

L^AT_EX: Online module 9

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August 19, 2011

Topics to be covered

- Typeset lengthy equations
- Matrices
- Types of integral notations
- Differential equations
- Examples
- Exercises

Typeset- long equations

- Lengthy equations can be split into smaller expressions extending to more than one line
- Alignment should be taken care off when wrapping an equation, as this improves readability

An example of a lengthy equation:

$$\begin{aligned} f(x) = & a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + a_{n-4} x^{n-4} \\ & + a_{n-5} x^{n-5} + a_{n-6} x^{n-6} + a_{n-7} x^{n-7} + a_{n-8} x^{n-8} \\ & + a_{n-9} x^{n-9} + a_{n-10} x^{n-10} + a_2 x^2 + a_1 x + a_0 \end{aligned}$$

contd...

- Equation should be within *equation array environment*, i.e., `\begin{eqnarray}` and `\end{eqnarray}`
- Part of an equation should end with the command `\nonumber`, followed by a double backslash at the place where equation is getting split
- If you want to number the equation, use the command `\nonumber` in the last line of the equation
- Alignment is controlled by an ampersand before and after the equal to sign in the first line; and double ampersands in the successive lines before the expression begins

contd...

Numbering is suppressed for the following equation:

```
\begin{eqnarray}
f(x) &=& a_n x^n+a_{n-1} x^{n-1}+a_{n-2} x^{n-2}+a_{n-3}
x^{n-3}+a_{n-4} x^{n-4} \nonumber \\
&& +a_{n-5} x^{n-5}+a_{n-6} x^{n-6}+a_{n-7} x^{n-7}+a_{n-8}
x^{n-8} \nonumber \\
&& +a_{n-9} x^{n-9}+a_{n-10} x^{n-10}+a_2x^2+
a_1x+a_0 \nonumber
\end{eqnarray}
```

Observe the following in the above code:

- position of double ampersands used in first line of the equation and in consecutive lines
- commands: double backslash and no number

contd...

Errors that can be made when you miss any commands in the code:

- Output without alignment, i.e. when all the ampersands from the equation (in the previous slide) are removed:

$$\begin{aligned} f(x) = & a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + a_{n-4} x^{n-4} \\ & + a_{n-5} x^{n-5} + a_{n-6} x^{n-6} + a_{n-7} x^{n-7} + a_{n-8} x^{n-8} \\ & + a_{n-9} x^{n-9} + a_{n-10} x^{n-10} + a_2 x^2 + a_1 x + a_0 \end{aligned}$$

- Output without `\nonumber` command (deleted from all lines):

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + a_{n-4} x^{n-4} (1)$$

$$+ a_{n-5} x^{n-5} + a_{n-6} x^{n-6} + a_{n-7} x^{n-7} + a_{n-8} x^{n-8} (2)$$

$$+ a_{n-9} x^{n-9} + a_{n-10} x^{n-10} + a_2 x^2 + a_1 x + a_0 (3)$$

contd...

- \LaTeX will throw an error if you remove double backslash from any of the lines
- Output with all the commands included in the environment:

$$\begin{aligned} f(x) = & a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + a_{n-4} x^{n-4} \\ & + a_{n-5} x^{n-5} + a_{n-6} x^{n-6} + a_{n-7} x^{n-7} + a_{n-8} x^{n-8} \\ & + a_{n-9} x^{n-9} + a_{n-10} x^{n-10} + a_2 x^2 + a_1 x + a_0 \end{aligned}$$

contd...

An equation which is numbered:

```
\begin{eqnarray}
f(x) &=& a_n x^n+a_{n-1} x^{n-1}+a_{n-2} x^{n-2}+a_{n-3}
x^{n-3}+a_{n-4} x^{n-4} \nonumber \\
&& +a_{n-5} x^{n-5}+a_{n-6} x^{n-6}+a_{n-7} x^{n-7}+a_{n-8}
x^{n-8} \nonumber \\
&& +a_{n-9} x^{n-9}+a_{n-10} x^{n-10}+a_2x^2+
a_1x+a_0
\end{eqnarray}
```


contd...

Output of the previous slide:

$$\begin{aligned} f(x) = & a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + a_{n-4} x^{n-4} \\ & + a_{n-5} x^{n-5} + a_{n-6} x^{n-6} + a_{n-7} x^{n-7} + a_{n-8} x^{n-8} \\ & + a_{n-9} x^{n-9} + a_{n-10} x^{n-10} + a_2 x^2 + a_1 x + a_0 \end{aligned} \quad (4)$$

contd...

