Bernoulli Trials Problems for 2015

1: The number of positive integers whose digits occur in strictly decreasing order is \(2(2^9 - 1)\).

2: Let \(n\) be the smallest positive integer such that \(7^n \equiv 1 \mod 2015\). Then \(n \geq 100\).

3: The number \(\sqrt[3]{7} + 5\sqrt{2} + \sqrt{11 - 6\sqrt{2}}\) is rational.

4: For every field \(F\) and every square matrix \(A\) with entries in \(F\), \(\text{Row}(A) \cap \text{Null}(A) = \{0\}\).

5: For each \(n \in \mathbb{Z}^+\), let \(x_n\) be the number of matrices \(A \in M_{3 \times n}(\mathbb{Z}_3)\) with no two horizontally or vertically adjacent entries equal. Then there exists \(n \in \mathbb{Z}^+\) such that \(x_n\) is a square.

6: \(\prod_{k=1}^{50} \frac{2k}{2k-1} > 12\).

7: \(\int_0^{\pi/2} \sqrt{2 \tan x} \, dx > \pi\).

8: A light at position \((0,0,4)\) shines down on the sphere of radius 1 centered at \((3,0,2)\) casting a shadow on the \(xy\)-plane. The area of the shadow is greater than 33.

9: There exists a continuous function \(f : [0,1] \to [0,1]\) such that for every \(y \in [0,1]\) the number of \(x \in [0,1]\) for which \(f(x) = y\) is finite and even.

10: There exists a polynomial \(f \in \mathbb{Q}[x,y]\) such that the map \(f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}\) is bijective.

11: There exists a bijective map \(f : \mathbb{Z}^+ \to [0,1] \cap \mathbb{Q}\) such that \(\sum_{n=1}^{\infty} \frac{f(n)}{n}\) converges.

12: For every sequence of real numbers \(\{a_n\}\), if \(\sum_{n=1}^{\infty} a_n\) converges then so does the series \(a_1 + a_2 + a_4 + a_3 + a_8 + a_7 + a_6 + a_5 + a_{16} + a_{15} + \cdots + a_9 + a_{32} + a_{31} + \cdots + a_{17} + a_{64} + \cdots\)

13: Initially, \(n = 2\). Two players, \(A\) and \(B\), take turns with \(A\) going first. At each turn, the player whose turn it is can either replace \(n\) by \(n + 1\) or by \(2n\). The first player to replace \(n\) by a number larger than 130 loses. In this game, player \(A\) has a winning strategy.