

An essential component of modern cancer therapies is external-beam radiation therapy (XRT). Despite being well-studied and widely used, XRT still has plenty of room for improvement. In particular, incorporation of spatial optimization techniques in the planning of cancer therapies has yet to be considered. Clinically, a radiation beam of uniform strength is used over the volume of a tumour even though the density of tumour cells is often non-uniform. This raises the question of whether a non-uniform radiation beam could produce better results.

Clinically speaking, the problem addressed in my research is whether or not XRT can be improved by allowing the radiation beam to vary spatially; and if so, what is the optimal beam shape for a given tumour? Mathematically, the question is essentially a large and complex optimization problem with many cases to consider.

The work begins with the well-established PI (proliferation/invasion) model for tumour growth. From there, the effect of XRT is incorporated by adding an extra term to the differential equation. This makes the model take the form

$$\frac{\partial n}{\partial t} = K\nabla^2 n(\vec{x}, t) + G(n, \vec{x}, t) - \gamma f(\vec{x}, t)D(n, \vec{x}, t)$$

where $n(\vec{x}, t)$ is the tumour cell-density, $G(n, \vec{x}, t)$ is the growth law of the tumour, $D(n, \vec{x}, t)$ is the death law, and $f(\vec{r}, t)$ is the radiation profile. Given the initial shape of a tumour and the parameters governing its growth (which can be found through medical imaging techniques), the goal is to determine the radiation profile which will produce the minimum total number of cells remaining in the tumour after radiation therapy. In the two papers resulting from my work on this problem, several biologically important cases are considered including different growth laws, death laws, initial tumour shapes, and number of radiation fractions. The papers also examine different optimization techniques which include both analytical and computational tools. In addition, a computational implementation of the augmented PI model using a pseudo-spectral method is used to visualize the results and observe overall trends.

The results of all the work show that by including spatial optimization in radiation beams, the overall number of cells killed by XRT can be dramatically increased from the uniform clinical case. It is my hope that one day, spatial optimization techniques can be incorporated into real-life clinical treatments.