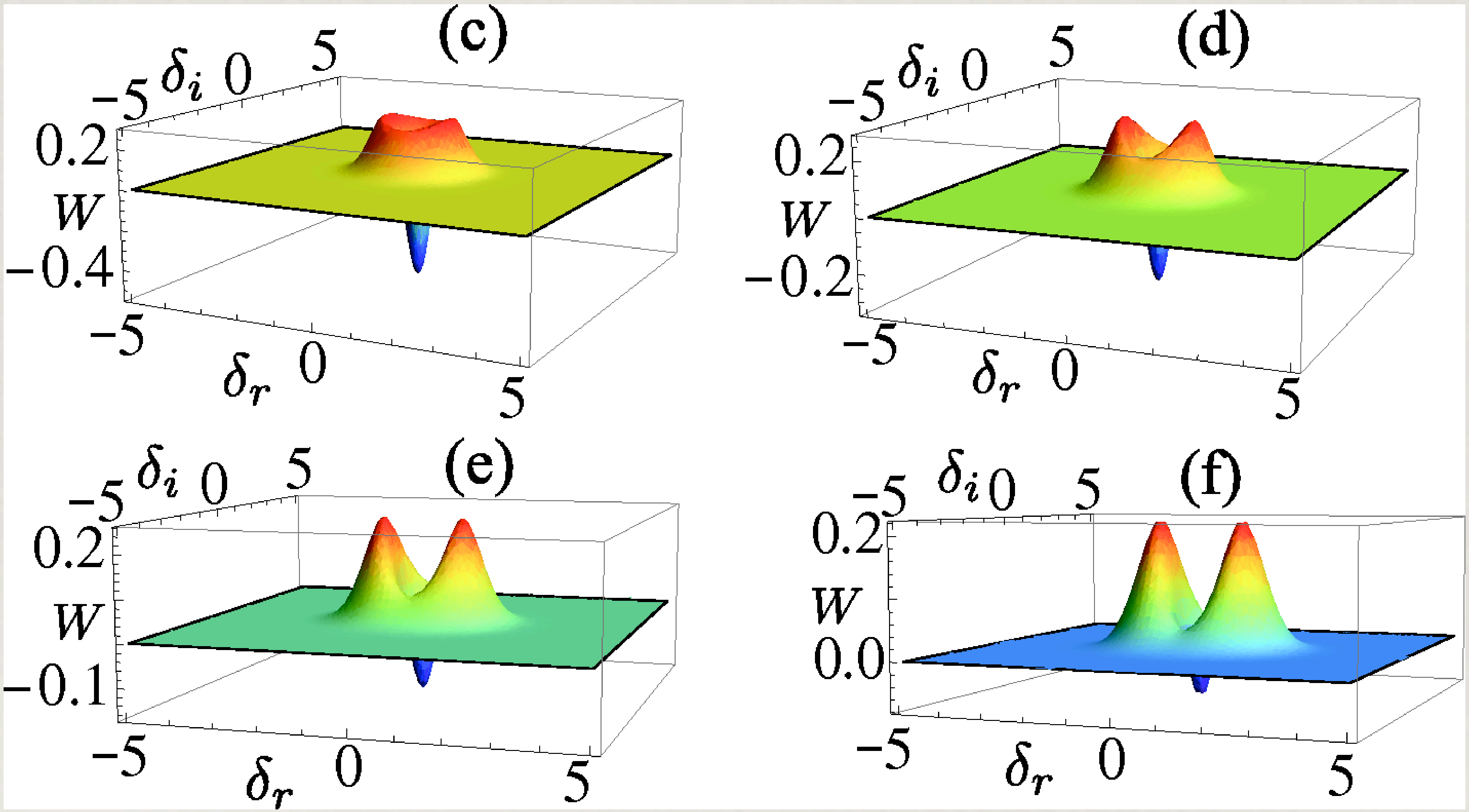


Temperature is the key



Temperature is the key

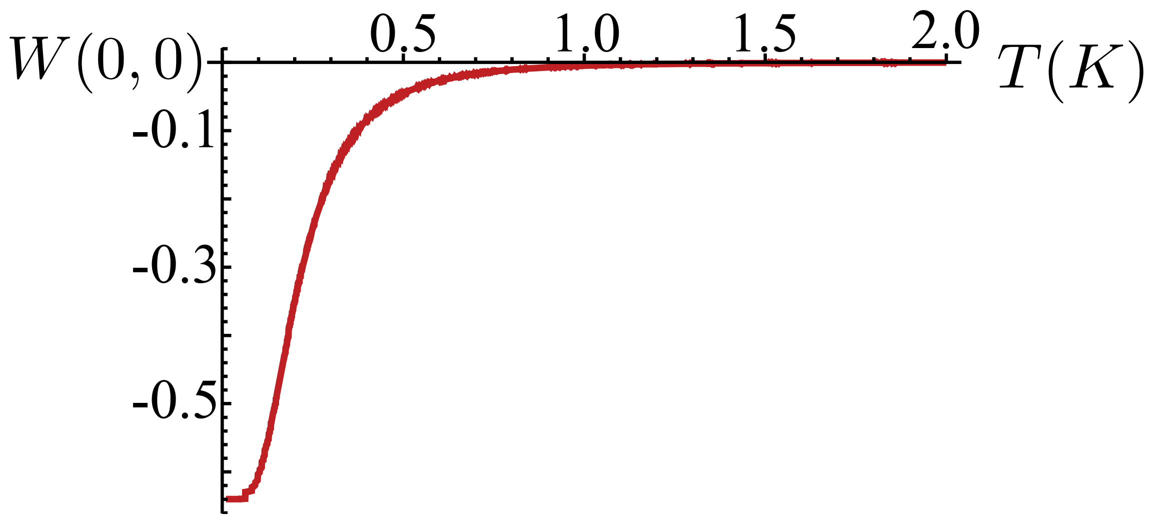
QUANTUM TECHNOLOGY at QUEEN´S
w w w . q t ϵ q . i n f o



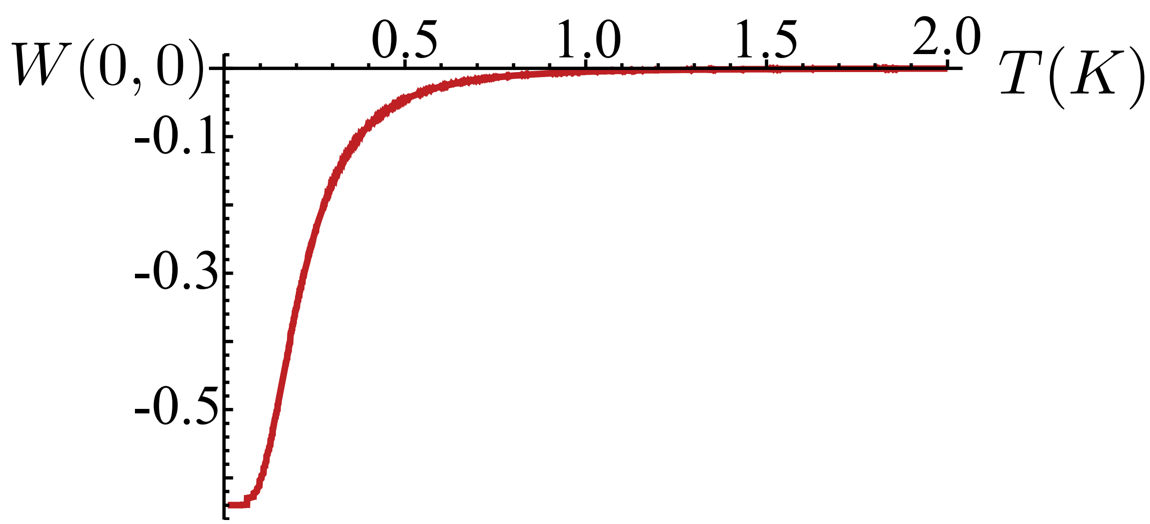
A few remarks

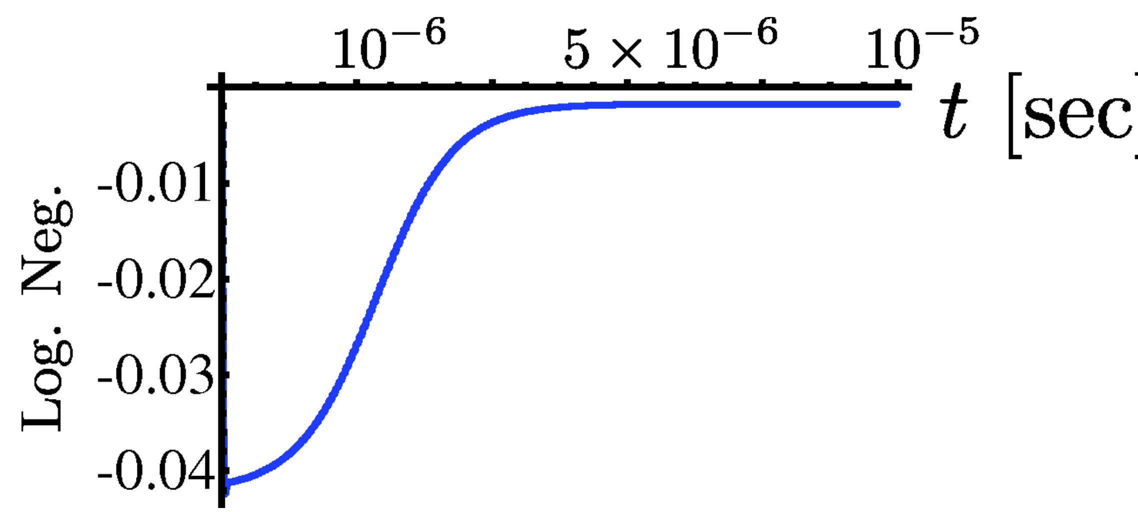
Wednesday, 8 June 2011

A few remarks



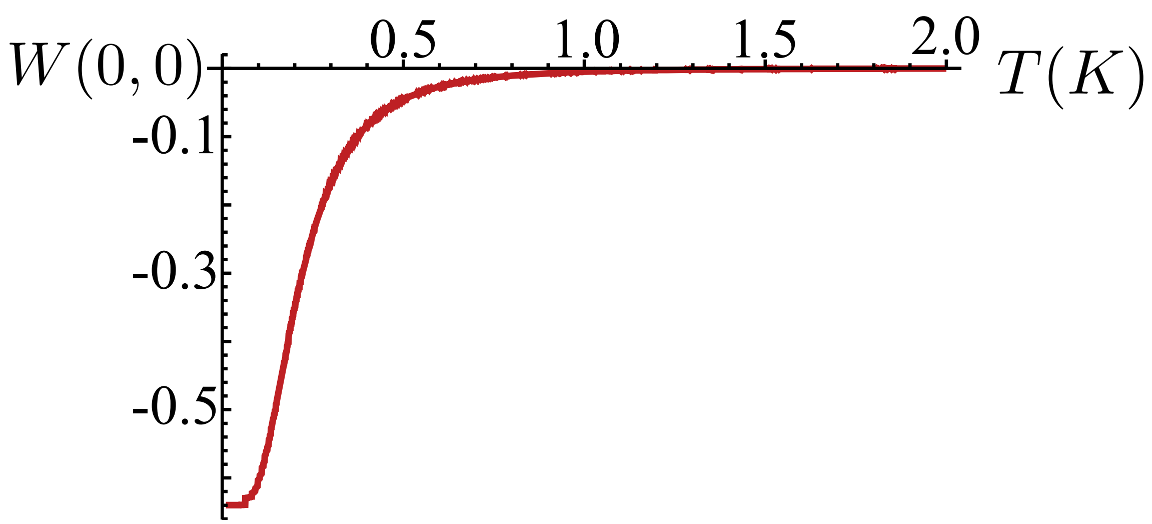
A few remarks

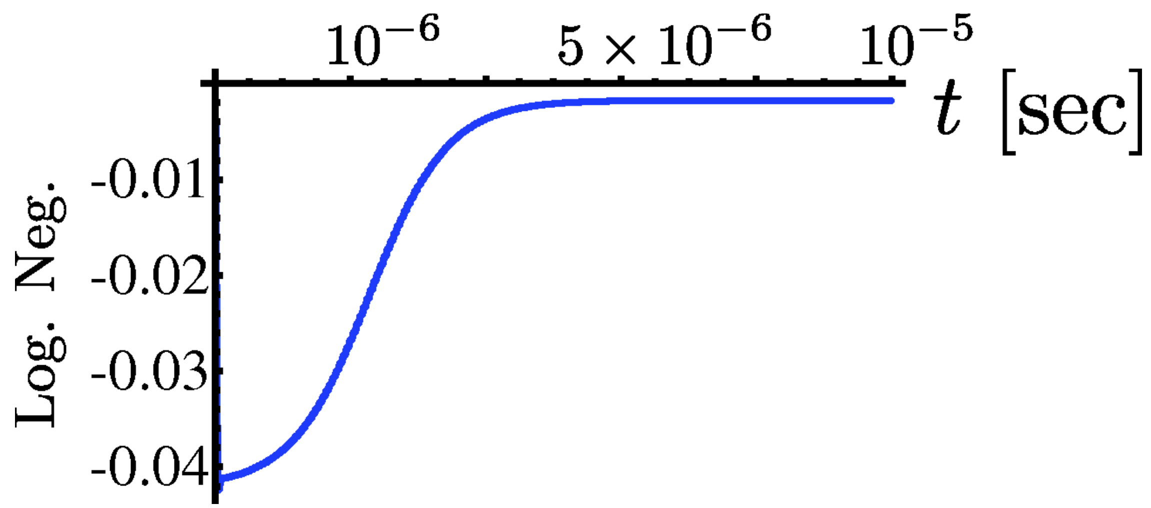


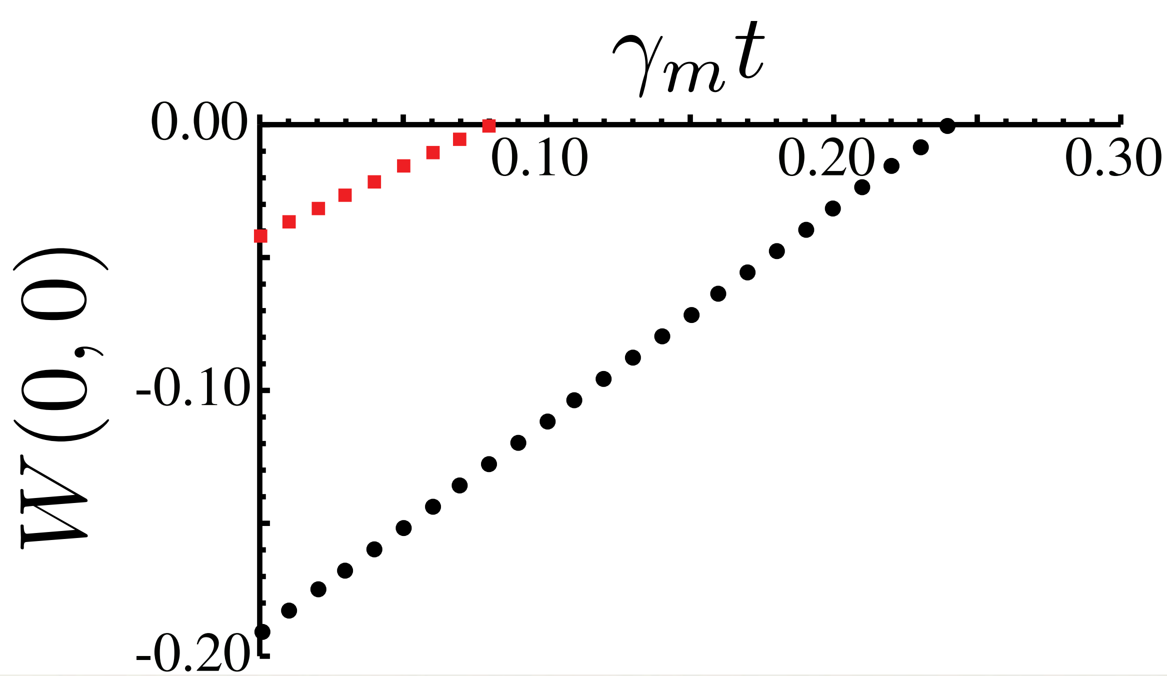


QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

A few remarks



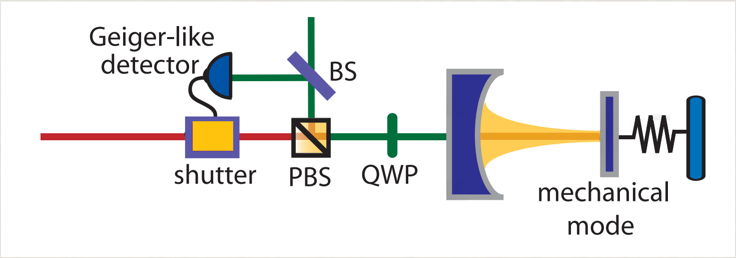




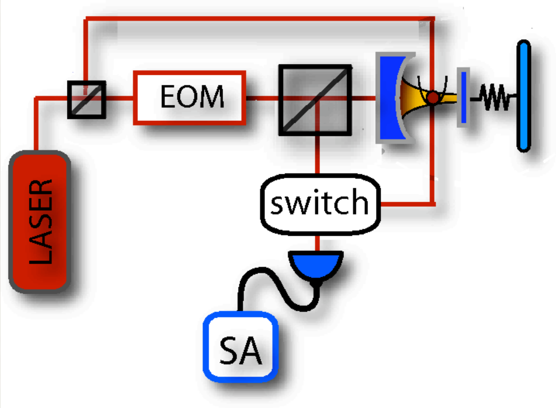
Second course

QTEQ QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

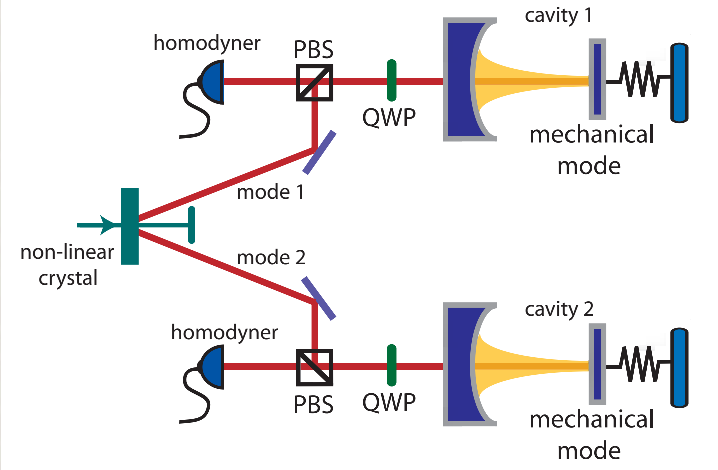
1. Non-classicality by photon-subtraction



