We studied the set of local dimensions occuring in self-similar probability measures, known as the multifractal spectrum. It is well known that for measures whose self similarity is sufficiently non-overlapping (those measures satisfying the Open Set Condition), that the multifractal spectrum is a closed interval. Recently, Hu and Lau discovered that when the measure is the 3-fold convolution of the standard Cantor measure, that the multifractal spectrum is a closed interval union a single isolated point (the dimension of the endpoints of the support). Our research focused around showing that this is indeed a general phenonemon for self-similar measures.

One of our first results was to show that the dimension of the endpoints being isolated in high convolution powers is in fact a very general result. Using elementary measure theory methods, we were able to show that as long as the support of the measure summed to a full interval, that for high convolution powers, the dimension(s) of the endpoints of the support are isolated points in the multifractal spectrum. This does not require an assumption of self-similarity.

With the above result in hand, we turned to showing that in the case of self-similar measures, the dimensions of the remaining points forms a closed interval. Similar to other work done in the area, we required that the self-similarity ratio be 1/integer; we were, however, able to weaken the assumptions on the probabilities associated with the measure. We found a set of "Good" points, and were able to prove that the multifractal spectrum of these points was a closed interval. In the case when all the probabilities are positive, the set of "Good" points is simply the support of the measure less the two endpoints, which gives us the desired result (that the multifractal spectrum is a closed interval union at most two points).

The next problem was to actually calculate the endpoints of the interval. We were able to do this for small convolution powers of the regular 1/d Cantor measure. Our method of calculation for this case clearly generalizes, though actually performing it for larger convolution powers becomes quite difficult and technical.

We also considered what happens when we consider the quotient measure on the torus. It was shown by Fong, Hare and Johnstone that in the case of the threefold convolution of the standard Cantor measure, that the multifractal spectrum was a closed interval, though their method of proof showed little hope of extending. We were able to show that in the case where all of the probabilities are positive, that the multifractal spectrum is indeed a closed interval in general.

Through the course of the research, many interesting examples and counterexamples were found. A particularly interesting example was the construction of a self-similar measure, whose multifractal spectrum was the union of two closed intervals. Other examples included: measures with countably many isolated points in its multifractal spectrum; measures with open multifractal spectrum; a measure whose endpoint dimensions are not isolated for any convolution powers (the support never sums to a full interval); and a measure on the torus whose multifractal spectrum is an interval, then an interval union a point, then an interval again as more and more convolutions are taken.

The research answered many questions asked in papers by other researchers, as well as greatly generalizing (often with much cleaner proofs) known results. There are many natural directions to continue the research, including exact spectrum calculations in more general cases, and considering the case when the self-similarity ratio is not of the form 1/integer.