Bernoulli Trials Problems for 2015

1: The number of positive integers whose digits occur in strictly decreasing order is $2(2^9 - 1)$.

2: Let n be the smallest positive integer such that $7^n \equiv 1 \bmod 2015$. Then $n \geq 100$.

3: The number $\sqrt[3]{7 + 5\sqrt{2}} + \sqrt{11 - 6\sqrt{2}}$ is rational.

4: For every field F and every square matrix A with entries in F, $\mathrm{Row}(A) \cap \mathrm{Null}(A) = \{0\}$.

5: For each $n \in \mathbf{Z}^+$, let x_n be the number of matrices $A \in M_{3\times n}(\mathbf{Z}_3)$ with no two horizontally or vertically adjacent entries equal. Then there exists $n \in \mathbf{Z}^+$ such that x_n is a square.

6: $\prod\limits_{k=1}^{50} \frac{2k}{2k-1} > 12.$

7: $\displaystyle\int_0^{\pi/2} \sqrt{2\tan x}\, dx \ > \pi.$

8: A light at position $(0, 0, 4)$ shines down on the sphere of radius 1 centered at $(3, 0, 2)$ casting a shadow on the xy-plane. The area of the shadow is greater than 33.

9: There exists a continuous function $f : [0, 1] \to [0, 1]$ such that for every $y \in [0, 1]$ the number of $x \in [0, 1]$ for which $f(x) = y$ is finite and even.

10: There exists a polynomial $f \in \mathbf{Q}[x, y]$ such that the map $f : \mathbf{N} \times \mathbf{N} \to \mathbf{N}$ is bijective.

11: There exists a bijective map $f : \mathbf{Z}^+ \to [0, 1] \cap \mathbf{Q}$ such that $\displaystyle\sum_{n=1}^{\infty} \frac{f(n)}{n}$ converges.

12: For every sequence of real numbers $\{a_n\}$, if $\sum\limits_{n=1}^{\infty} a_n$ converges then so does the series

$$a_1 + a_2 + a_4 + a_3 + a_8 + a_7 + a_6 + a_5 + a_{16} + a_{15} + \cdots + a_9 + a_{32} + a_{31} + \cdots + a_{17} + a_{64} + \cdots$$

13: Initially, $n = 2$. Two players, A and B, take turns with A going first. At each turn, the player whose turn it is can either replace n by $n + 1$ or by $2n$. The first player to replace n by a number larger than 130 loses. In this game, player A has a winning strategy.