2. Entanglement and diagnostics through cold-atom induced dynamics



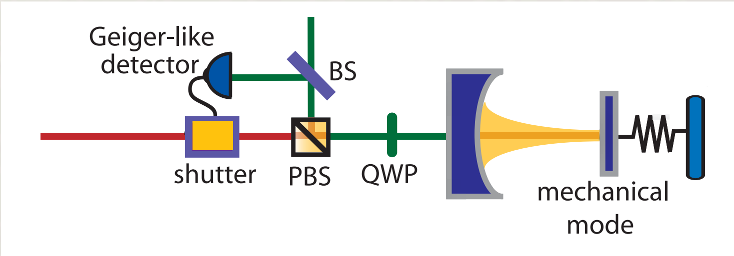
3. Quantum opto-mechanical networking



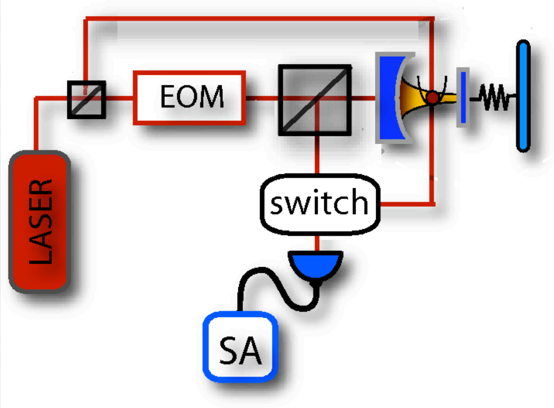
Second course

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

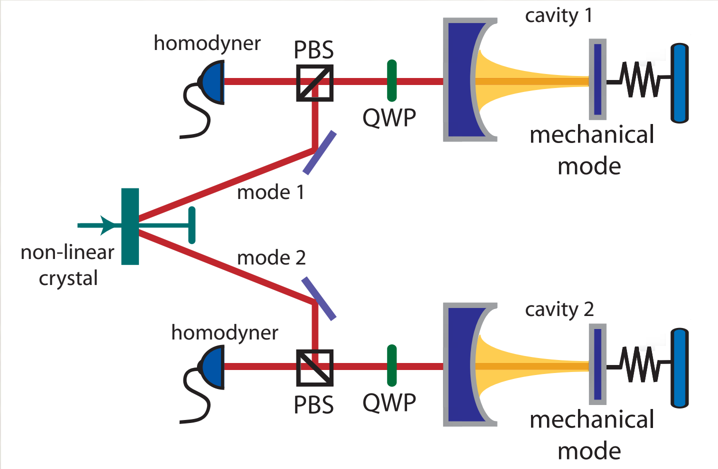
1. Non-classicality by photon-subtraction



2. Entanglement and diagnostics through cold-atom induced dynamics

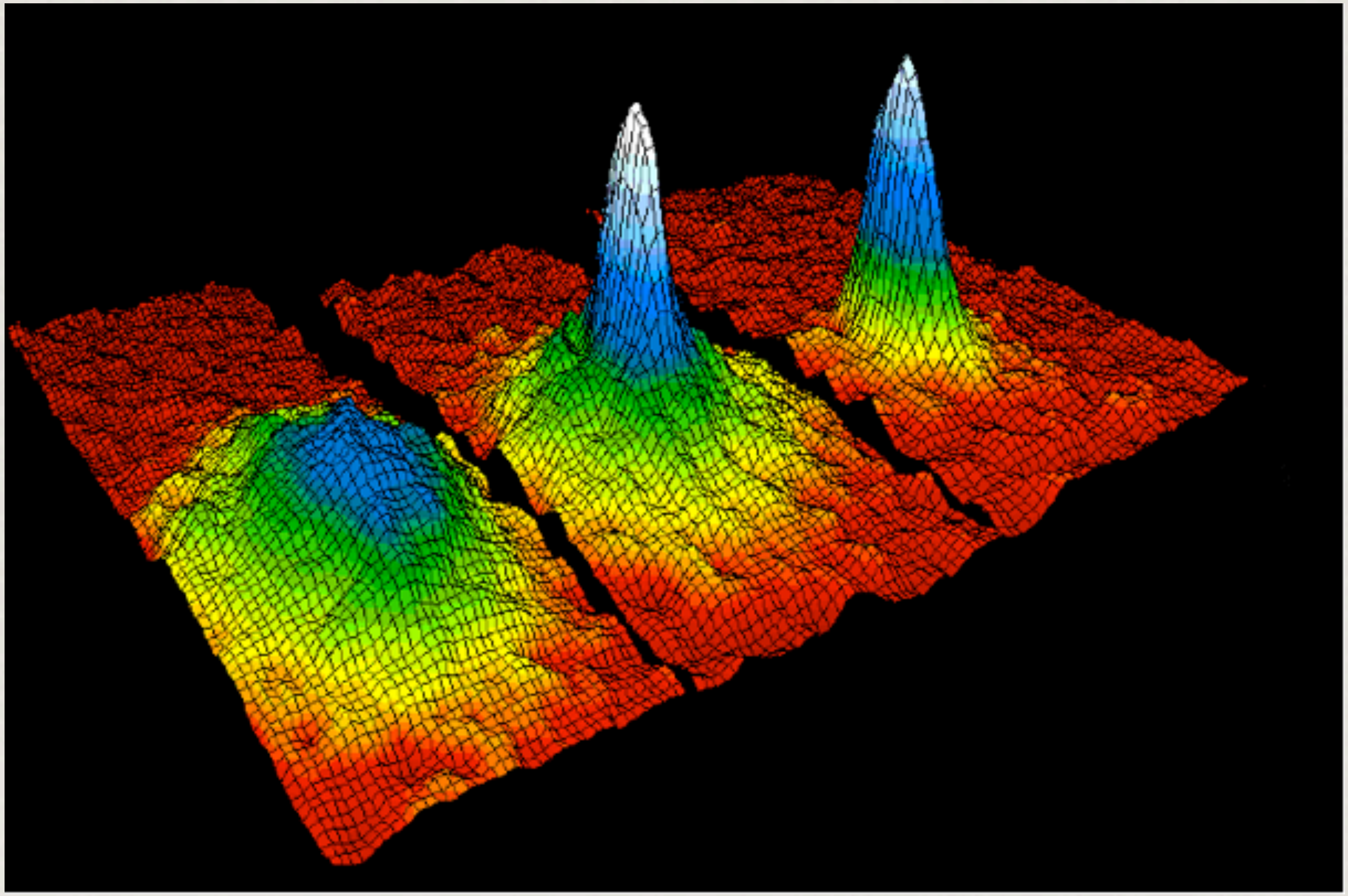


3. Quantum opto-mechanical networking

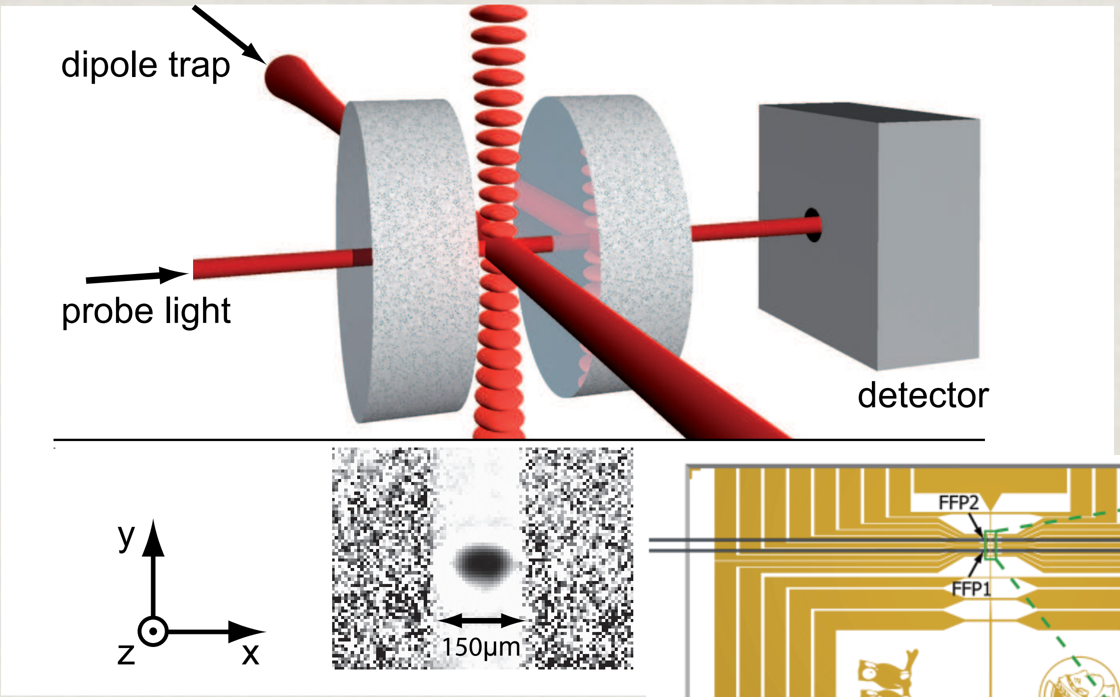


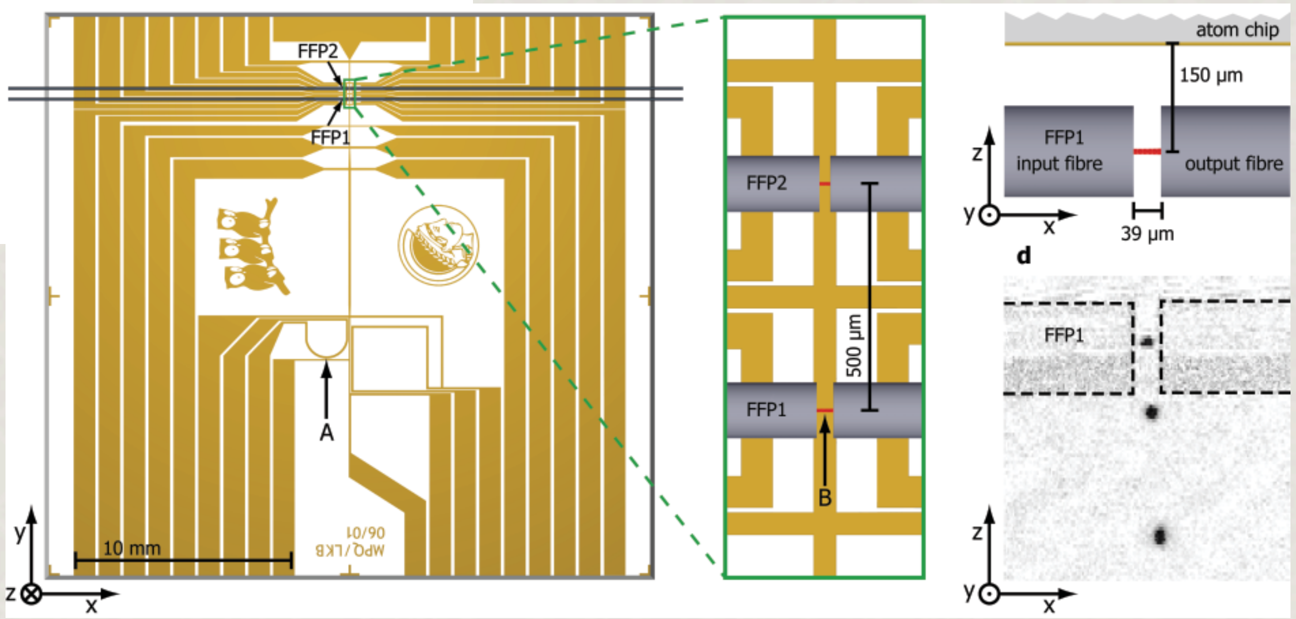
QTEQ QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Cold atoms for nonclassicality



BEC in a cavity

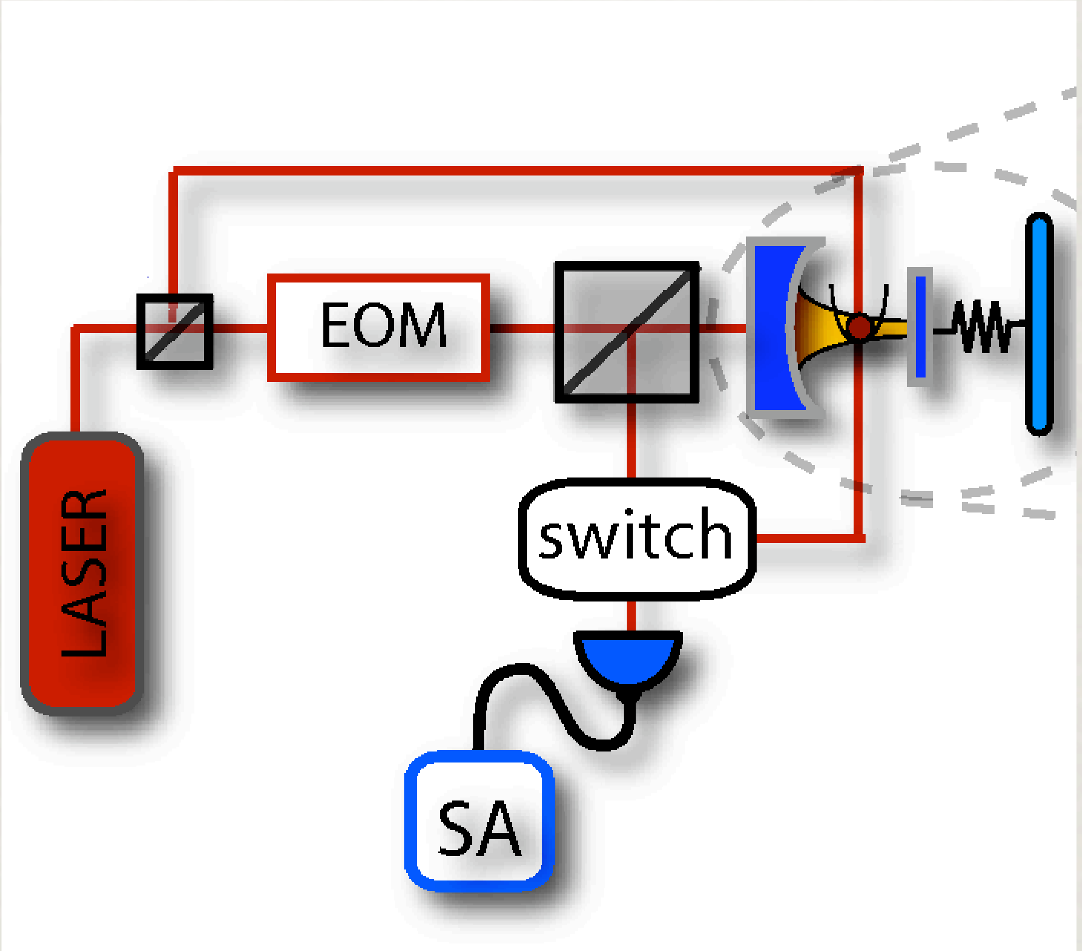




F. Brennecke et al., Nature 450, 268 (2007); Y. Colombe et al., Nature 450, 272 (2007)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