- Some expressions might extend to several lines in a derivation
- Alignment is controlled in the following way:

```
\begin{eqnarray}
\Psi(x) &=& F(x)+G(x)+H(x)\nonumber \\
&=& a_n x^n+a_{n-1} x^{n-1}+a_{n-2} x^{n-2}+a_{n-3}
x^{n-3}+a_{n-4} x^{n-4} \nonumber \\
&& +a_{n-5} x^{n-5}+a_{n-6} x^{n-6}+a_{n-7} x^{n-7}
+a_{n-8}x^{n-8}+a_{n-9} x^{n-9} \nonumber \\
&& +a_{n-10} x^{n-10}+a_2x^2 \nonumber \\
&=& \phi(x) + \psi(x) + \theta(x)
\nonumber
\end{eqnarray}
```

contd...

Output of the previous slide:

$$\begin{aligned}\Psi(x) &= F(x) + G(x) + H(x) \\ &= a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + a_{n-4} x^{n-4} \\ &\quad + a_{n-5} x^{n-5} + a_{n-6} x^{n-6} + a_{n-7} x^{n-7} + a_{n-8} x^{n-8} + a_{n-9} x^{n-9} \\ &\quad + a_{n-10} x^{n-10} + a_2 x^2 \\ &= \phi(x) + \psi(x) + \theta(x)\end{aligned}\tag{5}$$

Matrices

Several matrix environments that can be used such as:

- `\matrix`
- `\bmatrix`
- `\pmatrix`
- `\vmatrix`
- `\Vmatrix`

Matrix environment

- In this environment, displayed matrix is not enclosed within any braces
- Environment is the following:

```
 $\begin{matrix}  
 1 & 10 & 1 \\ 1 & 1 & 1 \\ 1 & 10 & 1 \\ \end{matrix} $
```

- Output:

$$\begin{matrix} 1 & 10 & 1 \\ 1 & 1 & 1 \\ 1 & 10 & 1 \end{matrix}$$

bmatrix environment

- In this environment, displayed matrix is enclosed within square braces
- Environment is the following:

```

 $\begin{bmatrix}$ 
1 & 10 & 1 \\
1 & 1 & 1 \\
1 & 10 & 1
 $\end{bmatrix}$ 

```

- Output:

$$\begin{bmatrix} 1 & 10 & 1 \\ 1 & 1 & 1 \\ 1 & 10 & 1 \end{bmatrix}$$

pmatrix environment

- In this environment, displayed matrix is enclosed within open braces
- Environment is the following:

```

 $\begin{pmatrix}$ 
1 & 10 & 1 \\
1 & 1 & 1 \\
1 & 10 & 1
 $\end{pmatrix}$ 

```

- Output:

$$\begin{pmatrix} 1 & 10 & 1 \\ 1 & 1 & 1 \\ 1 & 10 & 1 \end{pmatrix}$$

vmatrix environment

- In this environment, displayed matrix is enclosed within vertical lines
- Environment is the following:

```
\begin{vmatrix}
1 & 10 & 1 \\
1 & 1 & 1 \\
1 & 10 & 1
\end{vmatrix}
```

- Output:

$$\begin{vmatrix} 1 & 10 & 1 \\ 1 & 1 & 1 \\ 1 & 10 & 1 \end{vmatrix}$$

Vmatrix environment

- In this environment, displayed matrix is enclosed within double vertical lines
- Environment is the following:

```

 $\begin{Vmatrix}$ 
1 & 10 & 1 \\
1 & 1 & 1 \\
1 & 10 & 1
 $\end{Vmatrix}$ 

```

- Output:

$$\begin{Vmatrix} 1 & 10 & 1 \\ 1 & 1 & 1 \\ 1 & 10 & 1 \end{Vmatrix}$$

General matrix

- Below is the code for a general $m \times n$ matrix whose entries are ones and zeros, generated in `bmatrix` environment:

```
$M = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$
```

- Output:

$$M = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

contd...

- Below is the code for any general $m \times n$ matrix, generated in `bmatrix` environment:

```
$ M = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$
```

- Output:

$$M = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Differential equations

- Differential equations are easy to generate using L^AT_EX
- Example1: First order differential equation

```
$ \displaystyle{\frac{dy}{dx}} = f(x,y)$
```

Output: $\frac{dy}{dx} = f(x,y)$

- Example2: Newtons second law

```
$M\frac{d^2 x(t)}{dt^2} = f(x(t))$
```

Output: $M\frac{d^2x(t)}{dt^2} = f(x(t))$

- Example3:

```
 $\displaystyle{\frac{d^2}{dx^2}}(f(x)g(x))+ $  
 $b\displaystyle{\frac{dy}{dx}}+c=0$
```

Output: $\frac{d^2}{dx^2}(f(x)g(x)) + b\frac{dy}{dx} + c = 0$

contd...

