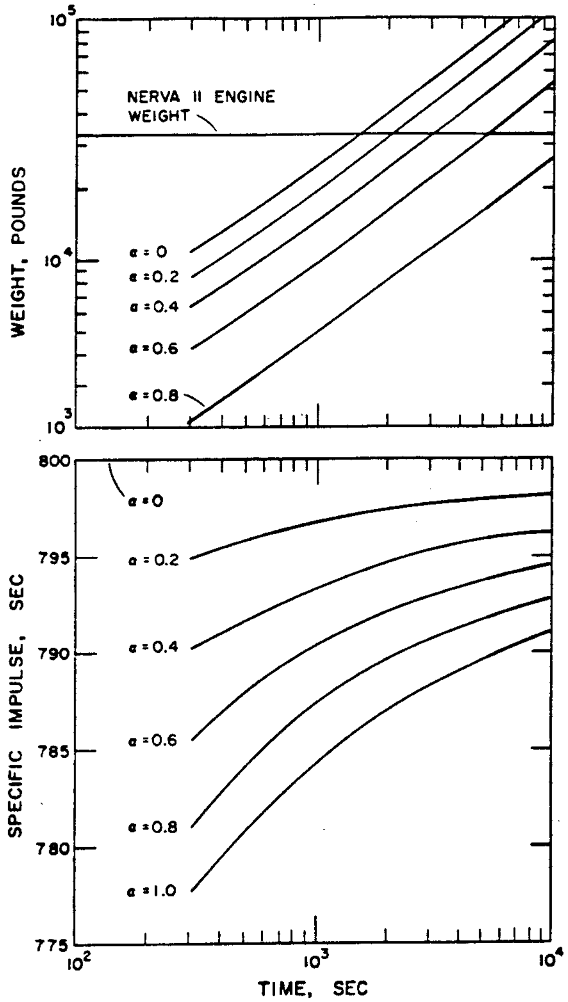

Fig. 5 Top: Weight of cooldown propellant plus additional stage weight (excluding engine weight) vs operating time for 5000-Mw engine, with the thrust-producing fraction, alpha, as the parameter. The estimated weight of the engine is shown.
Bottom: Average specific impulse vs operating time with alpha, the thrust-producing fraction of the cooldown propellant, as a parameter (exit temperature during cooldown is assumed to be 2000°R).

calculated using a tank mass fraction (ratio of usable propellant to stage weight) estimate of 0.8.[1] For reference, the estimate of 34,000 lb for the engine weight[1] is shown. For t_0 = 1800 sec and α of 0.1, the engine weight equals the non-thrust-producing coolant and stage weight, which suggests, if this consideration is taken alone, a tradeoff between carrying additional engines in lieu of cooldown weights rather than restarting the engine.

Use of a fraction of the cooldown hydrogen as a propellant leads to a degradation of specific impulse for any given maneuver that can be calculated:

$$\bar{I}_{sp} = (W_0 \times I_{sp_0} + \alpha W_{cd} \bar{I}_{spc})/(W_0 + \alpha W_{cd})$$

where $I_{sp} \equiv$ average specific impulse, sec; $I_{sp_0} \equiv$ design specific impulse = 80 sec; $I_{spc} \equiv$ cooldown specific impulse = 533 sec; $W_0 \equiv$ propellant used at 800 sec; and $W_{cd} \equiv$ cooldown propellant. It was assumed that I_{sp} scaled as the square root of chamber temperature, although it is recognized that other factors such as chamber pressure and nozzle expansion ratio enter. Figure 5 also shows I_{sp} vs operating time. For $0 < \alpha < 1$, and t_0 = 1800 sec, I_{sp} degradation is from $2\frac{1}{2}$ to $13\frac{1}{2}$ sec.

While it is clear that the sensitivity of the mission application to variations in specific impulse, or the so-called "exchange factors" calculated from the Mars mission in Ref. 1, cannot be applied to other missions inasmuch as they represent partial derivatives of a particular function evaluated at a specific value of the function, they can nevertheless be used to obtain an estimate of the effects for similar missions. Exchange factors calculated in Ref. 1 indicate that a change of 15 sec in I_{sp} leads to a change in initial total weight of the vehicle of 3.5%, which is about 70,000 lb for this mission.