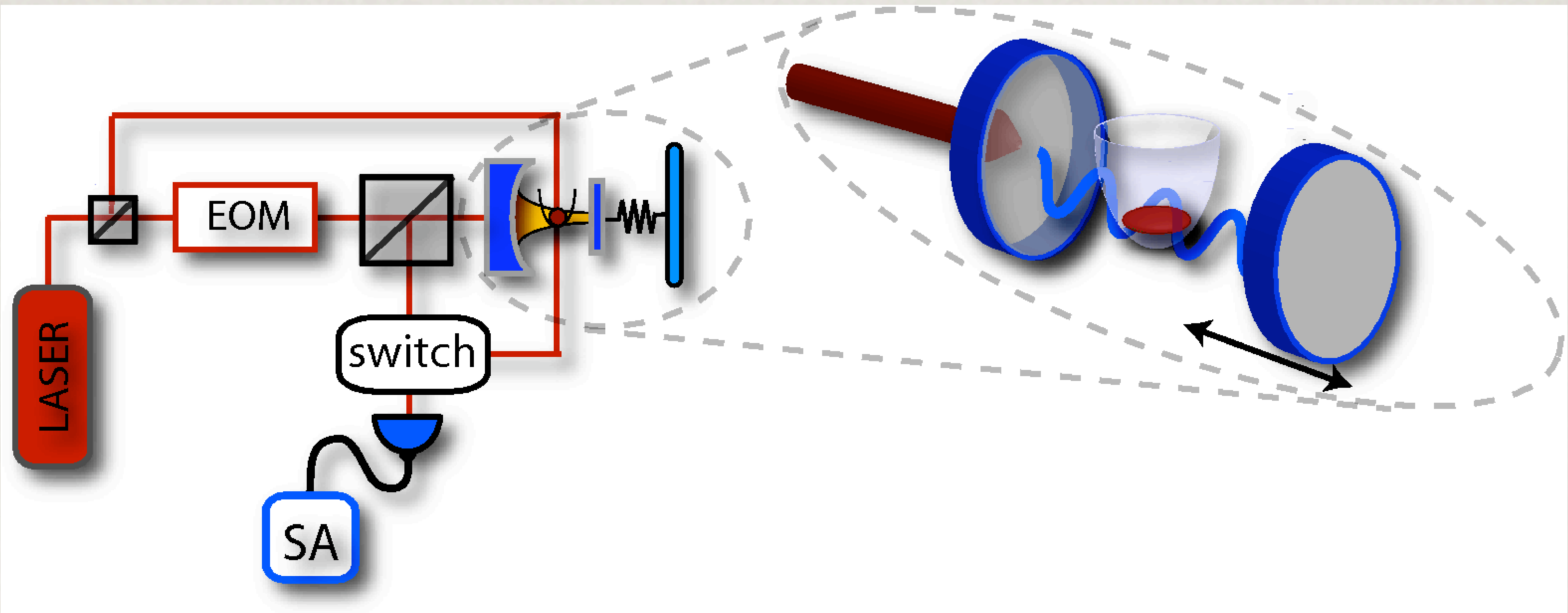
Hybridizing



M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

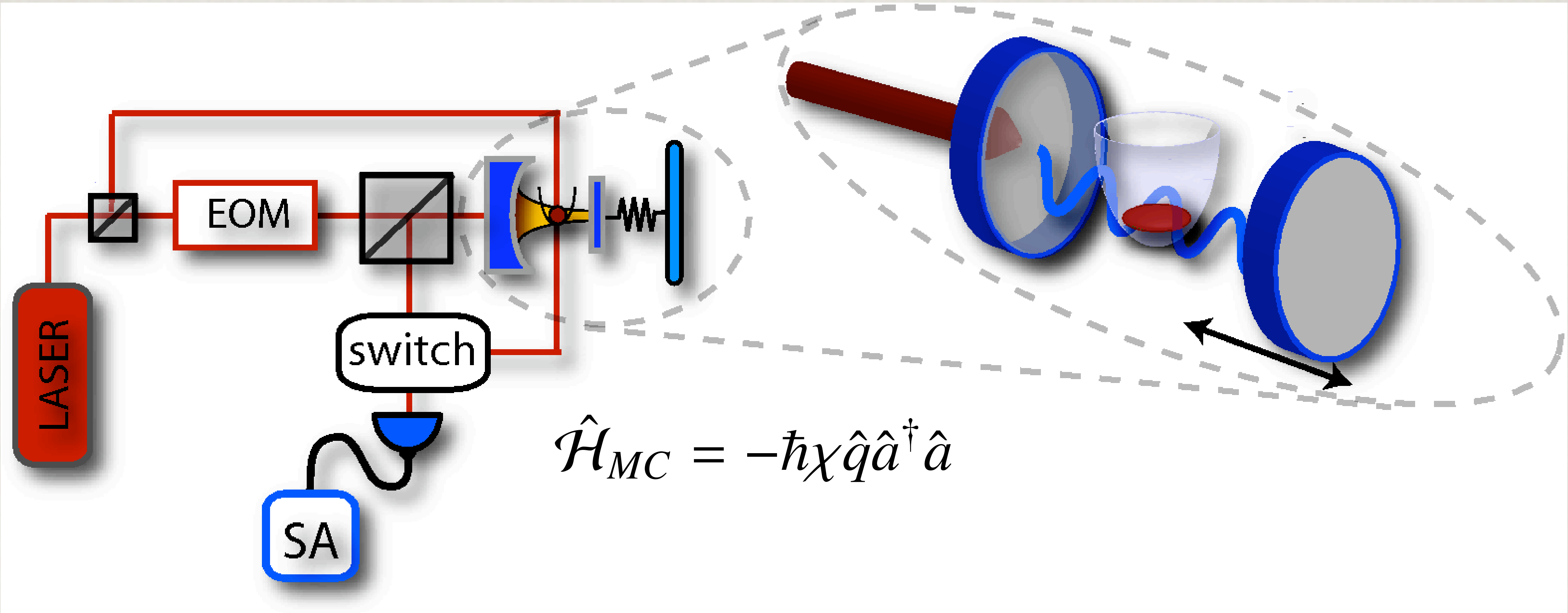
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Hybridizing



M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

Hybridizing

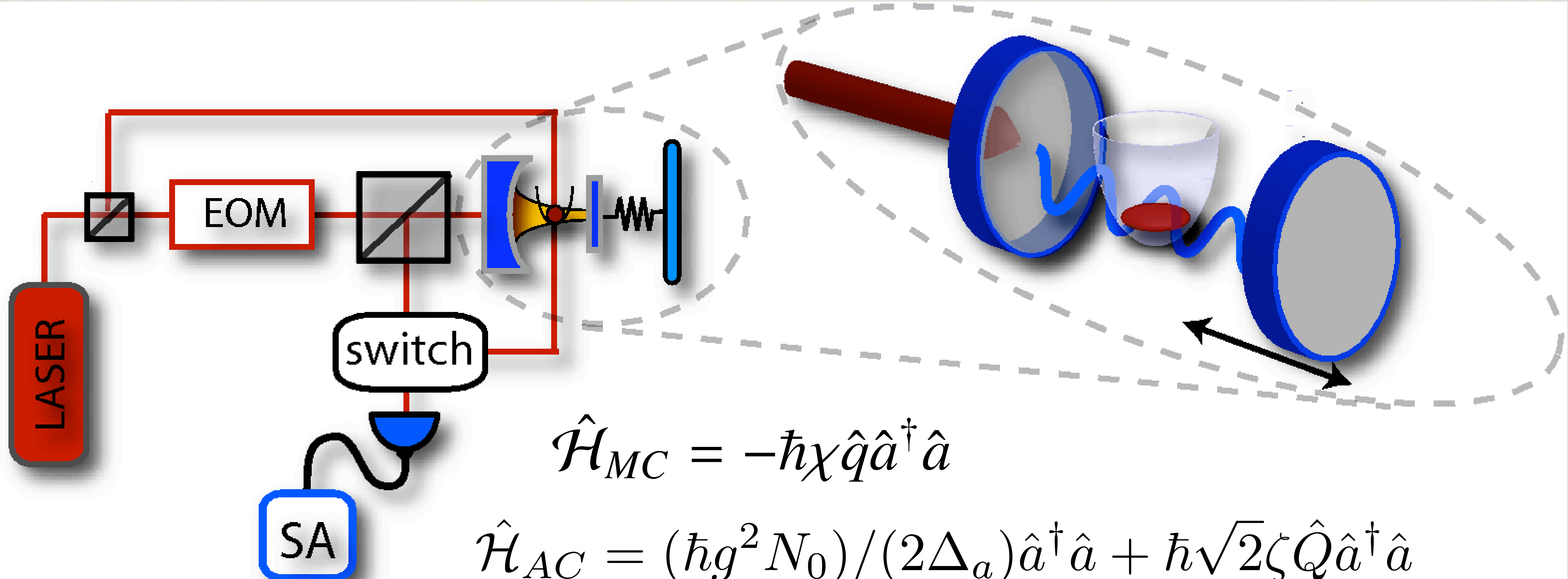


$$\hat{\mathcal{H}}_{MC} = -\hbar\chi\hat{q}\hat{a}^{\dagger}\hat{a}$$

M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

Hybridizing

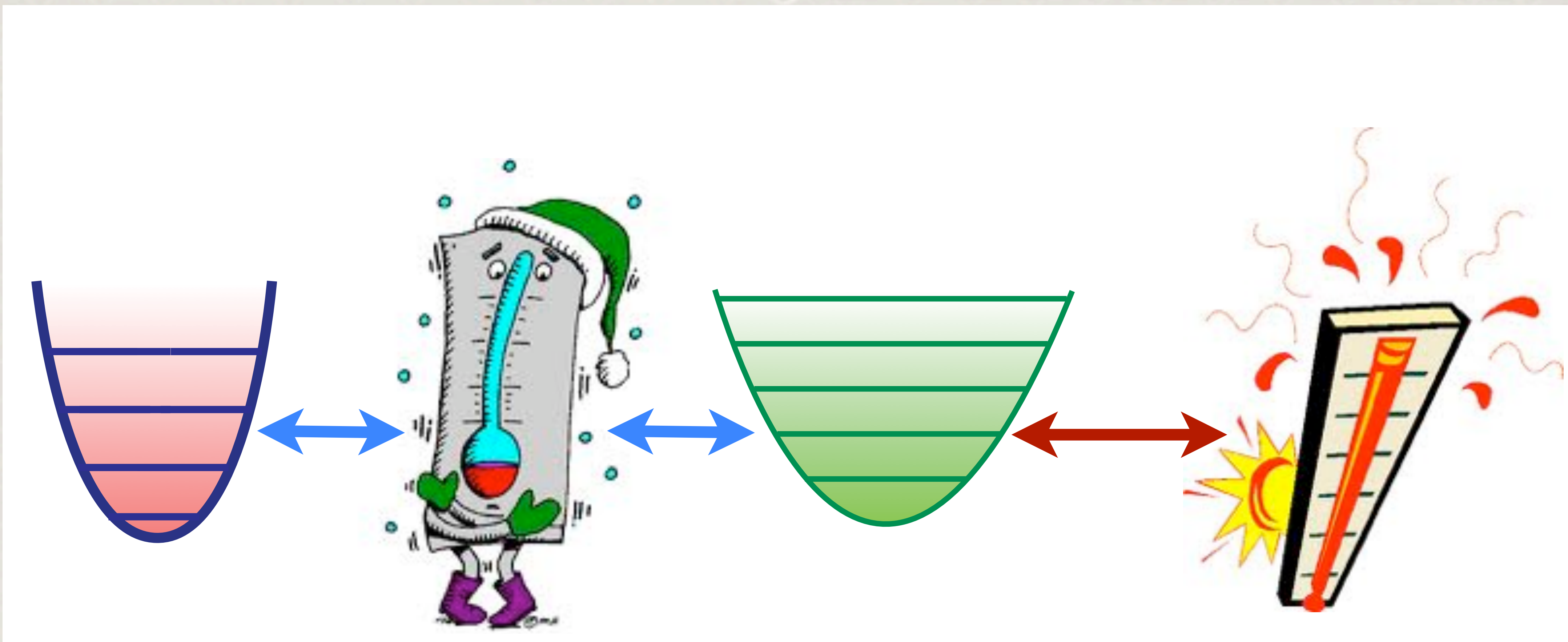


$$\hat{\mathcal{H}}_{MC} = -\hbar\chi\hat{q}\hat{a}^{\dagger}\hat{a}$$

$$\hat{\mathcal{H}}_{AC} = (\hbar g^2 N_0)/(2\Delta_a)\hat{a}^{\dagger}\hat{a} + \hbar\sqrt{2}\zeta\hat{Q}\hat{a}^{\dagger}\hat{a}$$

M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

Hybridizing



Mimicked Opto-mechanical coupling

Opto-mechanical coupling

Brownian motion at temperature T

M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

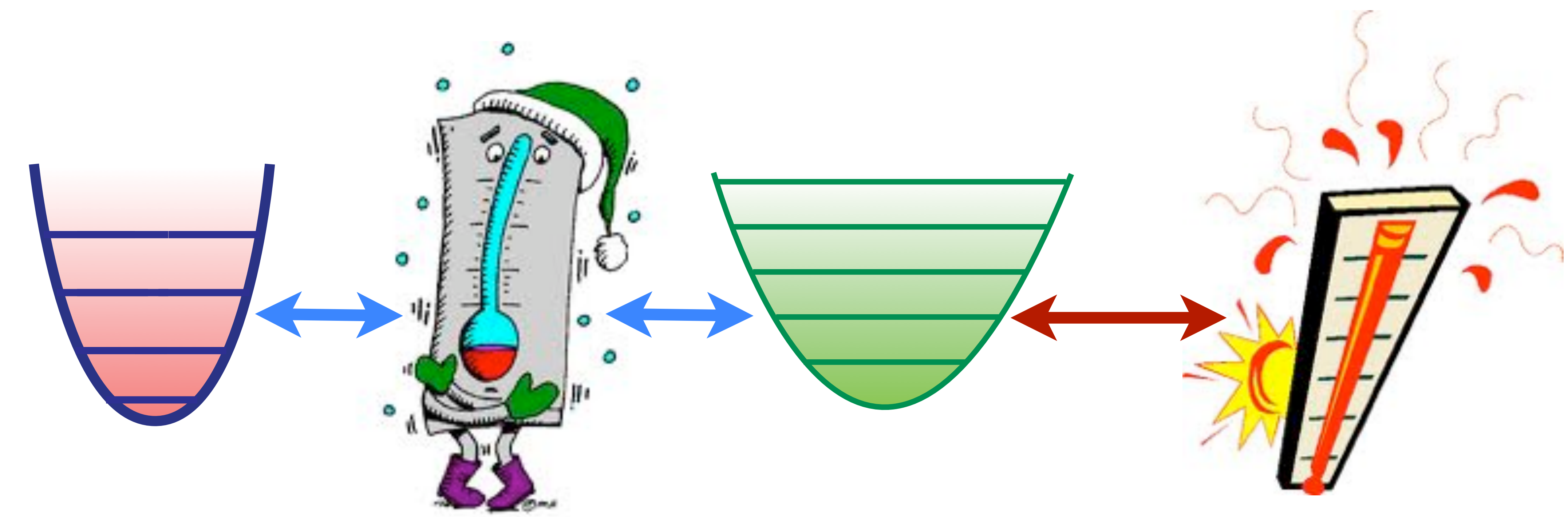
QTEQ

QUANTUM TECHNOLOGY at QUEEN'S

www.qteq.info

Hybridizing

$$S_{\mathcal{O}}(\omega)=(1/4\pi)\int d\Omega\, e^{-i(\omega+\Omega)t}\langle\hat{\mathcal{O}}(\omega)\hat{\mathcal{O}}(\Omega)+\hat{\mathcal{O}}(\Omega)\hat{\mathcal{O}}(\omega)\rangle$$



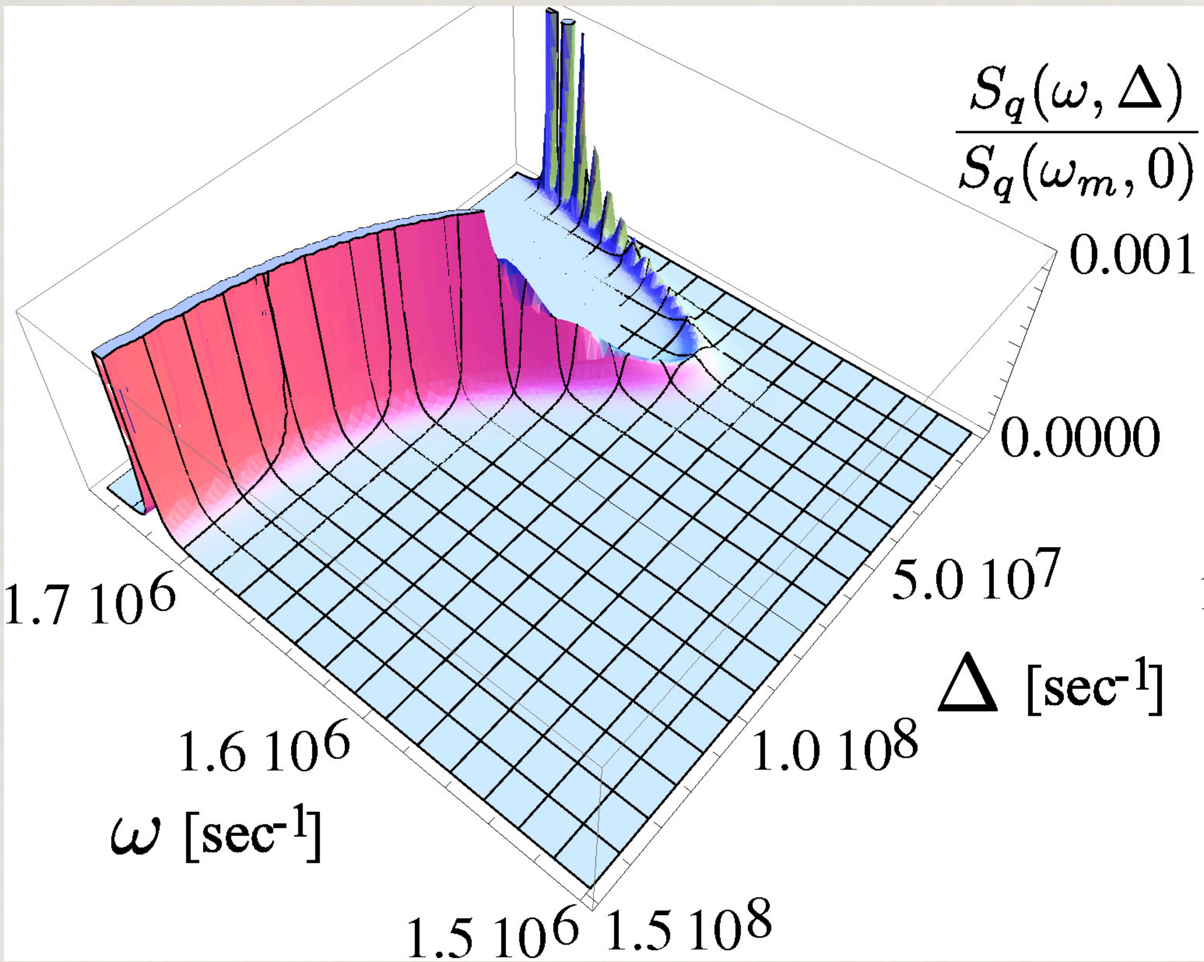
Mimicked Opto-mechanical coupling

Opto-mechanical coupling

Brownian motion at temperature T

M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

Mechanical DNS

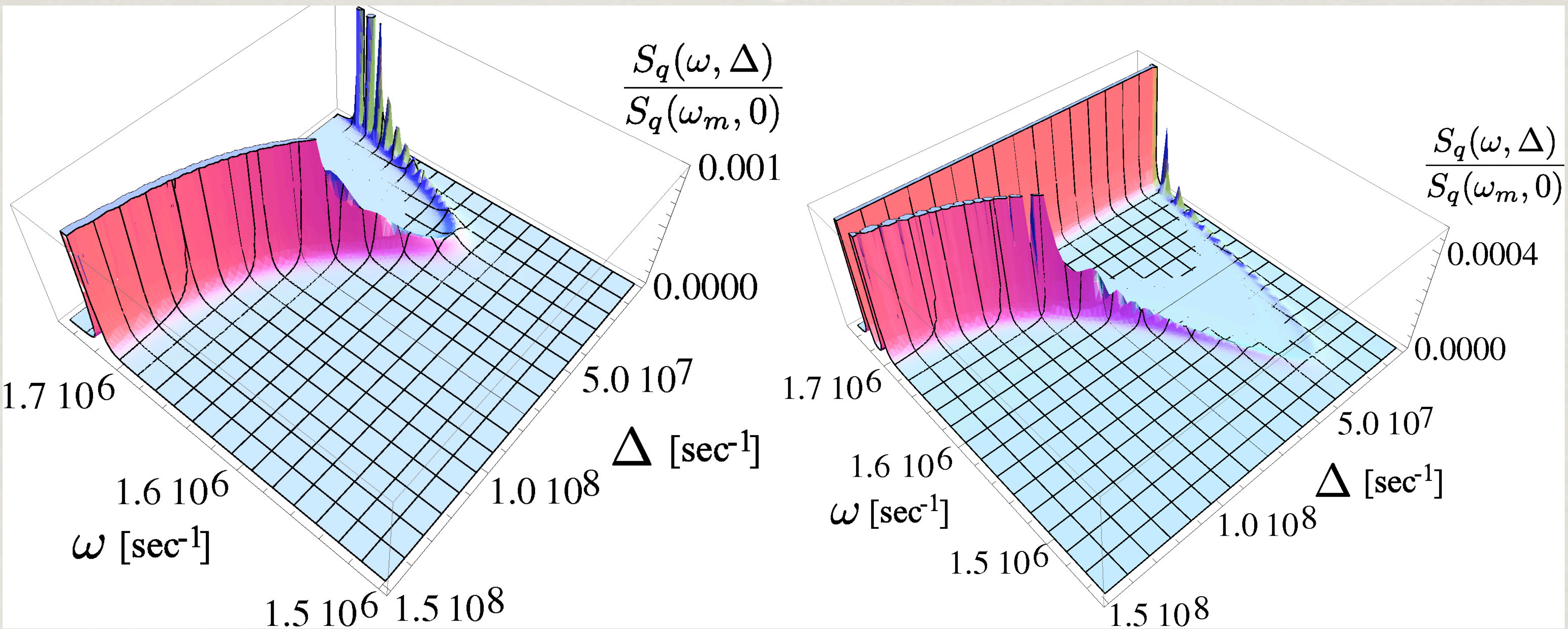


$\zeta=0$

M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Mechanical DNS



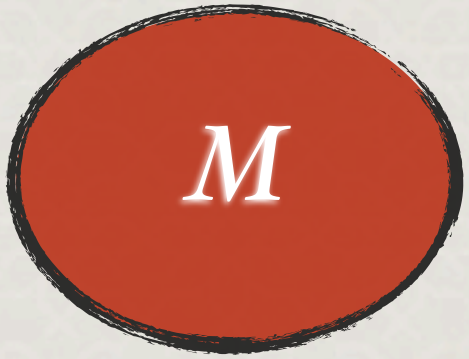
$\zeta=0$ $\qquad$ $\zeta\simeq0.7\chi\sqrt{\hbar/(m\omega_m)}$

