DESCRIPTION OF RESEARCH

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I have held two NSERC Undergraduate Research Assistantships under the supervision of Rahim Moosa. During the first assistantship, working collaboratively with Jack Klys, I used the methods of non-standard analysis to prove the existence of bounds in differential algebra. We published our results in the Journal of Algebra (Harrison-Trainor, M., Klys, J, and Moosa, R. *Nonstandard methods for bounds in differential polynomial rings*. Journal of Algebra, 360 (2012), 71–86.). The primality of ideals in polynomial rings is known to be definable because there are uniform bounds on the polynomials required to check whether an ideal is prime. One proof of these bounds by van den Dries and Schmidt uses a construction called ultraproducts. The ultraproduct of a sequence of structures has exactly the properties that almost all of those structures had, for an appropriate interpreation of almost all. It turns out that by choosing an appropriate sequence of structures, statements about ultraproducts are equivalent to statements about bounds.

We attempted to adapt these non-standard methods to the case of fields equipped with a derivation. The natural analogue of polynomials is differential polynomials, which are polynomials in which derivatives of the variables are allowed. A differential ideal is an ideal closed under the derivation. The question we considered is whether there is a bound on the polynomials that one must check to see if a differential ideal is prime. We were unable to prove or disprove the existence of a bound, but we found an existence-of-bounds analogue of a theorem that had been proved by ... about algorithms in differential fields. That theorem showed the equivalence of various problems in algorithmic differential algebra. One of the equivalent problems is the Kolchin problem, which is one of the most important open problems in differential algebra, while another is checking whether a differential ideal is prime. We proved that the existence of analagous bounds were equivalent, including an existence-of-bounds analogue of the Kolchin problem. Our work on ultraproducts also produced a proof of the existence of bounds for characteristic sets of prime differential ideals and a new proof of the effective differential Nullstellensatz. I gave an invited talk on these research results at the City University of New York during a two-day meeting of the Kolchin Seminar in Differential Algebra.

During the second assistantship, I studied differential jet spaces of varieties. These generalize tangent spaces to higher orders, and in some sense act like a power series, as the variety is entirely characterized by the sequence of jet spaces. The jet spaces, like tangent spaces, are linear and hence reduce the possibly non-linear information of the variety to linear information. I worked on trying to determine whether jet spaces are Moishezon over the field of constants. Moishezon types have a rather technical model-theoretic definition, but part of it is that after naming some independent parameters, they look like they are in the constants. So far, we have been unable to show that the jet spaces are Moishezon in all cases, but we have solved the problem for some special cases, and we have some ideas about how we might be able to combine the special cases together, but nothing concrete as yet. It is not known whether there are non-trivial Moishezon types in differential fields, but jet spaces are a natural candidate based on what happens in other contexts. An answer to this question would be an important part of knowing what kinds of Moishezon types exist in differential fields. I presented our results in the model theory seminar at McMaster University this fall.