SPECIAL K
Saturday November 1, 2014
10:00 am - 1:00 pm

1: Three circles, of radii 1, 2 and 3, are tangent in pairs at the points A, B and C. Find the area of triangle ABC.

2: Find the number of ways to represent 10! as a sum of consecutive positive integers.

3: Given $a \geq 1$, find the area of the square with one vertex at $(a, 0)$, one vertex above the curve $y = \sqrt{x}$, and the other two vertices on the curve $y = \sqrt{x}$.

4: Let n be an odd integer with $n > 3$. Let k be the smallest positive integer such that $kn+1$ is a square, and let l be the smallest positive integer such that ln is a square. Show that n is prime if and only if $n < \min\{4k, 4l\}$.

5: A **zigzag** is a set of the form $Z = \{ta + (1-t)b \big| 0 \leq t \leq 1\} \cup \{a + tu | t \geq 0\} \cup \{b - tu | t \geq 0\}$ for some $a, b, u \in \mathbf{R}^2$ with $u \neq 0$ (Z is the union of the line segment between a and b with a ray at a in the direction of u and a ray at b in the direction $-u$). Given a positive integer n, find the maximum number of regions into which n zigzags divide the plane.

6: Let $\{a_n\}$ be a sequence of real numbers with the property that for every $r \in \mathbf{R}$ with $r > 1$, we have $\lim\limits_{k \to \infty} a_{\lfloor r^k \rfloor} = 0$. Show that $\lim\limits_{n \to \infty} a_n = 0$.

BIG E
Saturday November 1, 2014
10:00 am - 1:00 pm

1: Given $a \geq 1$, find the area of the square with one vertex at $(a, 0)$, one vertex above the curve $y = \sqrt{x}$, and the other two vertices on the curve $y = \sqrt{x}$.

2: There are n closed (non-degenerate) line segments in $\mathbf{R}^3$. The sum of the lengths of the line segments is equal to 2014. Show that there is a plane in $\mathbf{R}^3$, which is disjoint from all of the line segments, such that the distance from the plane to the origin is less that 600.

3: Let n be an odd integer with $n > 3$. Let k be the smallest positive integer such that $kn+1$ is a square, and let l be the smallest positive integer such that ln is a square. Show that n is prime if and only if $n < \min\{4k, 4l\}$.

4: Let $f : [0, 1] \to \mathbf{R}$. Suppose f is continuous on $[0, 1]$ with $f(0) = f(1) = 0$ and $f(x) > 0$ for all $x \in (0, 1)$, and f'' exists and is continuous in $(0, 1)$. Show that

$$\int_0^1 \left| \frac{f''(x)}{f(x)} \right| dx > 4 \, .$$

5: Let $\{a_n\}$ be a sequence of real numbers with the property that for every $r \in \mathbf{R}$ with $r > 1$, we have $\lim_{k\to\infty} a_{\lfloor r^k \rfloor} = 0$. Show that $\lim_{n\to\infty} a_n = 0$.

6: Let $f : \mathbf{R}^n \to \mathbf{R}^n$ be bijective. Suppose that f maps connected sets to connected sets and that f maps disconnected sets to disconnected sets. Prove that f and f^{-1} are both continuous.