M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

An intuitive model

M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

An intuitive model

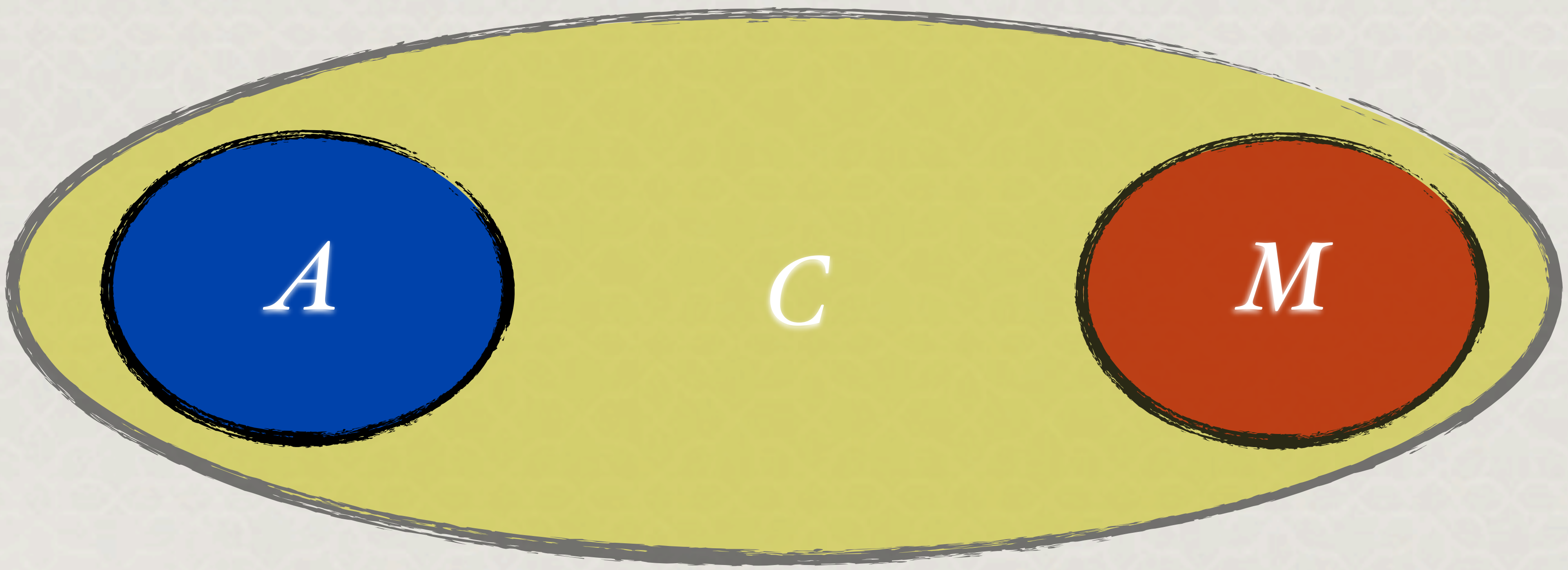


M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

An intuitive model

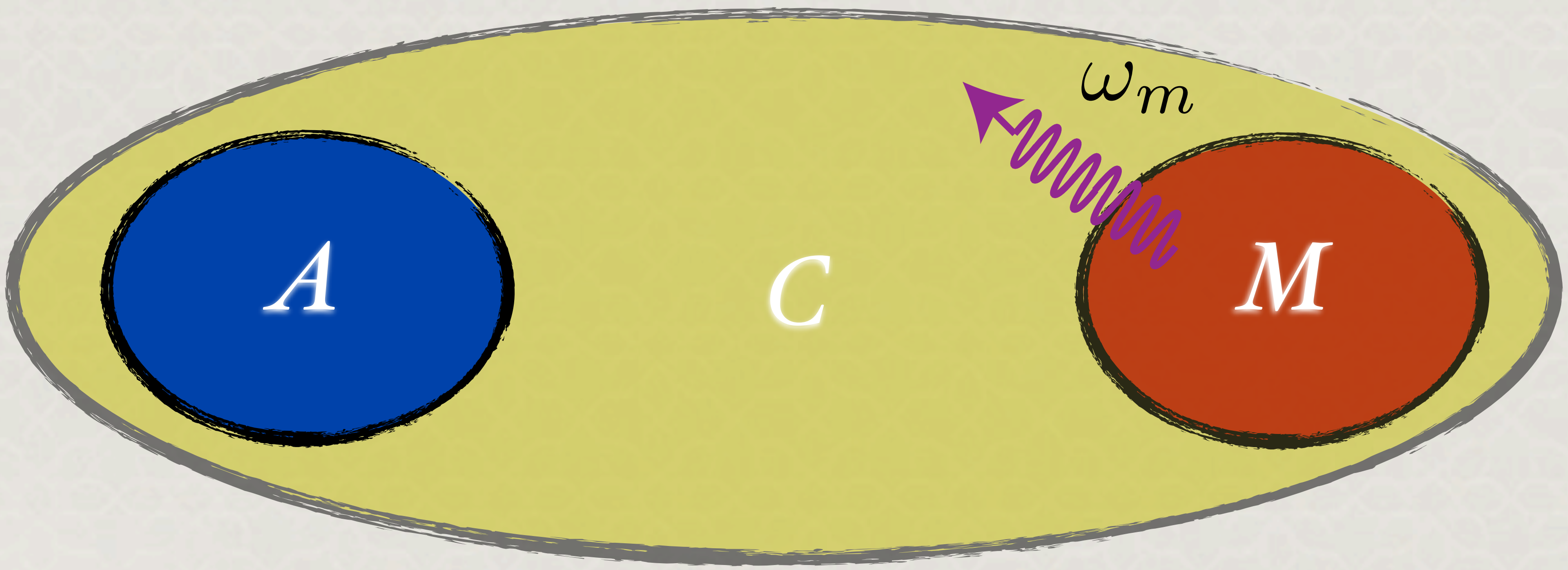
M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

An intuitive model



M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

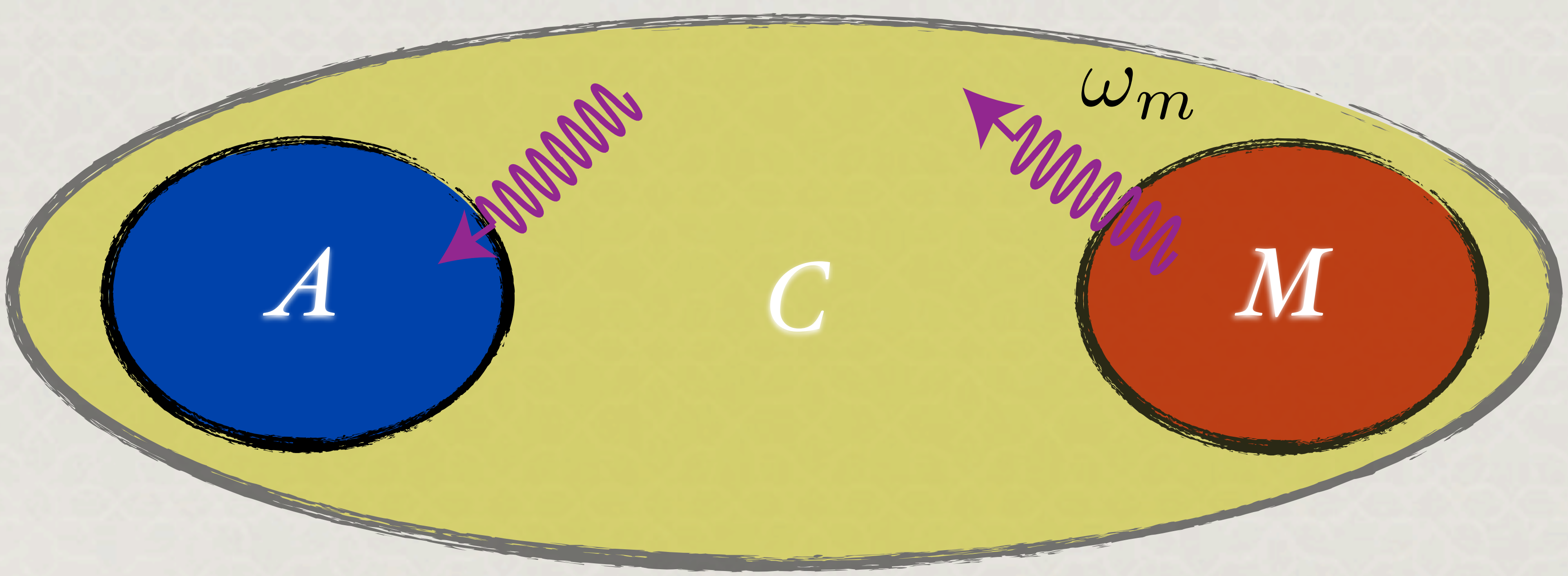
An intuitive model



M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

Wednesday, 8 June 2011

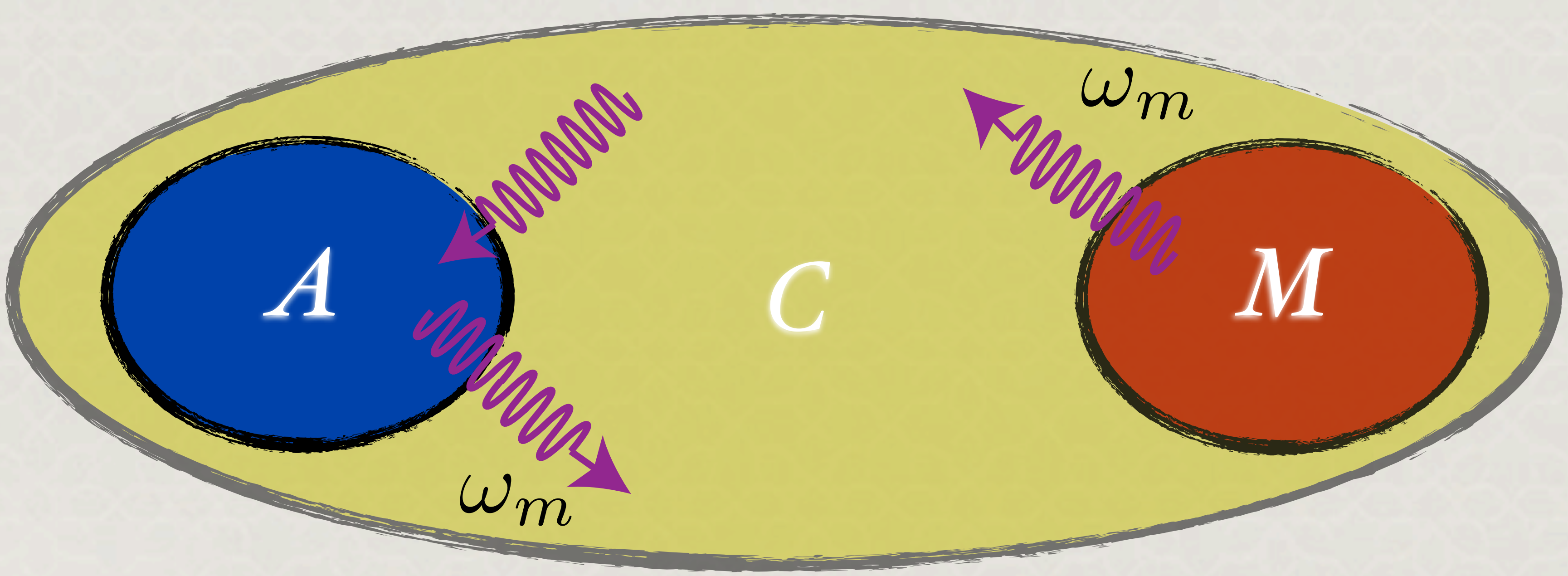
An intuitive model



M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

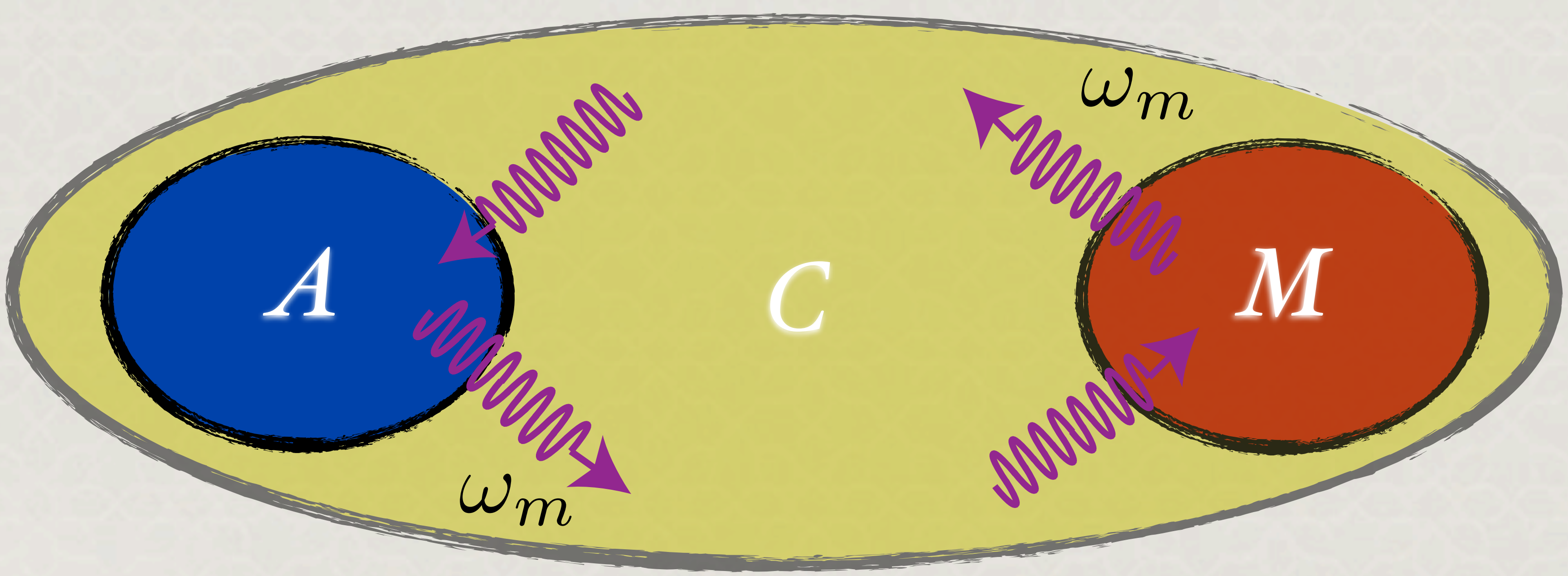
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

An intuitive model



M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

An intuitive model



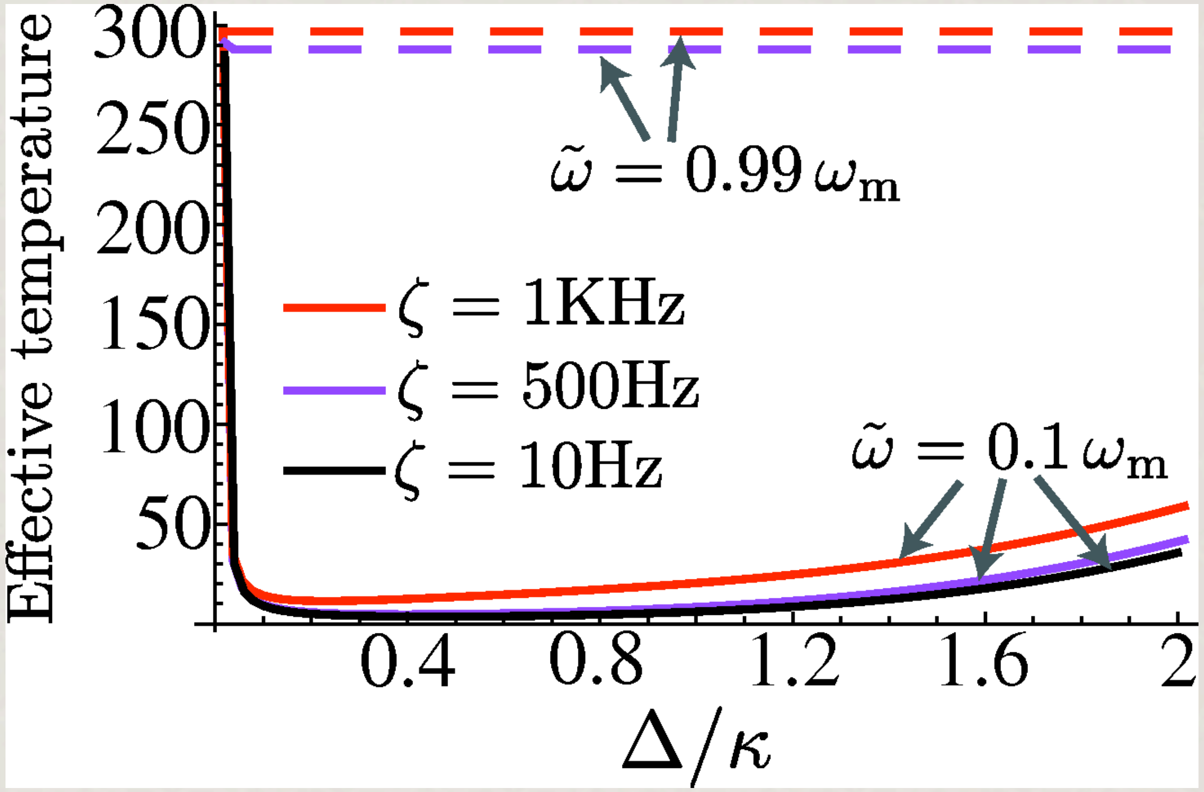
M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

