The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents

The Fifteenth Annual Small c Competition

for First and Second Year Students

Friday 25 September 2015

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open this booklet until you are told to do so.

10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.

11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.

100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.

101. On your response form, print your name, plan, and ID number.

110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.

111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.

1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.

1001. Diagrams are *not* drawn to scale. They are intended as aids only.

1010. Als u dit kunt lezen, spreekt u het Nederlands.

1011. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

1100. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.

1001. The only website you may use during the contest is `www.theonion.com`.

1110. Data was scrambled during construction in the MC building. Try and find the flipped bit above.

1111. Turn off and put away your cell phones, tablets, laptops, desktops, satellites and quantum computers.

10000. Hint: The answer to at least one question is **B**.

Part A

1. $\sqrt{16\sqrt{100} - 9\sqrt{64}}$ equals

   (A) 4 **(B)** $6\sqrt{7}$ **(C)** 16 **(D)** $2\sqrt{22}$ **(E)** 9

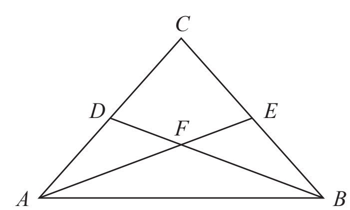
2. How many of the following four statements are true?

   - $6 > 3$
   - $\frac{1}{6} > \frac{1}{3}$
   - $6 \times 3 = 3 \times 6$
   - $6 \div 3 = 3 \div 6$

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

3. A Minion walks in the plane along three line segments from $(3, 7)$ to $(10, 7)$ to $(10, 2)$ to $(4, 2)$. What is the total distance walked by the Minion?

   (A) 17 **(B)** 4 **(C)** 5 **(D)** 18 **(E)** 13



4. In the diagram shown, $\angle BDC = 80°$, $\angle CEA = 80°$, and $\angle AFB = 130°$. What is the measure of $\angle ACB$?

   (A) $130°$ **(B)** $50°$ **(C)** $80°$ **(D)** $110°$ **(E)** $70°$

5. What is the value of $\dfrac{(-1)^{2015} - (-2016)^0}{(-1)^{2016} + 1^{2015}}$?

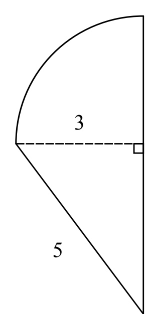
   (A) -1 **(B)** 0 **(C)** -0.5 **(D)** 1 **(E)** it is undefined

6. If $x \in \mathbb{R}$ and $\dfrac{9}{x} \cdot \dfrac{x^2}{6} \cdot \dfrac{18}{x^3} \cdot \dfrac{x^5}{3} = \dfrac{1}{3}$, what is the value of $\dfrac{1}{x}$?

   (A) $\frac{1}{3}$ **(B)** $\frac{1}{9}$ **(C)** 3 **(D)** 27 **(E)** $\frac{1}{27}$

7. Three fractions are equally spaced between $\frac{1}{8}$ and $\frac{3}{4}$. The largest of these three fractions is

   (A) $\frac{2}{3}$ **(B)** $\frac{5}{8}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{11}{16}$ **(E)** $\frac{19}{32}$



8. A figure composed of a quarter circle and right triangle is shown. A portrait of our new Dean of Math is not shown. What is the perimeter of the figure?

   (A) $12 + \frac{3\pi}{2}$ **(B)** $13 + \frac{3\pi}{2}$ **(C)** $15 + \frac{3\pi}{2}$ **(D)** $12 + 6\pi$ **(E)** $15 + 6\pi$

9. If 16 kilograms of goose food can feed 14 campus geese for 5 days, then at the same rate, how many kilograms would it take to feed 15 campus geese for 7 days?

   (A) 17 **(B)** 19 **(C)** 24 **(D)** 32 **(E)** 48

10. Donald Trump added together a list of 2015 four digit numbers to get a sum A. Then, he randomly chose one of the numbers in the list, rearranged its digits, placed it back in the list, and added the numbers in the new list to get a sum B. No matter what number Donald chose, $A - B$ will be divisible by

    (A) 2 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

Part B

11. If $(x-5)(2x-3)=0$, then what is the maximum possible value of $2x-3$?

(A) 0 **(B)** 7 **(C)** 13 **(D)** $\frac{3}{2}$ **(E)** $\frac{7}{2}$

12. Which of the following is not a divisor of $6!+7!+8!$?

Note: The notation $n!$ represents the product of the first n positive integers. For example:
$22! = 22\times21\times20\times19\times18\times17\times16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1 = 1124000727777607680000$.