Conclusions

A restart requirement for a nuclear engine mission application can lead to weight penalties comparable to the initial engine weight, and through I_{sp} degradation, to equivalent weight penalties exceeding the engine weight; this depends upon the mission and upon how successful one is in using the cooldown fraction to produce thrust.

Since cooldown thrust is well-defined in time and magnitude, its usefulness is limited. Indeed, for some precise maneuvers a non-thrust-producing cooldown mechanism would be required to prevent perturbations to the trajectory. Finally, the radiation fields produced by the cooled-down reactor would exist outside the shadow shield for considerable periods of time, and would limit extravehicular activities and could influence scientific experiments.

Reactor restart requirements exist for ground test development programs to maximize the amount of data per reactor. Even here, it should be pointed out that such a requirement can introduce problems that may never be faced in a non-restartable mission application, e.g., the separation of thermal recycle effects upon the operation of the reactor from other effects.

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Thermal Contact Resistance across Elastically Deformed Spheres

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Nomenclature

a	=	contact radius
A	=	area
E	=	modulus of elasticity
F	=	force
J	=	Bessel function
$k, \bar{K}$	=	thermal conductivities of metal and gas, respectively
K_n	=	Knudsen number
L	=	mean free path
M	=	molecular weight

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P = pressure
q, Q = heat flux and heat rate, respectively
r = coordinate axis
R = thermal resistance
R_0 = molar gas constant
S = stress
T = temperature
u = separation between sphere and plane in vicinity of contact
z = coordinate axis
α = accommodation coefficient
γ = specific heat ratio for gas, c_p/c_v
ϵ = ratio of contact to sphere radius
ρ = sphere radius
η = pure number
ν = Poisson's ratio
σ = Stefan-Boltzmann constant
ω = displacement in z direction

Subscripts

a = apparent
c = conduction
fm = free molecule
g = gas
p = plane
r = radiation
s = sphere, shear
∞ = continuum
1, 2 = planes 1 and 2, respectively

Problem

THERMAL control of a spacecraft involves maintaining the vehicle, or sections of it, and components at proper operating temperatures.

Requirements may be very severe, some types of reconnaissance equipment requiring temperature regulation within approximately 2°F and thermal gradient control within a fraction of a degree. This note deals with the thermal contact resistance due to the presence of ball bearings between the solar cell shield and the satellite flywheel of the Orbiting Solar Observatory satellite,[1] which would function in a circular, 300-naut-mile orbit. The modes of heat transfer between interfaces of the various components are conduction through the metal parts, conduction through the rarefied gas in the separation zone, and radiation between surfaces.

The heat-transfer problem to be investigated is that of smooth solid metallic spheres of radius ρ separating two smooth, rigid planes at uniform and constant temperatures T_1 and T_2 (Fig. 1a). The spheres are surrounded by a rarefied gas corresponding to the atmosphere for the satellite's orbit. It is assumed that the temperature difference $(T_1 - T_2)$ is small, the problem is axisymmetric, and that steady-state conditions prevail.

Under a load F, the elastic sphere and the rigid plane will be in physical contact over a radius a. The total displacement in the direction of loading is[2]

$$\omega(r) = \begin{cases} a^2(2 - r^2/a^2)/2\rho & r \leqslant a \\ \dfrac{a^2[(2 - r^2/a^2)\sin^{-1}a/r + (r^2/a^2 - 1)^{1/2}]}{\pi\rho} & r > a \end{cases} \quad (1)$$

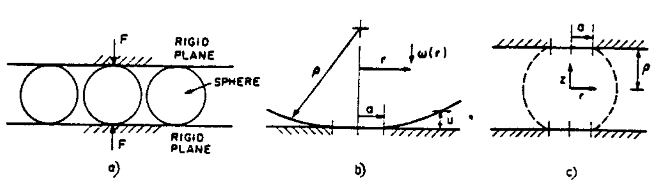

Fig. 1 a) Physical model, b) model used in deformation analysis, and c) model used in thermal analysis.