QTEQ QUANTUM TECHNOLOGY at QUEEN´S www.qteq.info

Cold-atom-induced heating effect

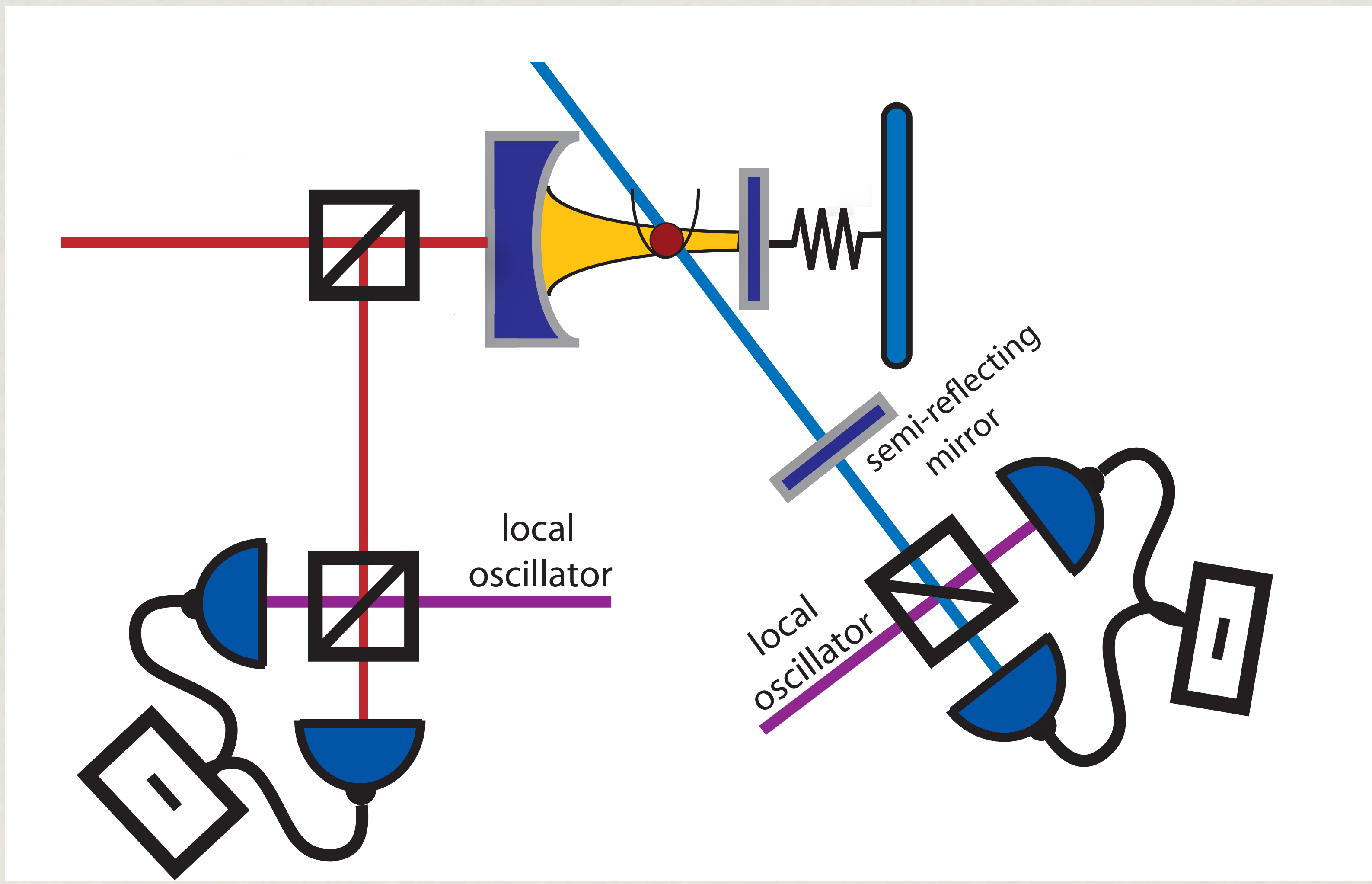
$$T_{\mathrm{eff}}=\langle U\rangle/k_B$$

$$\langle U\rangle=m\omega_m^2\langle\delta\hat{q}^2\rangle/2+\langle\delta\hat{p}^2\rangle/(2m)$$



M. Paternostro, G. De Chiara, and G. M. Palma, Phys. Rev. Lett. 104, 243602 (2010)

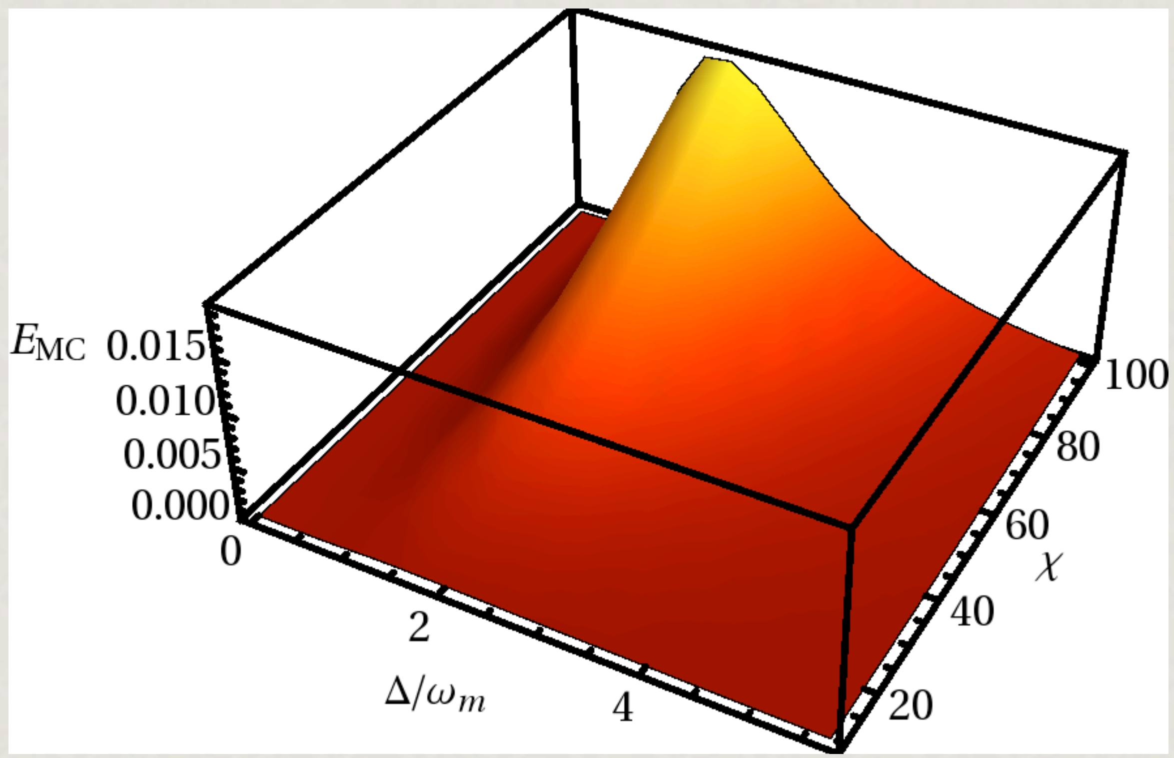
Entanglement?



G. De Chiara, M. Paternostro, and G. M. Palma, Phys. Rev. A. (to appear, 2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

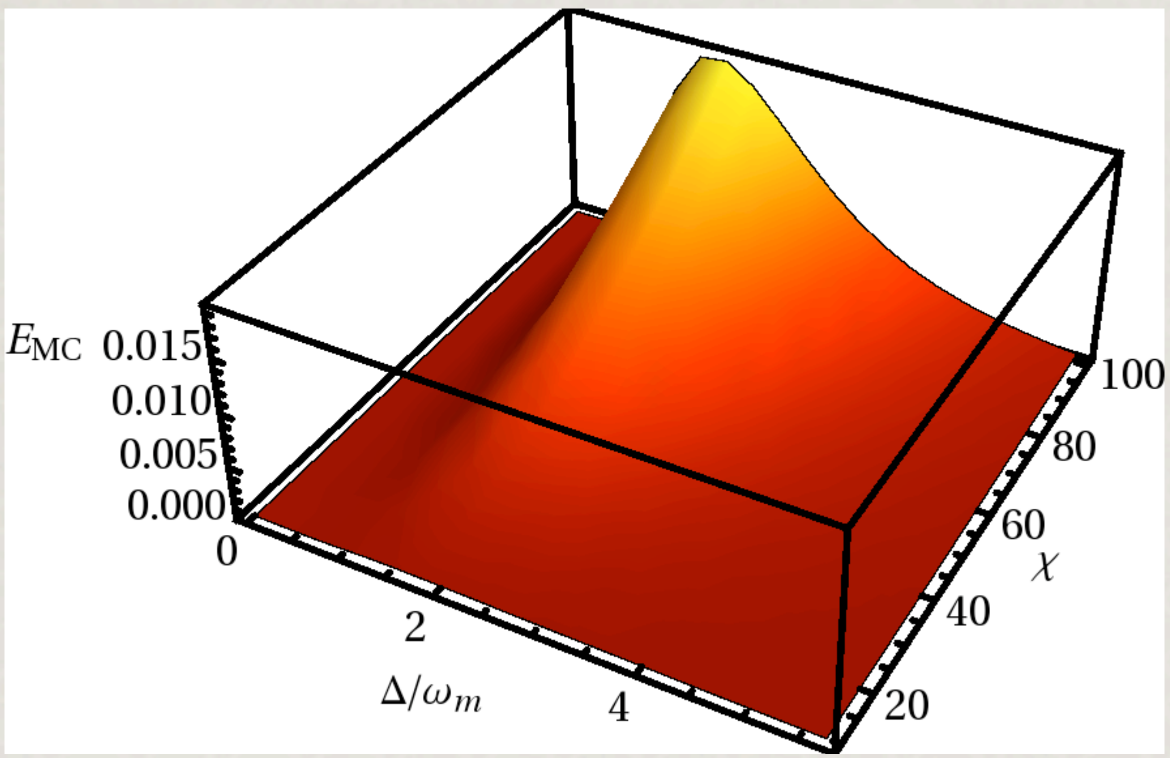
Diagnostics?



Equally-coupled subsystems

G. De Chiara, M. Paternostro, and G. M. Palma, Phys. Rev. A. (to appear, 2011)

Diagnostics?

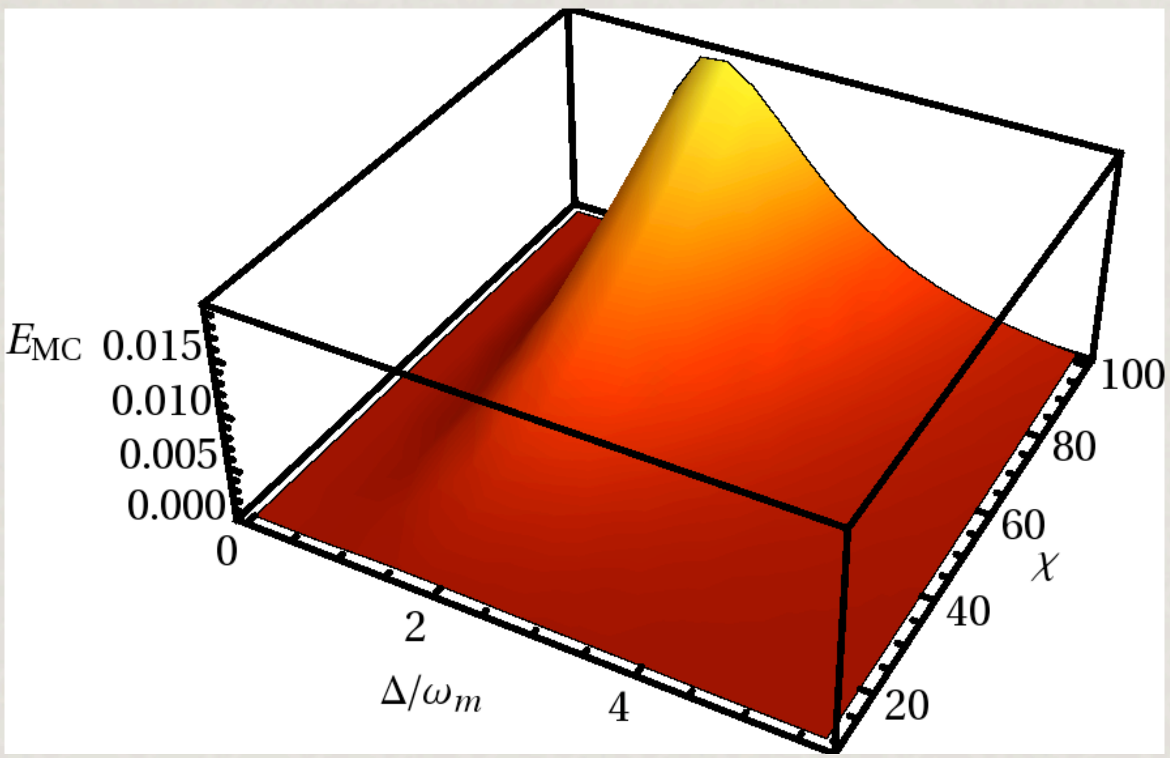


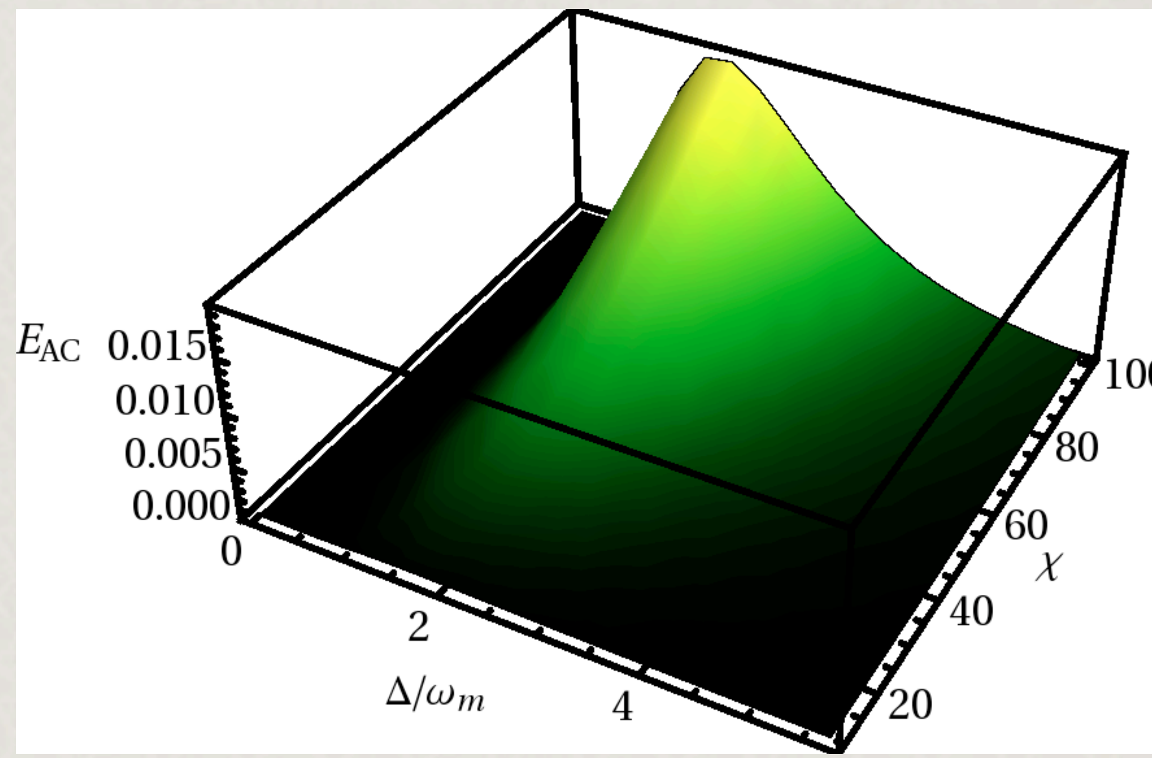
Equally-coupled subsystems

G. De Chiara, M. Paternostro, and G. M. Palma, Phys. Rev. A. (to appear, 2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
w w w . q t e q . i n f o

Diagnostics?



