The research I conducted was in the field of formal languages and automata; specifically, on the state complexity of regular languages and on atoms of regular languages. My main result was a set of necessary and sufficient conditions for a regular language to have the maximal number of atoms and for all of its atoms to have maximal state complexity.

To explain this result, I will introduce a few notions from language and automata theory. It is well known that a language is regular if and only if it can be recognized by a deterministic finite automaton (DFA). Furthermore, for each regular language L, there is a unique minimal DFA that recognizes it. The number of states in this minimal DFA is called the *state complexity* of L.

The *quotient* of L by a word w is the set $w^{-1}L = \{x \mid wx \in L\}$. If L is regular, then it has a finite number of quotients (in fact, the number of quotients equals the state complexity). The *atoms* of a regular language L are certain non-empty intersections of its quotients and complemented quotients. The atoms of L are pairwise disjoint, and it is possible to write any quotient of L (including L itself) and any quotient of any atom of L as a union of L's atoms. Thus, the atoms of L can be thought of as its basic building blocks. Tight upper bounds on the number of atoms of a regular language and the state complexities of atoms are known; a language which meets these bounds is called *maximally atomic.*

A DFA's transitions are normally thought of as a "transition function" which maps state-symbol pairs to states, but one can also think of them as a family of functions, each indexed by an alphabet symbol, which map states to states. The *transition semigroup* of a DFA is the semigroup of transformations (functions from a set into itself) generated by this family of functions. My result shows that the property of being maximally atomic is equivalent to some simple conditions on the transition semigroup of a regular language's minimal DFA:

A regular language with state complexity n is maximally atomic if and only if the transition semigroup of its minimal DFA contains a transformation whose image has size $n-1$, and permutations which can map any subset of the state set to any other subset of the same size.

In other words, the transition semigroup must contain a singular transformation of maximal rank and its subgroup of permutations must be k-set-transitive for $0 \leq k \leq n$.

This result connects maximally atomic languages to another measure of complexity called *syntactic complexity.* The syntactic complexity of a regular language is the size of its *syntactic semigroup*, which is isomorphic to the transition semigroup of its minimal DFA. If a language with state complexity n has the maximal possible syntactic complexity n^n, then its transition semigroup contains every transformation of the DFA's state set, and thus satisfies the conditions of the result. This gives as a corollary that every language with maximal syntactic complexity is maximally atomic.

Aside from this result, I also worked on a paper about the complexity of regular *right ideal* languages: languages L over an alphabet Σ that satisfy $L = L\Sigma^*$. We proved that there exists a "most complex" sequence of right ideals: a sequence $(R_n \mid n \geq 1)$ of right ideals such that R_n has state complexity n, and each R_n meets the known upper bounds for state complexity of basic operations (that is, languages like R_n^* and $R_m \cup R_n$ have the maximal possible state complexity), syntactic complexity, number of atoms, and state complexity of atoms. This was analogous to earlier work by Dr. Brzozowski in which he found a general "most complex" sequence of regular languages; the goal was to see if the same could be done for subclasses of the regular languages, and we were successful in the case of right ideals.