Assuming that the sphere and plane have the same elastic properties, and that Poisson's ratio is approximately 0.3, the Hertz contact radius can be determined from classical elasticity theory to be $a = 1.11[F\rho/E]^{1/3}$, which can be written in dimensionless form as

$$\epsilon = 1.62[P_a/E]^{1/3} \quad (2)$$

If it is assumed that the distance between the plane and a point on the surface of the sphere (Fig. 1b) can be represented with sufficient accuracy by $z = r^2/2\rho$, then the separation u, beyond the contact zone, can be determined from Eq. (1) for $r > a$ to be

$$u/\rho = \left\{\frac{r^2/2a^2 - [(2 - r^2/a^2)\sin^{-1}a/r + (r^2/a^2 - 1)^{1/2}]}{\pi}\right\}\frac{a^2}{\rho^2} \quad (3)$$

Since only elastic deformations of the sphere are being considered, it would be convenient to have an expression relating the maximum permissible ratio of the contact radius to sphere radius, $\epsilon_{max} = a/\rho$, to the physical properties of the sphere. The point with the maximum shearing stress is on the z axis at a depth equal to about $a/2$. This point must be considered as the weakest point in metals. The maximum shearing stress at this location (for $\nu = 0.3$) is about 31% of the maximum compressive stress at the center of contact,

$$S_s = 0.31(1.5F/\pi a^2) \quad (4)$$

Using Eqs. (2) and (4), the desired relationship is

$$\epsilon_{max}^2 = 84(S_s/E)^2 \quad (5)$$

where S_s is the maximum shear stress of the material.

Conduction through the Sphere

The mathematical model proposed for determining the resistance to heat flow through the sphere (Fig. 1c), is based upon Eqs. (1, 2, and 5), which state that for elastically deformed spheres, $\omega(r) \ll \rho$ and $a \ll \rho$. The problem is solved considering that the sphere is replaced by a conductor of infinite extent bounded by two parallel planes $z = \pm\rho$, with two circular isothermal contact spots applied to these planes, so that their centers lie in the z axis. The solution must fulfill the following conditions:

$$\partial^2T/\partial r^2 + (1/r)(\partial T/\partial r) + \partial^2T/\partial z^2 = 0 \qquad -\rho < z < +\rho \quad (6)$$

$$\frac{\partial T}{\partial z} = \begin{cases} 0 & z = \pm\rho \quad r > a \\ Q/2\pi ka(a^2 - r^2)^{1/2} & z = \pm\rho \quad r < a \end{cases} \quad (7)$$

The last boundary condition describes the distribution of the heat sources necessary to the circular surface in order to maintain its temperature uniform. The solution that satisfies Eq. (6) is

$$T = \int_0^\infty [\phi(m)e^{mz} + \varphi(m)e^{-mz}]J_0(mr)dm \quad (8)$$

where $\phi(m)$ and $\varphi(m)$ are arbitrary functions of m. Without loss of generality, T may be assumed to be zero when $z = 0$, and therefore $\phi(m) = -\varphi(m)$. Thus, Eq. (8) becomes

$$T = \int_0^\infty 2\phi(m)\sinh(mz)J_0(mr)dm \quad (9)$$

The other two boundary conditions can be satisfied by[3]

$$\int_0^\infty \sin(ma)J_0(mr)dm = \begin{cases} 0 & r > a \\ (a^2 - r^2)^{-1/2} & r < a \end{cases} \quad (10)$$

Hence, if we take

$$\phi(m) = (Q/4\pi ka)[\sin(ma)/\cosh(m\rho)]/m \quad (11)$$

both conditions will be satisfied and the solution is

$$T = \frac{Q}{2\pi ka}\int_0^\infty \frac{\sinh(mz)}{\cosh(m\rho)}\sin(ma)J_0(mr)\frac{dm}{m} \quad (12)$$

From this, one can obtain an approximation to the thermal resistance between the isothermal contact spots:

$$T_1 - T_2 = \frac{Q}{2ka} - \frac{Q}{\pi ka}\int_0^\infty \frac{2e^{-2m\rho}}{1+e^{-2m\rho}} J_0(mr)\sin(ma)\frac{dm}{m} \quad (13)$$