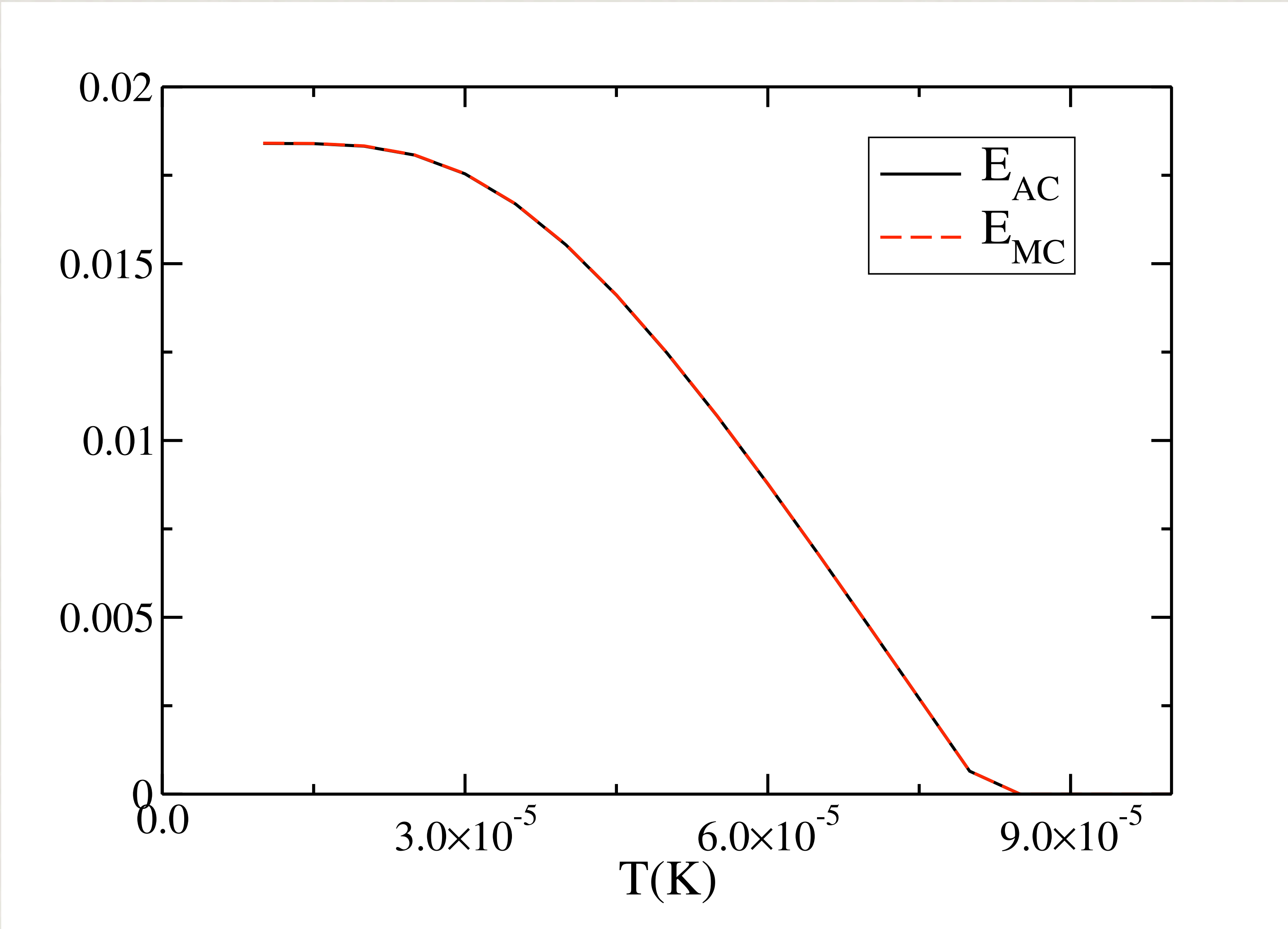


Equally-coupled subsystems

G. De Chiara, M. Paternostro, and G. M. Palma, Phys. Rev. A. (to appear, 2011)

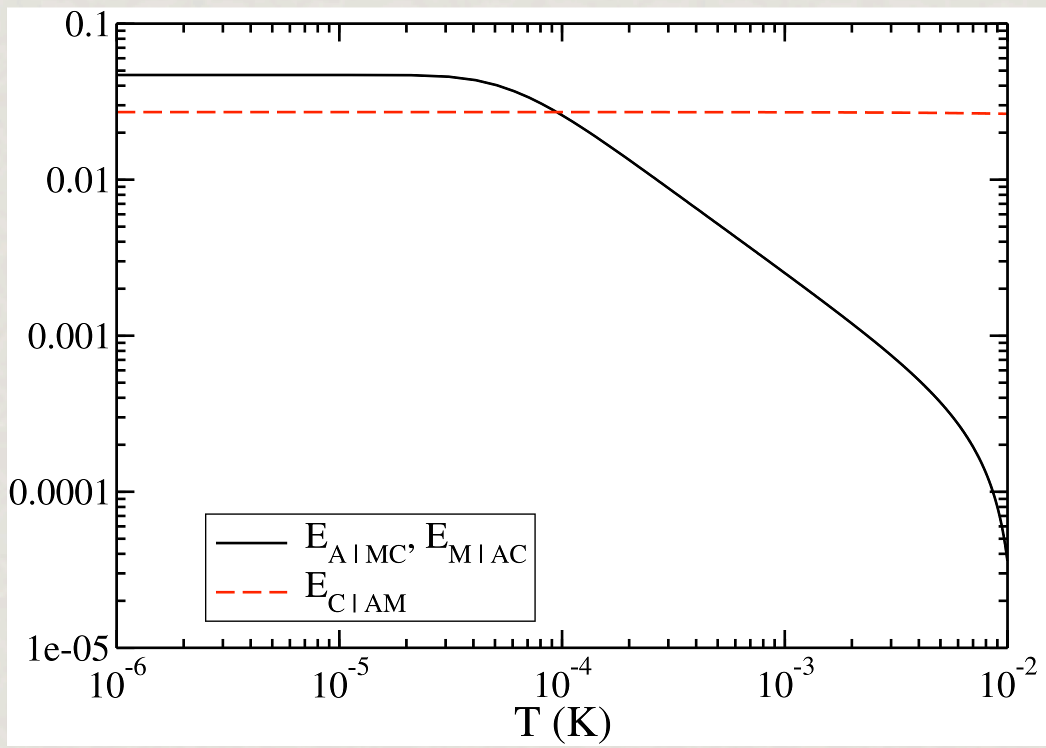
Somebody likes it cold...

QTEQ
QUANTUM TECHNOLOGY at QUEEN´S
www.qteq.info



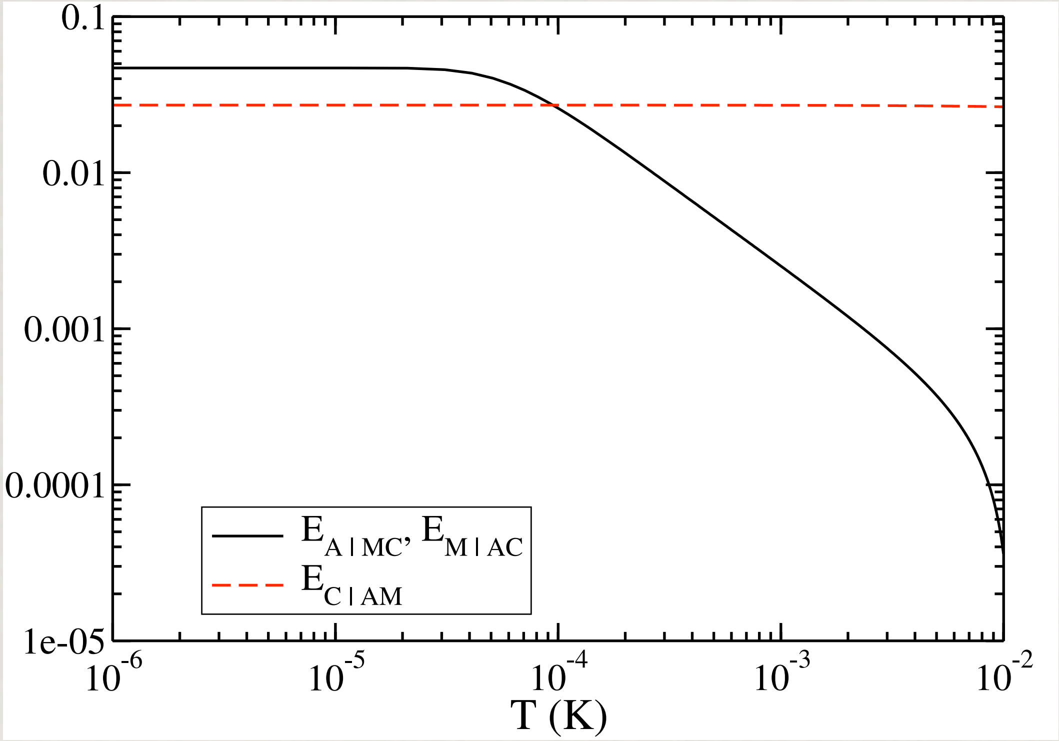
G. De Chiara, M. Paternostro, and G. M. Palma, Phys. Rev. A. (to appear, 2011)

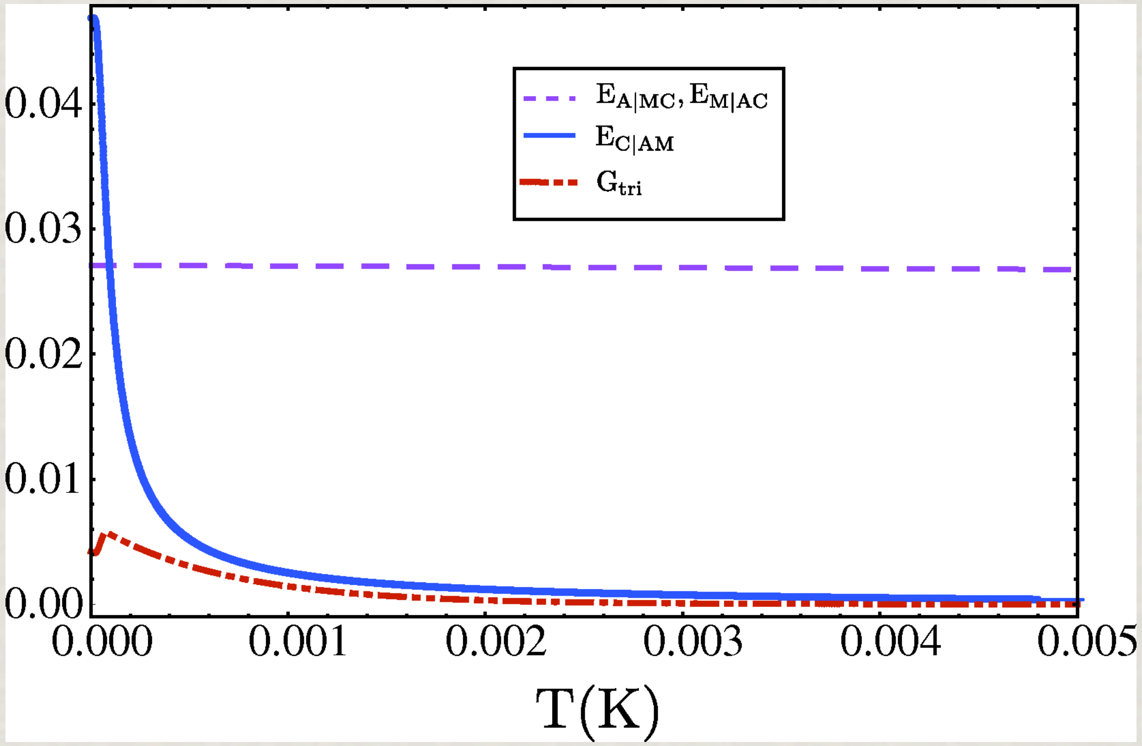
Tripartite entanglement



G. De Chiara, M. Paternostro, and G. M. Palma, Phys. Rev. A. (to appear, 2011)

Tripartite entanglement



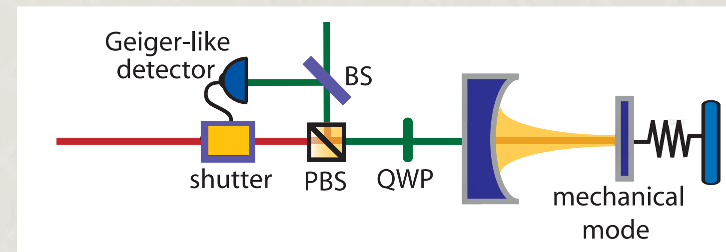


G. De Chiara, M. Paternostro, and G. M. Palma, Phys. Rev. A. (to appear, 2011)

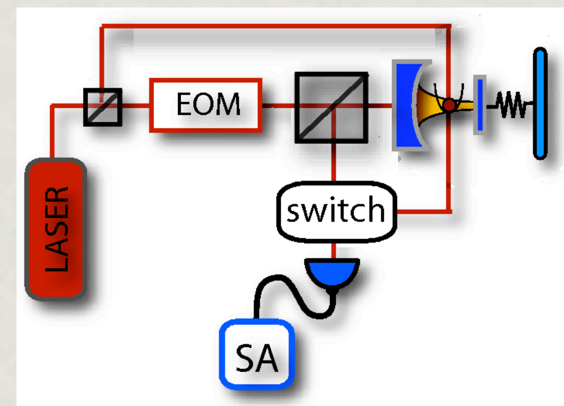
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Dessert

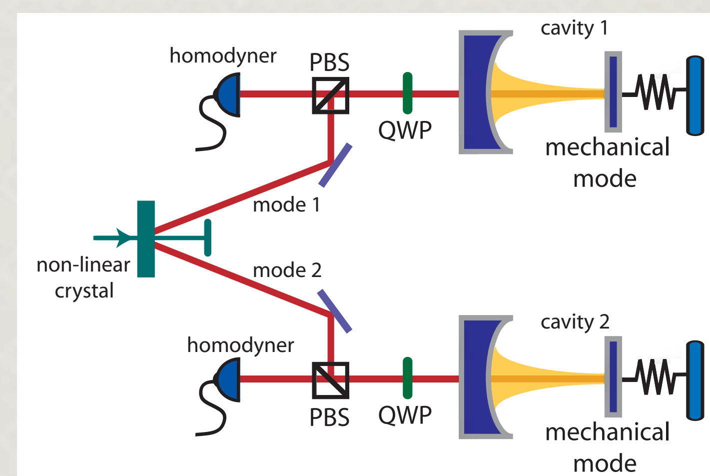
1. Non-classicality by photon-subtraction



2. Entanglement and diagnostics through cold-atom induced dynamics



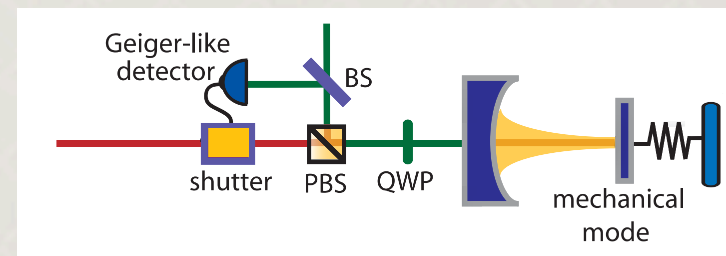
3. Quantum opto-mechanical networking



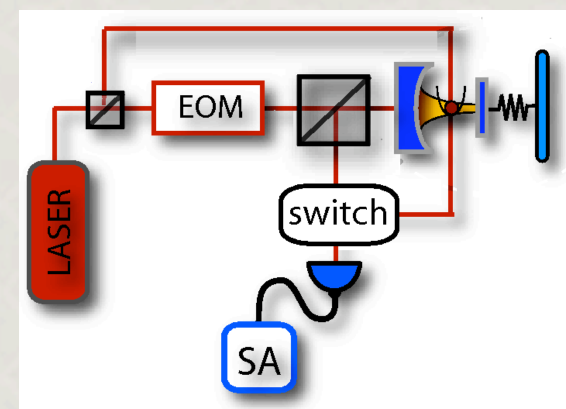
QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Dessert

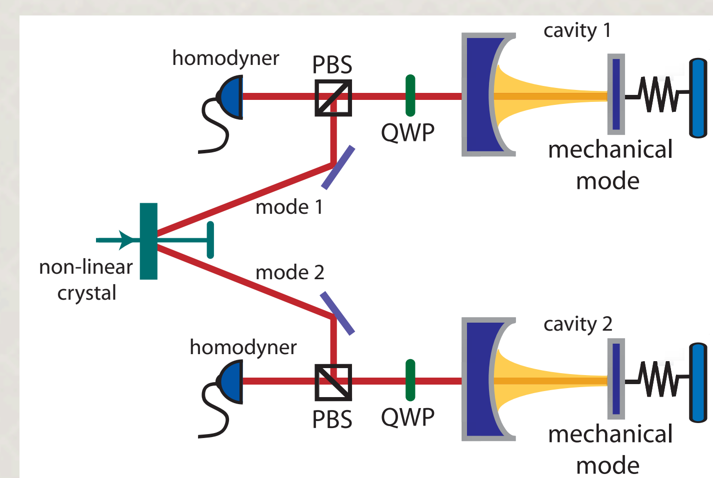
1. Non-classicality by photon-subtraction