- Easy way to denote derivative is by a prime

- Example1:

```
$ y^\prime = f(x,y)$
```

Output: $y' = f(x,y)$

- Example2:

```
$ y^{\prime \prime} = f(x,y)$
```

Output: $y'' = f(x,y)$

- Example3:

```
$ ay^{\prime \prime} + by^\prime + c = f(x,y)$
```

Output: $ay'' + by' + c = f(x,y)$

contd...

- Another way to denote derivative is by a dot

- Example1:

`$ \dot{y} = f(x,y)$`

Output: $\dot{y} = f(x,y)$

- Example2:

`$ \ddot{y} = f(x,y)$`

Output: $\ddot{y} = f(x,y)$

- Example3:

`$ a\ddot{y}+b\dot{y}+c = f(x,y)$`

Output: $a\ddot{y} + b\dot{y} + c = f(x,y)$

contd...

- Command `\partial` is used to write a partial differential equation

- Example1:

```
$ \displaystyle{\frac{\partial y}{\partial x}} = f(x,y)$
```

Output: $\frac{\partial y}{\partial x} = f(x,y)$

- Example2:

```
$$\displaystyle{\frac{\partial^2 y}{\partial t^2}} = f(t)$$
```

Output: $\frac{\partial^2 y}{\partial t^2} = f(t)$

- Example3:

```
$$\displaystyle{\frac{\partial^2}{\partial t^2}}(f(x)g(x))
```

```
b\displaystyle{\frac{\partial y}{\partial x}}+c=0$
```

Output: $\frac{\partial^2}{\partial x^2}(f(x)g(x)) + b\frac{\partial y}{\partial x} + c = 0$

Integrals

- Double and triple integrals can easily be generated
- Example1: double integrals

`$ \int_1^2 \int_0^3 F(x,y) \, dx \, dy $`

Output: $\int_1^2 \int_0^3 F(x,y) \, dx \, dy$

- Example2: for any general domain

`$ \iint_{\limits_{D}} F(x,y) \, dx \, dy $`

Output: $\iint_D F(x,y) \, dx \, dy$

contd...

- Example3: triple integrals

$\$ \int_1^2 \int_0^3 \int_{-1}^1 F(x,y,z) \, dx \, dy \, dz \$$

Output: $\int_1^2 \int_0^3 \int_{-1}^1 F(x,y,z) \, dx \, dy \, dz$

- Example4: for any general domain

$\$ \iint_{\text{limits}_D} F(x,y,z) \, dx \, dy \$$

Output: $\iint_D F(x,y,z) \, dx \, dy \, dz$

- Example5: limits belong to an interval

$\$ \int_{(x,y) \in I} ax^2 \, dx + b \exp\{y\} \, dy \$$

Output: $\iint_{(x,y) \in I} ax^2 \, dx + b \exp y \, dy$

contd...

- In complex analysis we come across contour integrals
- Replace the command `\int` by `\oint`
- Example6:

`$ \oint_{(x,y)\in I} ax^2, dx + by^2, dy$`

Output: $\oint_{(x,y)\in I} ax^2 dx + b \exp y dy$

Unions and intersections

- Union: use the command `\bigcup`
- Example1:

`$$\bigcup_{i=1}^{100} i$`

Output: $\bigcup_{i=1}^{100} i$

- Intersection: use the command `\bigcap`

`$$\bigcap_{i=1}^{100} i$`

Output: $\bigcap_{i=1}^{100} i$

Theorem environment

- Use the following environment:

```
\begin{theorem}
```

```
Statement of theorem is written here
```

```
\end{theorem}
```

Output:

Theorem

Statement of theorem is written here

Proof environment

- Use the following environment for proof:

```
\begin{proof}
```

```
Proof of theorem is written here
```

```
\end{proof}
```

Output:

Proof.

Proof of theorem is written here □

Notice that a square box is generated (to denote QED) by \LaTeX automatically when the proof ends

Exercises

Try to generate the following equations:

- Equation1:

$$\iint H(x, y, z) dx dy = \exp(x)$$

- Equation2:

$$\iiint H(x, y, z) dx dy dz = 0$$

Exercises

- Equation3:

$$H(x) = \begin{cases} \int_1^2 \int_0^3 F(x, y) dx dy & \text{if } x \geq 0 \\ \oint_{(x,y) \in I} ax^2 dx + by^2 dy & \text{if } y < 0 \end{cases}$$

- Equation4:

$$\bigcup_{i=1}^{100} \left(\bigcap_{j=1}^{100} j \right) i$$

contd...

- Ordinary differential equation1:

$$\log y \frac{d^2}{dt^2}(f(t)g(t)) + \frac{dy}{dt} = 0$$

- Ordinary differential equation2:

$$\exp(y)\ddot{y} + b\dot{y} + \dot{y} = 0$$

- Partial differential equation:

$$\frac{\partial^2}{\partial x^2}(\exp(x) \tan(x)) = 0$$