To obtain a better approximation, the integrand in Eq. (13) can be expanded in powers of r and a. If terms of the order $(a/\rho)^3$ and higher are neglected, the result is

$$R_c \equiv (T_1 - T_2)/Q = 1/2ka - \ln 2/\pi k\rho \quad (14)$$

which is the total resistance between the planes. The thermal resistance of the deformed sphere on a unit area basis can be expressed in terms of ϵ as

$$R_c = 0.885(1 - 0.433\epsilon)/k\epsilon \quad (15)$$

When the contact radius is small relative to the sphere radius ($\epsilon \ll 1$), which is true for elastically deformed spheres, the thermal contact resistance is due entirely to the constriction of the heat flow lines,[4] and the total resistance becomes $R_c = 0.885/k\epsilon$.

Conduction through the Gas

In the continuum limit ($Kn \to 0$), we may apply Fourier's heat conduction equation

$$q_\infty = \bar{K}(T_s - T_p)/u \quad (16)$$

where q_∞ is the heat conducted per unit time from unit surface area of the sphere to the plane or vice versa, and u is the separation. The gas thermal resistance between parallel planes in the continuum limit is defined as

$$R_\infty = u/\bar{K} \quad (17)$$

In the free molecule regime where the mean free path in the gas is large enough so that the number of collisions between molecules is small in comparison with the number of collisions between the molecules and the walls ($Kn \gg 1$), for small temperature differences,[5]

$$q_{fm} \cong \frac{\alpha}{2}\frac{(\gamma+1)}{(\gamma-1)}\left(\frac{R_0}{2\pi MT_p}\right)^{1/2} P(T_s - T_p) \quad (18)$$

where

$$\alpha = \alpha_1\alpha_2/\alpha_1 + \alpha_2 - \alpha_1\alpha_2 \quad (19)$$

where α_1 and α_2 are the values of the accommodation coefficient for the two surfaces.

The gas thermal resistance in the free molecule regime is dependent upon the gas properties, the accommodation coefficient, and the gas pressure:

$$R_{fm} = \{(R_0/2\pi MT_p)^{1/2}P\alpha(\gamma+1)/2(\gamma-1)\}^{-1} \quad (20)$$

The heat flux in the regime between the free molecule ($Kn \gg$ 1) and the continuum regime ($Kn \to 0$) can be determined from Sherman's[6] "universal" transition curve

$$q/q_\infty = 1/(1 + q_\infty/q_{fm}) \quad (21)$$

The gas thermal resistance in the transition regime can now be written as

$$R_g = R_\infty(1 + q_\infty/q_{fm}) \quad (22)$$

It can be shown that in the limit as $P \to P_\infty$, $q_{fm} \gg q_\infty$ and $R \to R_\infty$. Also in the limit as $P \to 0$, $q_{fm} \ll q_\infty$ and $R \to (q_\infty/q_{fm}) = R_{fm}$.

We can determine the average thermal resistance of the gas in the separation zone, assumed to be composed of many annuli having different separations, by determining the average continuum gas resistance over an area defined by $r/a = \eta$, where $\eta > 1$. The average continuum resistance, as a first approximation, can be evaluated from

$$\bar{R}_\infty = \frac{1}{\pi(\eta a)^2}\int_{r=a}^{r=\eta a}\left(\frac{u}{\bar{K}}\right)2\pi r\,dr \quad (23)$$

where u the separation is given by Eq. (3). Letting $\nu = r/a$, the expression for the average gas resistance takes the form

$$\bar{R}_\infty = \frac{2\epsilon^2}{\bar{K}\eta^2}\int_1^\eta \left\{\frac{\nu^2}{2} - \left[\frac{(2-\nu^2)\sin^{-1}}{\pi}\frac{1}{\nu} + \frac{(\nu^2-1)^{1/2}}{\pi}\right]\right\}\nu d\nu \quad (24)$$