2. Entanglement and diagnostics through cold-atom induced dynamics

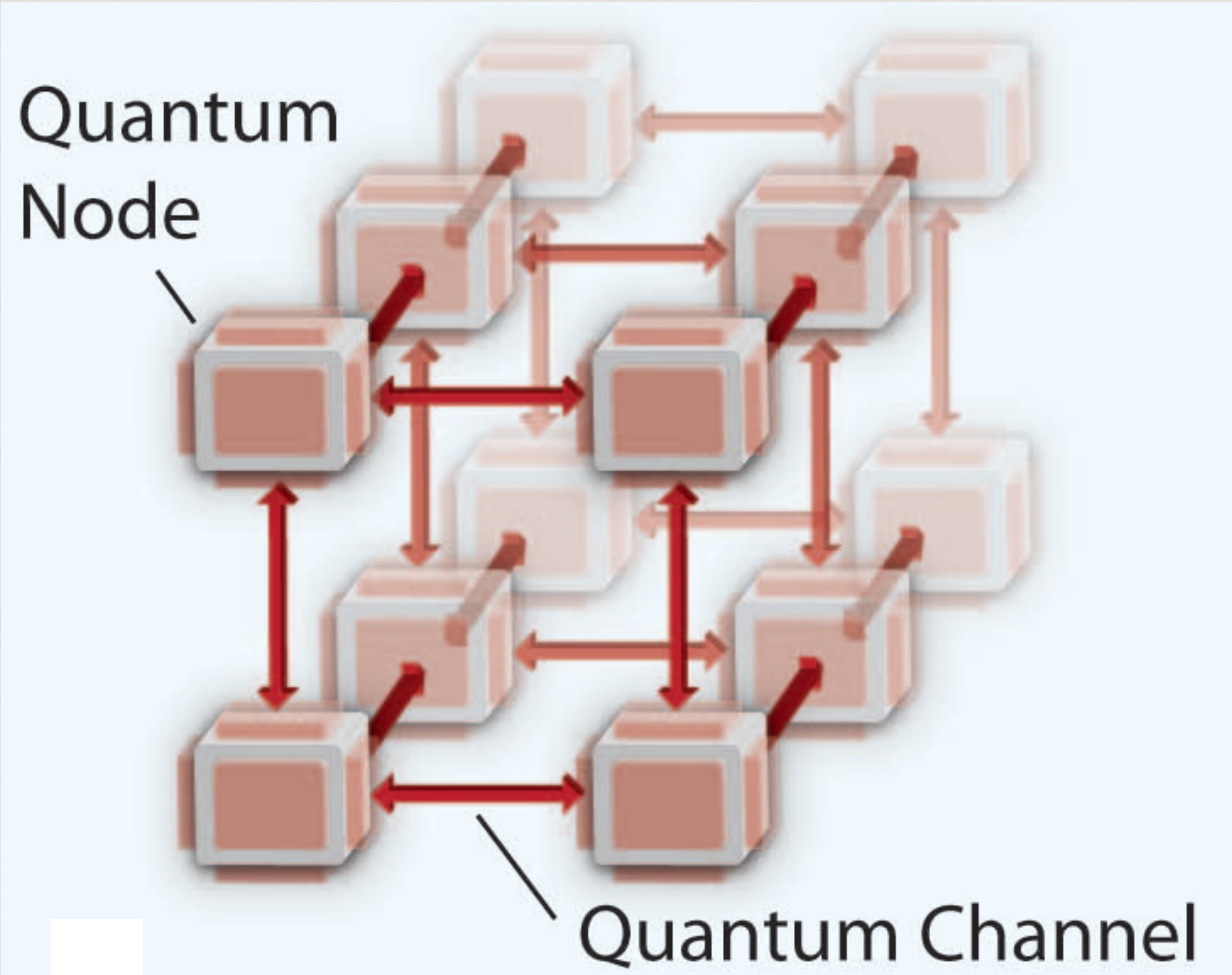


3. Quantum opto-mechanical networking



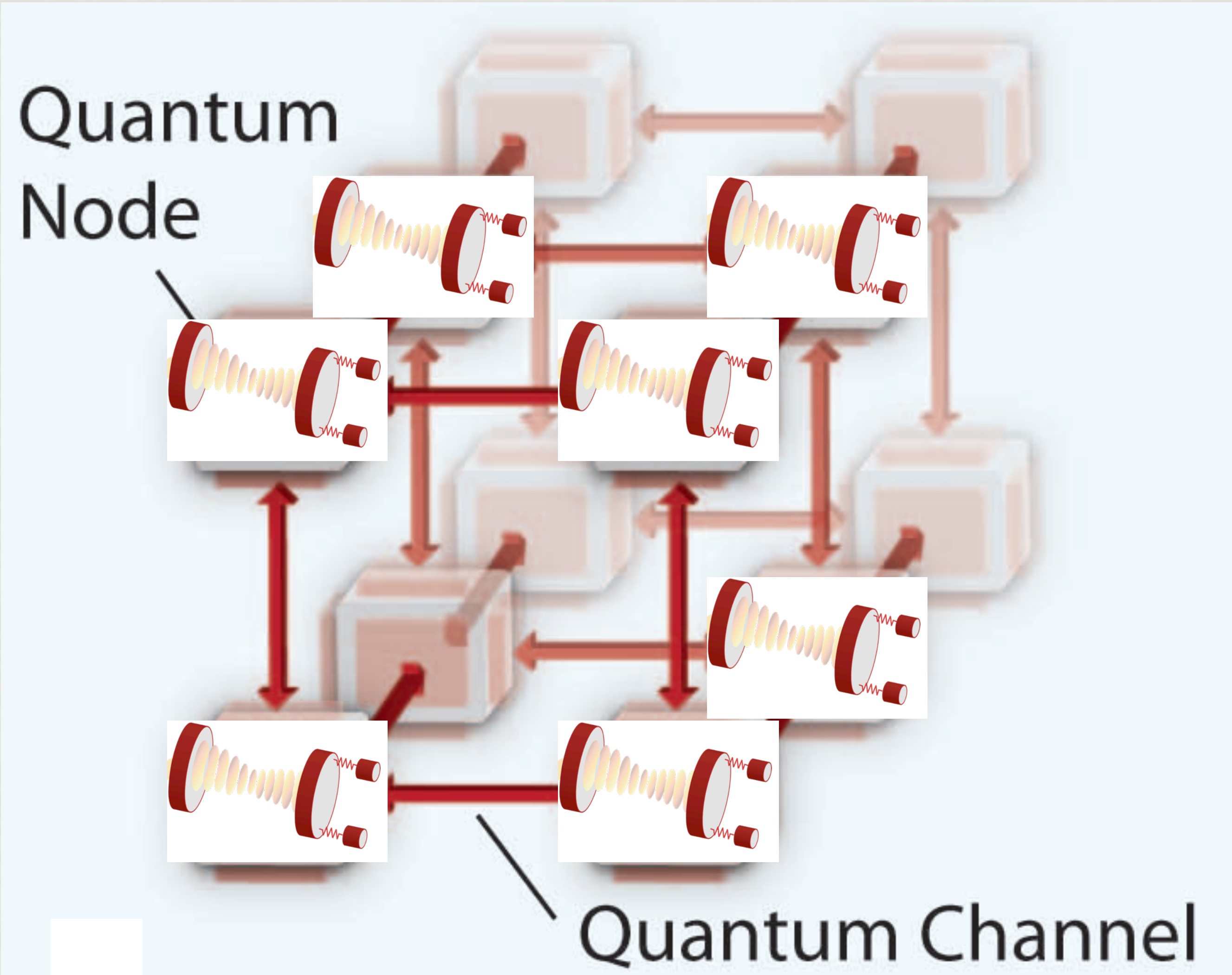
QTEQ
QUANTUM TECHNOLOGY at QUEEN´S
www.qteq.info

Distribute and conquer



H. J. Kimble, Nature 453, 1023 (2008)

Distribute and conquer



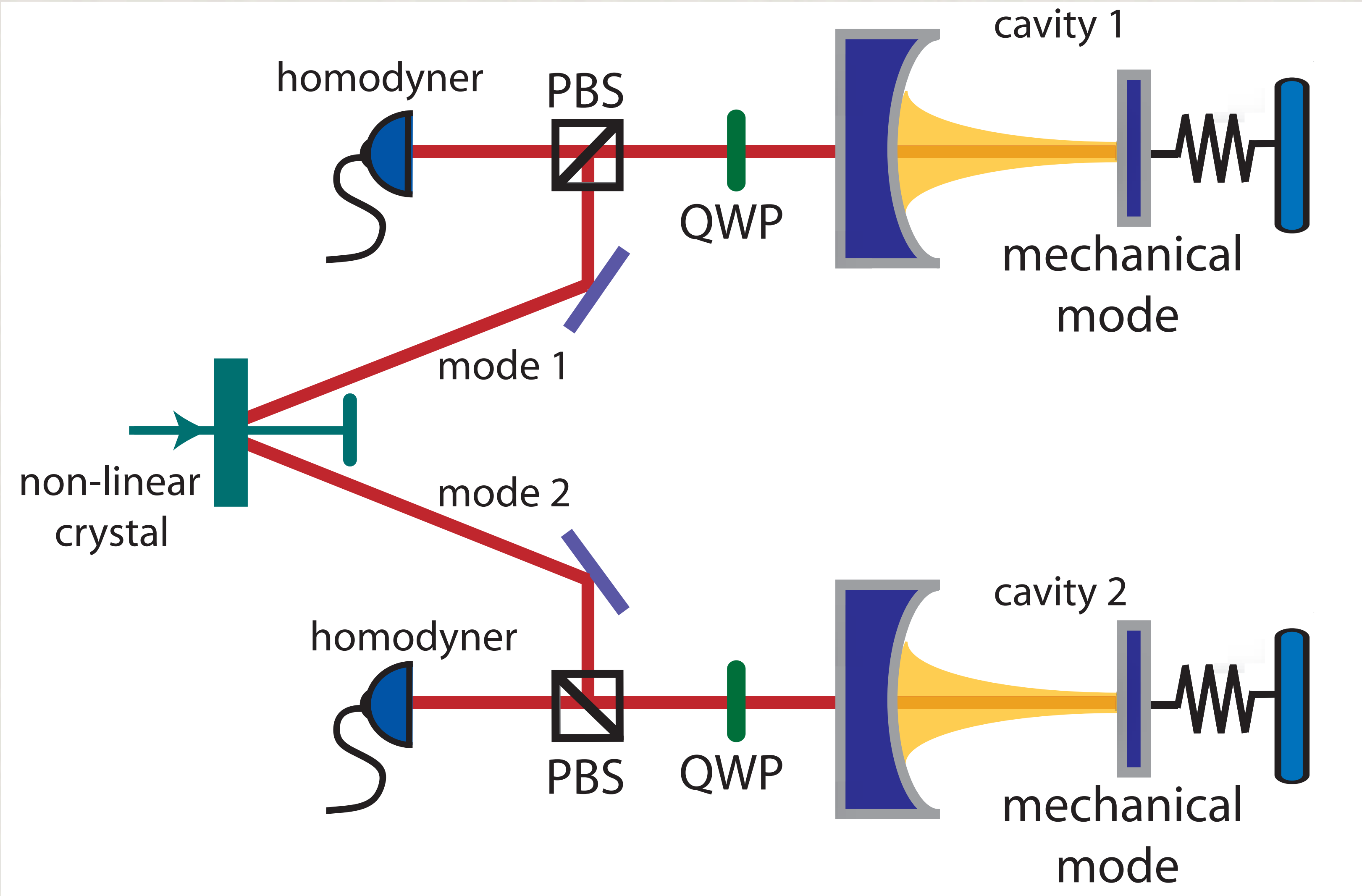
H. J. Kimble, Nature 453, 1023 (2008)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Quantum opto-networking

L. Mazzola, and M. Paternostro, arXiv:1104.0897 (2011)

Quantum opto-networking

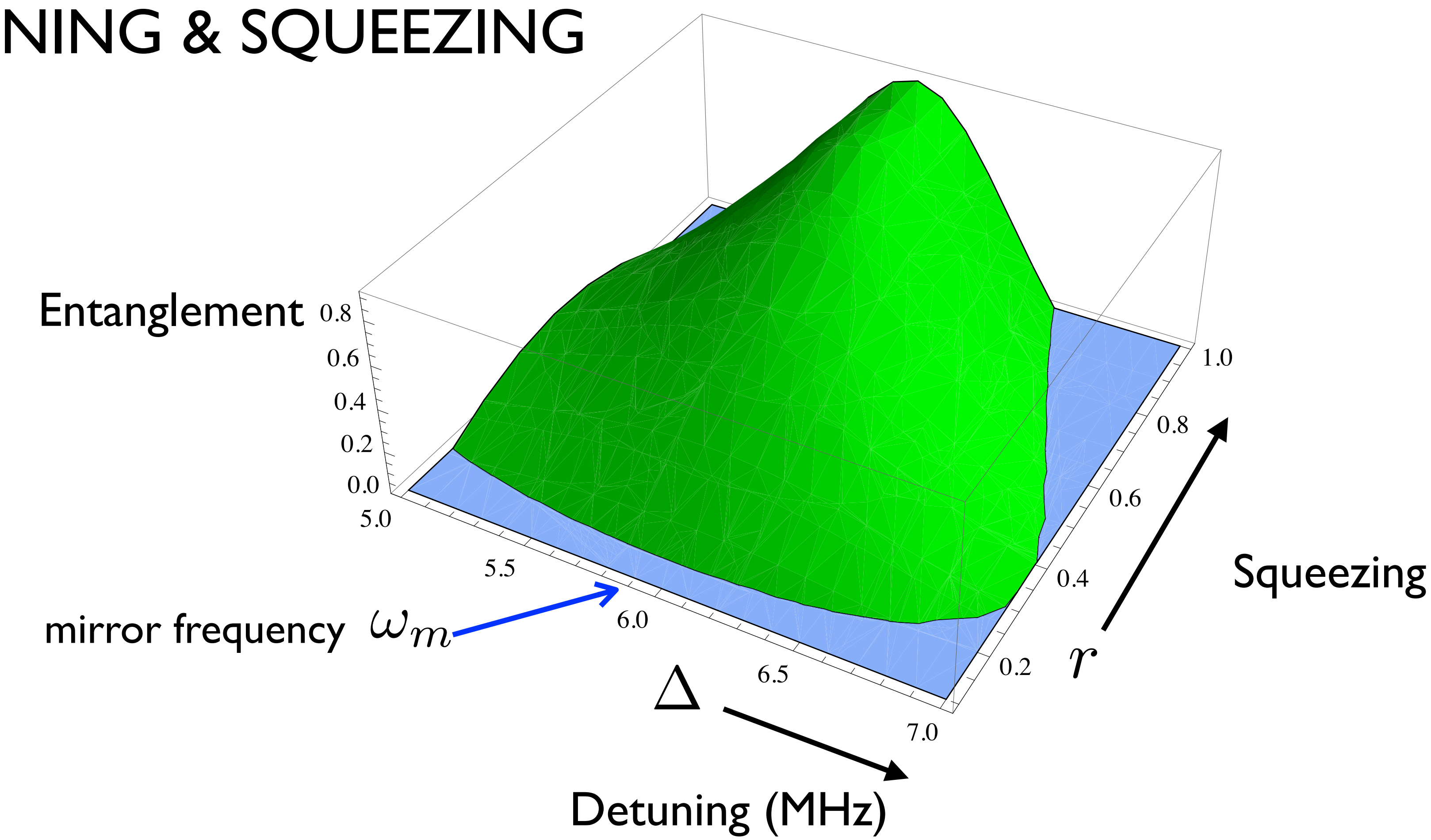


L. Mazzola, and M. Paternostro, arXiv:1104.0897 (2011)

Pouring entanglement..

QTEQ QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

ENTANGLEMENT vs DETUNING & SQUEEZING



L. Mazzola, and M. Paternostro, arXiv:1104.0897 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Discord in CV systems

$$\boldsymbol{\sigma} = \begin{bmatrix} \boldsymbol{\alpha}_1 & \gamma \\ \gamma^T & \boldsymbol{\alpha}_2 \end{bmatrix}$$

P. Giorda, and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010);
G. Adesso, and A. Datta, Phys. Rev. Lett. 105, 030501 (2010)

QTEQ QUANTUM TECHNOLOGY at QUEEN'S www.qteq.info

Discord in CV systems

$$\boldsymbol{\sigma} = \begin{bmatrix} \boldsymbol{\alpha}_1 & \gamma \\ \gamma^T & \boldsymbol{\alpha}_2 \end{bmatrix} \qquad A_2 = \det \boldsymbol{\alpha}_2$$

P. Giorda, and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010);
G. Adesso, and A. Datta, Phys. Rev. Lett. 105, 030501 (2010)

QTEQ QUANTUM TECHNOLOGY at QUEEN'S www.qteq.info

Discord in CV systems

$$\boldsymbol{\sigma} = \begin{bmatrix} \boldsymbol{\alpha}_1 & \gamma \\ \gamma^T & \boldsymbol{\alpha}_2 \end{bmatrix} \qquad A_2 = \det \boldsymbol{\alpha}_2$$

$$\mathcal{D} = f(\sqrt{A_2}) - f(\mu_-) - f(\mu_+) + \inf_{\boldsymbol{\sigma}_0} f(\sqrt{\det \boldsymbol{\epsilon}})$$

P. Giorda, and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010);
G. Adesso, and A. Datta, Phys. Rev. Lett. 105, 030501 (2010)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Discord in CV systems

$$\boldsymbol{\sigma} = \begin{bmatrix} \boldsymbol{\alpha}_1 & \gamma \\ \gamma^T & \boldsymbol{\alpha}_2 \end{bmatrix} \qquad A_2 = \det \boldsymbol{\alpha}_2$$

$$\mathcal{D} = f(\sqrt{A_2}) - f(\mu_-) - f(\mu_+) + \inf_{\boldsymbol{\sigma}_0} f(\sqrt{\det \boldsymbol{\epsilon}})$$

$$f(x) = (\tfrac{x+1}{2}) \log[\tfrac{x+1}{2}] - (\tfrac{x-1}{2}) \log[\tfrac{x-1}{2}]$$

P. Giorda, and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010);
G. Adesso, and A. Datta, Phys. Rev. Lett. 105, 030501 (2010)

Discord in CV systems

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

$$\boldsymbol{\sigma} = \begin{bmatrix} \boldsymbol{\alpha}_1 & \gamma \\ \gamma^T & \boldsymbol{\alpha}_2 \end{bmatrix} \qquad A_2 = \det \boldsymbol{\alpha}_2$$

$$\mathcal{D} = f(\sqrt{A_2}) - f(\mu_-) - f(\mu_+) + \inf_{\boldsymbol{\sigma}_0} f(\sqrt{\det \boldsymbol{\epsilon}})$$