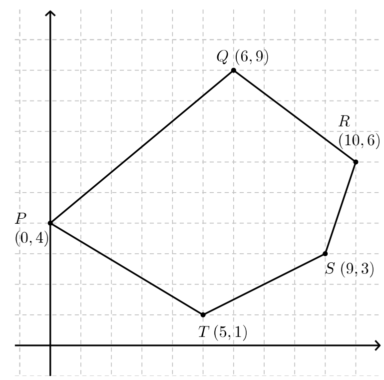
(A) 27 **(B)** 80 **(C)** 256 **(D)** 320 **(E)** 360

13. The number of real solutions to $(\ln x)^3 = \ln x$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

14. The number of integers x such that $-50 \le x \le 50$ where the fraction $\dfrac{x^2+25x-325}{275-13x}$ has a numerator greater than the denominator is

(A) 45 **(B)** 101 **(C)** 0 **(D)** 56 **(E)** 38



15. What is the area of the polygon formed by the five vertices $P(0,4)$, $Q(6,9)$, $R(10,6)$, $S(9,3)$ and $T(5,1)$?

(A) 36 **(B)** 38 **(C)** 44 **(D)** 52 **(E)** 74

16. If θ is an acute angle such that $\sin\theta\cos\theta = \frac{1}{8}$, then $\sin\theta + \cos\theta$ equals

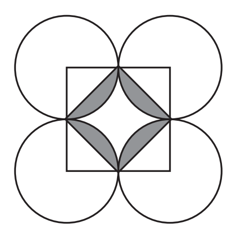
(A) $\frac{1}{2}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** $\frac{\sqrt{5}}{2}$ **(E)** $\frac{\sqrt{7}}{2}$

17. Given

$$xy = 13$$
$$yz^3 = 31$$
$$x^3y^2z = 5$$

what is the value of $(xyz)^2$?

(A) $\sqrt[4]{2015}$ **(B)** $\sqrt{2015}$ **(C)** 2015 **(D)** 2015^2 **(E)** 2015^4



18. Each corner of a square lies at the centre of a different circle of radius 1. The four circles are congruent and mutually tangent as shown. What is the shaded area?
(Important Note: Nothing was shaded using invisible ink.)

(A) $\pi-2$ **(B)** $4\pi-4$ **(C)** $\pi-4$ **(D)** π^2-2 **(E)** π^2-4

19. How many pairs of positive integers (m,n) satisfy $\dfrac{2}{3} < \dfrac{m+n}{mn} < 1$?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 7 **(E)** 8

20. Let a be an integer such that $b = \dfrac{2a^2+3a-8}{2a+37}$ is a positive integer. The difference between the largest and the smallest possible value of b is

(A) 300 **(B)** 568 **(C)** 620 **(D)** 417 **(E)** 585

Part C

21. The sum of the digits of 2015 is a perfect cube since $2+0+1+5=2^3$. How many numbers between 2000 and 3000 have the property that the sum of their digits is a perfect cube?

(A) 34 **(B)** 27 **(C)** 24 **(D)** 15 **(E)** 6

22. Three people play a game. George HW rolls a fair regular normal typical standard six-sided die. Then Jeb rolls the die. Then George W rolls the die. The winner is the one who first rolls an even number. If no even number is rolled after all three have rolled once, the process is repeated until someone wins. What is the probability that Jeb will be the winner?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2}{7}$ **(D)** $\frac{1}{27}$ **(E)** $\frac{4}{7}$

23. Stephen Mulcair places four different non-zero digits in a 2-by-2 grid. The two rows and two columns form four two-digit numbers. For example, the two different grids shown form the two-digit numbers 12, 34, 13 and 24. In how many ways can Stephen Mulcair place digits in this way so that each of the four two-digit numbers is prime?

1	2
3	4

1	3
2	4

(A) 18 **(B)** 20 **(C)** 22 **(D)** 24 **(E)** 32

24. Elizabeth Trudeau puts an x or y in each of the empty 22 boxes below. This gives a sequence of 24 x's and y's that begins with xy. In how many ways can Elizabeth Trudeau do this so that the entire sequence is some pattern fully repeated at least twice?

x y □□□□□□□□□□□□□□□□□□□□□□

(A) 1078 **(B)** 1080 **(C)** 1082 **(D)** 1084 **(E)** 1086

25. A *miraculous* triple is a triple of positive integers (a, b, c) satisfying

$$\sum_{k=1}^{2014}\left(\frac{\sin x}{\sin(kx)\sin(kx+x)}\right)=\frac{\sin x}{\sin x\sin 2x}+\frac{\sin x}{\sin 2x\sin 3x}+\frac{\sin x}{\sin 3x\sin 4x}+\cdots+\frac{\sin x}{\sin 2014x\sin 2015x}=\frac{\sin ax}{\sin bx\sin cx}.$$

for all real values of x with $0 < x < \frac{\pi}{2015}$. The authors of this contest are pretty sure they know what all the miraculous triples are, but are certain they know the miraculous triple (a_0, b_0, c_0) such that c_0 is as small as possible. What is the sum $(a_0)^2 + 2(b_0)^2 + 3(c_0)^2$?

(A) 12172620 **(B)** 12188740 **(C)** 12184709 **(D)** 12184711 **(E)** 12176649