For $\nu \gg 1$, and since the integrand is zero at $\eta = 1$,

$$\bar{R}_\infty = \frac{\epsilon^2(\eta^2 - 5.1/\eta)}{7.1\bar{K}} \quad (25)$$

We can now write the average gas resistance between the sphere and plane or vice versa over the entire gas pressure range, from the free molecule to continuum regime, as

$$\bar{R}_g = \epsilon^2(1 + q_\infty/q_{fm})(\eta^2 - 5.1/\eta)/7.1\bar{K} \quad (26)$$

Radiation Transfer between Sphere and Plane

For small temperature differences, the radiation heat flow between the isothermal plane and the sphere can be written in a linearized form,[7]

$$Q = A_pF_{ps}4\sigma T_{avg}^3(T_s - T_p) \quad (27)$$

where A_p is the projected area of the sphere on the plane, T_{avg} is the average absolute temperature between sphere and plane, and F_{ps} is the radiation exchange factor between the plane and sphere (or vice versa) which depends upon the surface conditions of the plane and sphere, the geometry of contact, and the interfaces that exist on the sides of the sphere. It is further assumed that one plane sees very little of the second plane because the separation between spheres is small. The thermal resistance for the radiation mode can now be written as

$$R_r = 1/A_pF_{ps}4\sigma T_{avg}^3 \quad (28)$$

Summary and Conclusions

When the thermal resistances of the three modes (conduction through the sphere, conduction through the gas) and radiation between sphere and plane are of the same order of magnitude, the over-all thermal resistance between isothermal planes can be determined from

$$R = 1/(1/R_c + 1/2R_g + 1/2R_r) \quad (29)$$

The Knudsen number, for which the gas resistance is an order of magnitude larger than the metal resistance, can be determined from Eqs. (15) and (26). The ratio q_∞/q_{fm} can be approximated by $Kn(3/\alpha)(T_p/T_{\bar{K}})^{1/2}$. For small temperature differences, $T_{\bar{K}}$, at which $\bar{K}$ is the heat conductivity, is very close to the mid-temperature of T_s and T_p. The factor $(T_{\bar{K}}/T_p)^{1/2}$ can be expected to range between 0.7 and 0.9.

The Knudsen number can now be related to the gas and metal thermal conductivities as well as the accommodation coefficient

$$Kn = \frac{\alpha}{3}\left[1460\frac{\bar{K}}{k-1}\right]\left(\frac{T_{\bar{K}}}{T_p}\right)^{1/2} \quad (30)$$

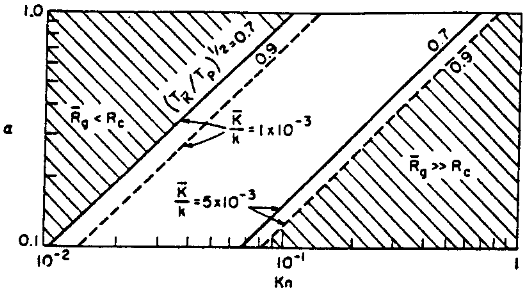

Fig. 2 Limits on Kn for α and $\bar{K}/k$.

Figure 2 shows the relationship between Kn, α, and the ratio $\bar{K}/k$ with $(T_{\bar{K}}/T_p)$ as an additional parameter. For all values of $\bar{K}n$ to the right of the curves, the gas resistance relative to the metal resistance can be neglected.

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Viscosity Correlation for Para-Hydrogen in the Gaseous and Liquid States

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Nomenclature

- h = Planck's const, 6.6240×10^{-27} erg-sec/molecule
- m = mass/molecule, g
- P, P_c = pressure and critical pressure, respectively, atm
- P_R = reduced pressure P/P_c
- P_t = pressure at triple point
- R = gas const, 82.055 cm³ atm/g-mole °K
- T, T_c = temperature and critical temperature, respectively, °K
- T_R = reduced temperature T/T_c
- T_t = triple point temperature, °K
- v_c = critical volume, cm³/g-mole
- z_c = critical compressibility factor P_cv_c/RT_c
- ϵ = maximum energy of attraction for Lennard-Jones potential, ergs/molecule
- Λ^* = quantum mechanical parameter, $h/\sigma(m\epsilon)^{1/2}$
- μ, μ^* = viscosity and gas viscosity at 1 atm, respectively, ~cp
- μ_c, μ_R = critical viscosity, cp, and reduced viscosity μ/μ_c, respectively
- ρ, ρ_c = density and critical density, respectively, g/cm³
- ρ_R = reduced density ρ/ρ_c
- σ = collision diameter, cm