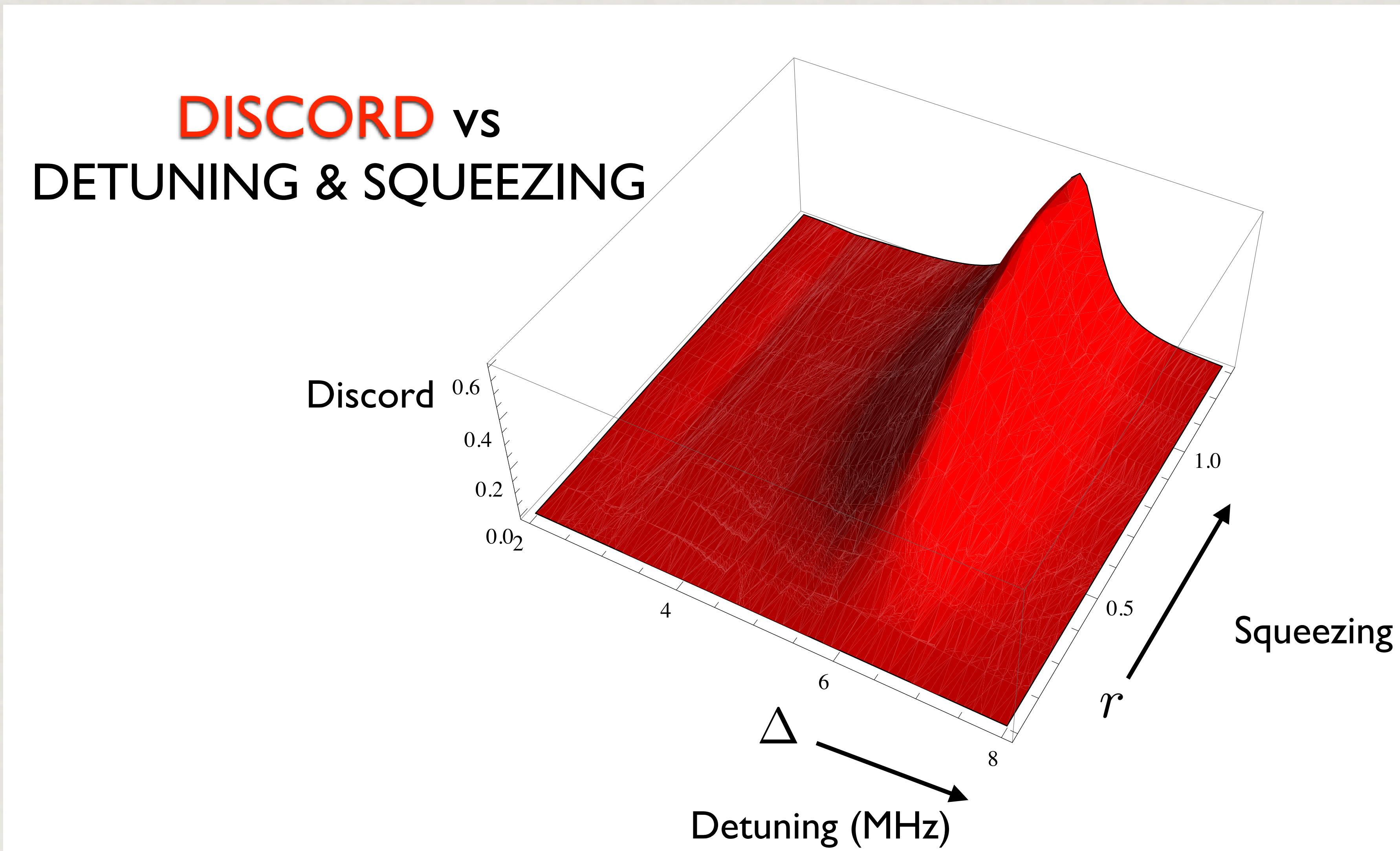
$$f(x) = \left(\tfrac{x+1}{2}\right) \log\left[\tfrac{x+1}{2}\right] - \left(\tfrac{x-1}{2}\right) \log\left[\tfrac{x-1}{2}\right]$$

$$\boldsymbol{\epsilon} = \boldsymbol{\alpha}_1 - \gamma(\boldsymbol{\alpha}_2 + \boldsymbol{\sigma}_0)^{-1}\gamma^T \quad \textit{Schur's complement of } \boldsymbol{\alpha}_2$$

P. Giorda, and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010);
G. Adesso, and A. Datta, Phys. Rev. Lett. 105, 030501 (2010)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

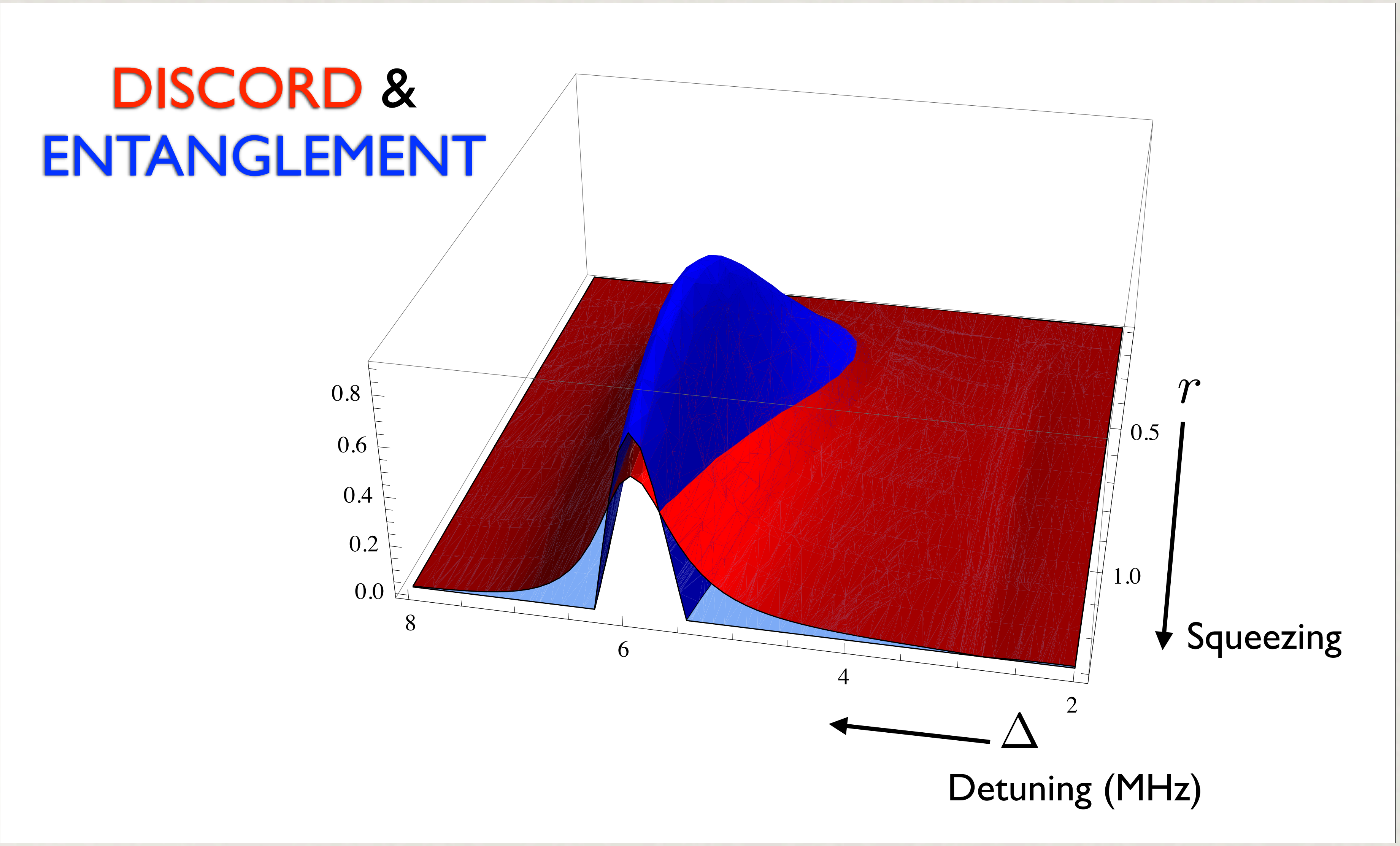
There's life beyond entanglement...



L. Mazzola, and M. Paternostro, arXiv:1104.0897 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

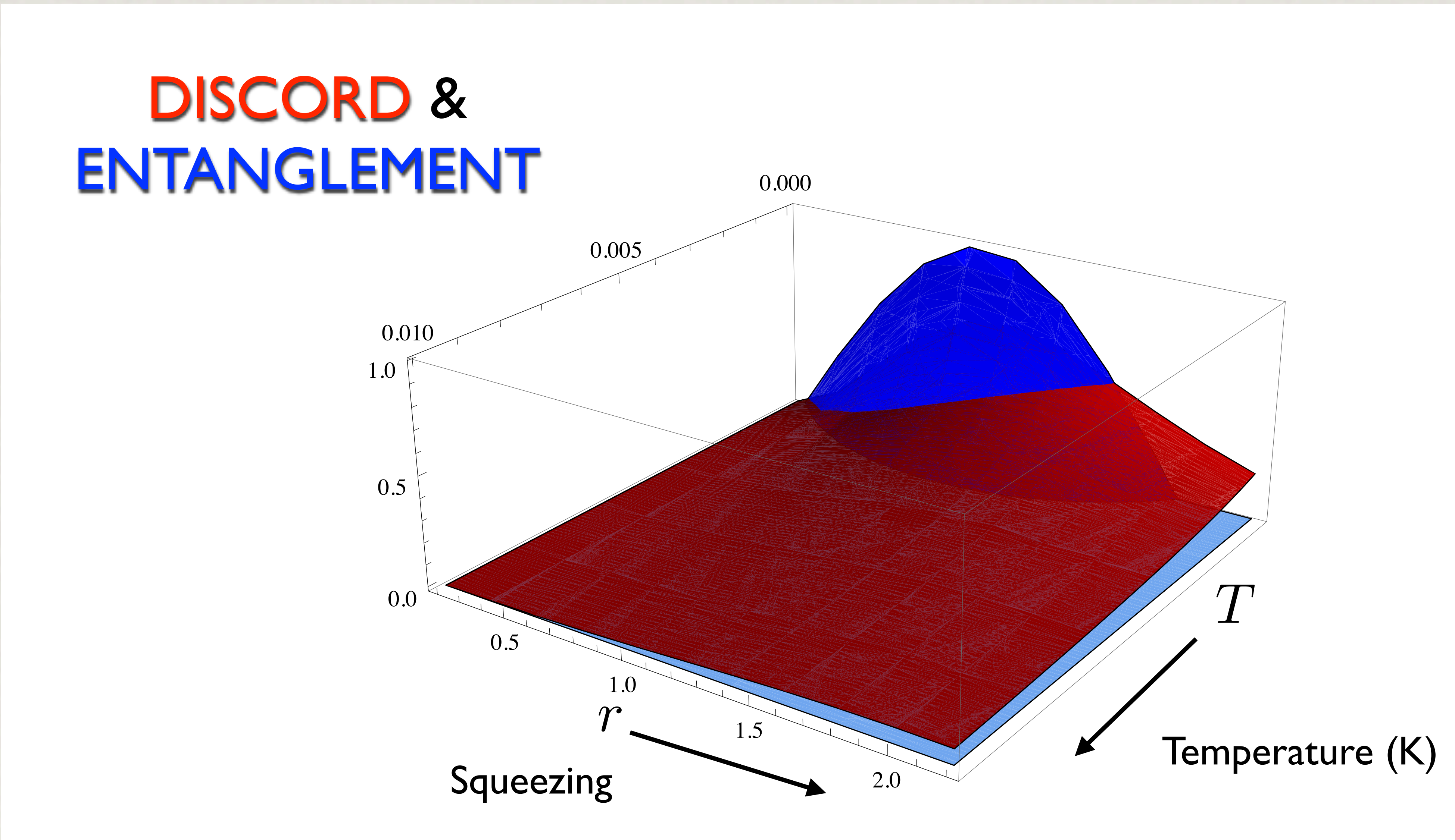
There's life beyond entanglement...



L. Mazzola, and M. Paternostro, arXiv:1104.0897 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Somebody likes it hot



L. Mazzola, and M. Paternostro, arXiv:1104.0897 (2011)

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S
www.qteq.info

Tanks!

I don't know what the fuss is,
but it's quite a lot of fun

EPSRC
Pioneering research and skills

BRITISH COUNCIL

Alice through the looking glass,
tribute to L. Carroll, Guilford (UK)

Dear Mr. Referee...

$$m_{11}=\frac{-\kappa G^4(\gamma_m+2\kappa)\omega_m^2\Delta+2G^2\omega_m\kappa\Delta^2\left[\kappa\gamma_m^2+2\gamma_m\kappa^2+2\kappa^3+\omega_m^2(\gamma_m+\kappa)\right]+G^2\omega_m\left[2\kappa^3(\gamma_m+\kappa)\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)-\gamma_m\kappa\overline{N}\omega_m\Delta\left(3\gamma_m\kappa+4\kappa^2+2\omega_m^2\right)\right]}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left({\gamma_m}^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]}$$

$$+\frac{\gamma_m G^2\overline{N}\omega_m^2\Delta^3(\gamma_m+4\kappa)+2\kappa^2G^2\Delta^4\omega_m+2\omega_m\gamma_m\kappa\overline{N}\left(\kappa^2+\Delta^2\right)\left[\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+\Delta^4\right]}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left({\gamma_m}^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]},$$

$$m_{22}=\frac{G^2\left[2\kappa^2\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)+\gamma_m\overline{N}\omega_m\Delta(\gamma_m+2\kappa)+2\kappa\Delta^2(\gamma_m+\kappa)\right]+2\gamma_m\kappa\overline{N}\left[\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+\Delta^4\right]}{2G^2\omega_m\Delta(\gamma_m+2\kappa)^2+4\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+4\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+4\gamma_m\kappa\Delta^4},$$

$$f_{11}=\frac{2\gamma_m\kappa\omega_m\Delta^4\left(G^2\overline{N}+\gamma_m^2+2\gamma_m\kappa+3\kappa^2-2\omega_m^2\right)+\kappa G^2\Delta^3\left(5\omega_m^2\gamma_m+4\omega_m^2\kappa-2\gamma_m^3-4\gamma_m^2\kappa-4\gamma_m\kappa^2\right)}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]}$$

$$+\frac{G^2\kappa\Delta\left[\kappa\omega_m^2\left(-2\gamma_m^2+\gamma_m\kappa+4\kappa^2\right)-2\gamma_m\kappa^2(\gamma_m+\kappa)^2-\gamma_m\omega_m^4\right]-2G^2\gamma_m\kappa\Delta^5+2\gamma_m\kappa^3\omega_m\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\omega_m\Delta^6}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]}$$

$$+\frac{\omega_m\Delta^2\left\{-\kappa G^4(\gamma_m+2\kappa)+G^2\gamma_m\overline{N}\left[2\kappa(\gamma_m+\kappa)^2+\gamma_m\omega_m^2\right]+2\gamma_m\kappa\left[2\gamma_m^2\kappa^2+2\gamma_m\kappa\left(2\kappa^2+\omega_m^2\right)+3\kappa^4+\omega_m^4\right]\right\}}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]},$$

Dear Mr. Referee...

$$f_{22}=\frac{G^4\omega_m\left[-\gamma_m\overline{N}\omega_m\Delta(\gamma_m+2\kappa)+2\kappa^2(\gamma_m+\kappa)^2+\gamma_m\kappa{\omega_m}^2-\kappa\Delta^2(3\gamma_m+4\kappa)\right]+G^2\kappa\Delta^3\left[7\gamma_m\omega_m^2+4\kappa\omega_m^2-2\gamma_m\left({\gamma_m}^2+2\gamma_m\kappa+2\kappa^2\right)\right]}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]}$$
$$+\frac{G^2\left[\gamma_m\overline{N}\omega_m\Delta^2\left(\omega_m^2\gamma_m+2\kappa\omega_m^2+2\kappa^3\right)+2\gamma_m\kappa^2\overline{N}\omega_m(\gamma_m+\kappa)\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)\right]}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]}$$
$$+\frac{-\kappa G^2\Delta\left[2\gamma_m\kappa^2(\gamma_m+\kappa)^2+\kappa\omega_m^2(2\gamma_m-\kappa)(3\gamma_m+4\kappa)+3\gamma_m\omega_m^4\right]-2\gamma_m\kappa G^2\Delta^5}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]}$$
$$+\frac{2\gamma_m\kappa\omega_m\left(\kappa^2+\Delta^2\right)\left[\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+\Delta^4\right]}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]},$$

$$f_{12}=\frac{G^2\kappa\omega_m\left(\Delta\left(G^2(\gamma_m+\kappa)(\gamma_m+2\kappa)+\gamma_m\overline{N}\left(2\kappa(\gamma_m+\kappa)^2+\gamma_m{\omega_m}^2\right)\right)+2\gamma_m\kappa\overline{N}\Delta^3+\gamma_m\kappa\omega_m\left(\kappa(\gamma_m+\kappa)+{\omega_m}^2\right)-\gamma_m\omega_m\Delta^2(\gamma_m+3\kappa)\right)}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]},$$

$$c_{11}=\frac{G\omega_m\left\{\kappa\Delta\left[2\kappa^2(\gamma_m+\kappa)\left(G^2+\gamma_m^2\overline{N}+\kappa\gamma_m\overline{N}\right)-\gamma_m\omega_m^2\left(G^2+2\kappa^2\overline{N}\right)\right]+2\gamma_m\kappa^3\omega_m\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)+2\gamma_m\kappa\overline{N}\Delta^5-2\gamma_m\kappa\omega_m\Delta^4\right\}}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]}$$
$$+\frac{G\omega_m\left\{2\kappa\Delta^3\left(G^2(\gamma_m+\kappa)+\gamma_m\overline{N}\left[\gamma_m^2+2\gamma_m\kappa+2\kappa^2-\omega_m^2\right)\right]+\gamma_m\omega_m\Delta^2\left[\gamma_m\left(G^2\overline{N}+2\kappa^2\right)+2\kappa\left(G^2\overline{N}+\omega_m^2\right)\right]\right\}}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]},$$

$$c_{12}=\frac{G^3\kappa\omega_m\left\{-\gamma_m\overline{N}\omega_m\Delta(\gamma_m+2\kappa)+\left[2\kappa^2(\gamma_m+\kappa)^2+\gamma_m\kappa\omega_m^2\right]+2\Delta^2(\gamma_m+\kappa)^2\right\}}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]}$$
$$+\frac{2\gamma_m G\kappa\omega_m\left(\kappa^2+\Delta^2\right)\left[\overline{N}(\gamma_m+\kappa)\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)-\omega_m\Delta(\gamma_m+2\kappa)+\kappa\overline{N}\Delta^2\right]}{2\left(\omega_m\kappa^2+\omega_m\Delta^2-G^2\Delta\right)\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]},$$

$$c_{21}=\frac{G\gamma_m\kappa\left[-\overline{N}\omega_m\Delta(\gamma_m+2\kappa)+\kappa\left(\kappa\gamma_m+\kappa^2+{\omega_m}^2\right)+\Delta^2(\gamma_m+\kappa)\right]}{G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4},$$

$$c_{22}=-\frac{G^3\kappa\omega_m(\gamma_m+2\kappa)+2\gamma_m G\kappa\left[\gamma_m\kappa\overline{N}\omega_m+\kappa^2(\overline{N}\omega_m+\Delta)+(\omega_m^2-\Delta^2)(\overline{N}\omega_m-\Delta)\right]}{2\left[G^2\omega_m\Delta(\gamma_m+2\kappa)^2+2\gamma_m\kappa\Delta^2\left(\gamma_m^2+2\gamma_m\kappa+2\kappa^2-2\omega_m^2\right)+2\gamma_m\kappa\left(\kappa\gamma_m+\kappa^2+\omega_m^2\right)^2+2\gamma_m\kappa\Delta^4\right]}.$$