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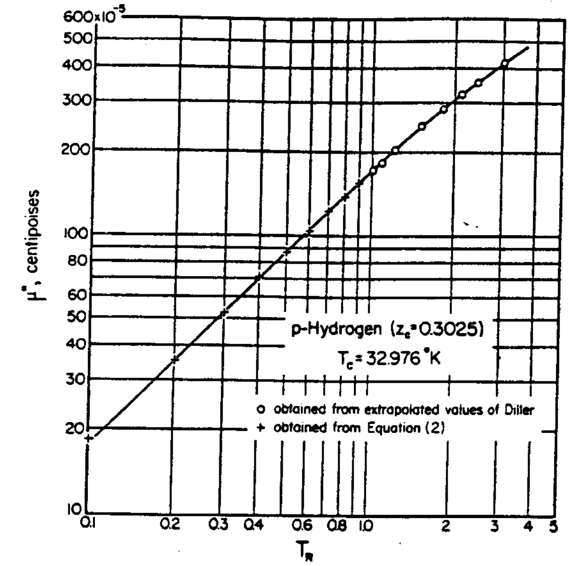

Fig. 1 Viscosity of gaseous p-hydrogen at atmospheric pressure and cryogenic temperatures.

Introduction

At cryogenic temperatures, liquefied normal hydrogen (n-H_2) transforms to the para-form (p-H_2) with the evolution of heat which is dissipated through the vaporization of liquid hydrogen. Therefore, for rocket use, the ortho-form (o-H_2) present in normal hydrogen is catalytically converted to the para-form before liquefaction, and knowledge of the physical properties of p-H_2 is important to designers. The viscosity behavior of p-H_2 is of particular interest with regard to pumping requirements. This note reports a reduced state viscosity correlation based on recent measurements for p-H_2 reported by Diller.[1] Coremans et al.,[2] using an oscillating disk viscometer, obtained measurements

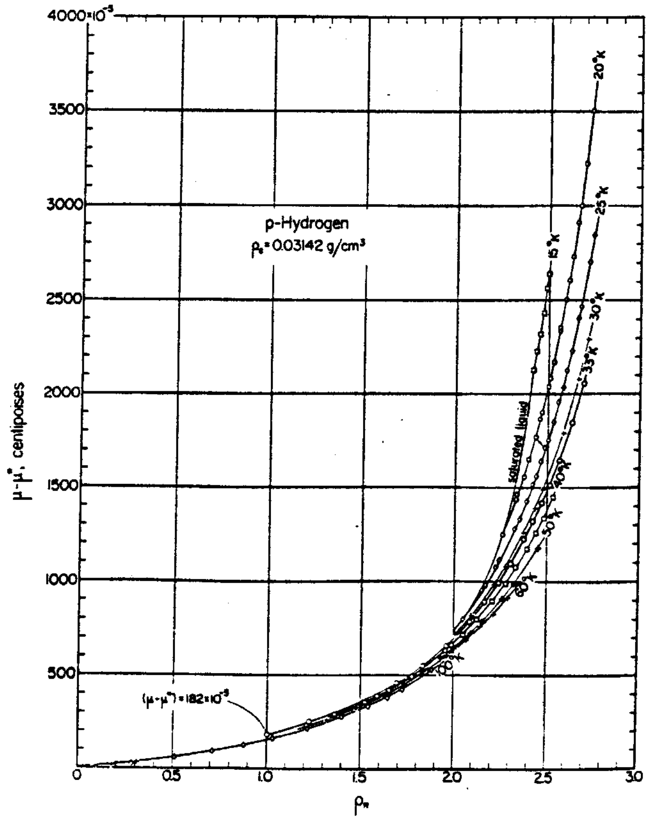

Fig. 2 Constant temperature relationships between μ-μ^* and ρ_R for the gaseous and liquid states of para-hydrogen.