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M.M. Yovanovich, Univ. of Waterloo
Waterloo, Ontario, Canada

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NATURAL CONVECTION FROM ISOTHERMAL SPHEROIDS

IN THE CONDUCTIVE TO LAMINAR FLOW REGIMES

M.M. Yovanovich[1]
Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada
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Abstract

A simple correlation equation based on the linear superposition of the diffusive and laminar boundary-layer limits is presented for prolate and oblate spheroids and spheres. The characteristic body length is based upon the diffusive limit characteristic length which is the square root of the total surface area. The accuracy of the simple correlation equation is as accurate or more accurate than the more complex correlation equations proposed by Raithby-Hollands and Hollands-Hassani for these bodies.

Nomenclature

A	surface area of the body
$\sqrt{A}$	characteristic length of the body
AR	aspect ratio of oblate and prolate spheroids
B	correlation coefficient or major axis of spheroid
C	correlation coefficient or minor axis of spheroid
$C_{\sqrt{A}}$	correlation coefficient for air based on $\mathcal{L} = \sqrt{A}$
$\overline{C_\ell}$	Raithby-Hollands parameter
$\overline{C_t}$	Hassani-Hollands parameter
$\hat{C}_t$	$\overline{C_t}\sqrt{A}/H$
e	eccentricity of the oblate and prolate spheroids $[e = \sqrt{1-u^2}]$
f_o	conversion factor for oblate spheroids
f_p	conversion factor for prolate spheroids
$F(Pr)$	function of Prandtl number $[0.670/[1 + (0.492/Pr)^{9/16}]^{4/9}]$
g	scalar gravitational acceleration
$G_{\sqrt{A}}$	laminar boundary layer body-gravity function $\left[\frac{1}{A}\int\int_A \left(\frac{P}{\sqrt{A}}\sin\theta\right)^{1/3} dA\right]^{3/4}$
$Gr_{\mathcal{L}}$	Grashof number $[g\beta(T_0 - T_\infty)\mathcal{L}^3/\nu^2]$
h	heat transfer coefficient
H	Hassani-Hollands characteristic length in Rayleigh number only
k	thermal conductivity
$\mathcal{L}$	characteristic length of the body
m	correlation coefficient
n	Churchill-Usagi parameter in Hassani-Hollands correlation
$Nu_{\mathcal{L}}$	Nusselt number $[Nu_{\mathcal{L}} = h\mathcal{L}/k]$
Nu_ℓ	thin laminar boundary layer Nusselt number of Raithby-Hollands
Nu_t	turbulent boundary layer Nusselt number of Raithby-Hollands
Nu^∞	diffusive limit Nusselt number as $Ra \to 0$
$Nu^\infty_{\sqrt{A}}$	diffusive limit Nusselt number based on $\sqrt{A}$
p	Churchill-Usagi parameter in Hassani-Hollands correlation
P	local perimeter of the body
Pr	Prandtl number $[\nu/\alpha]$
$Ra_{\mathcal{L}}$	Rayleigh number $[Gr_{\mathcal{L}}Pr = g\beta(T_0 - T_\infty)\mathcal{L}^3/\nu\alpha]$
T_0	uniform boundary temperature
T_∞	fluid temperature remote from the body
u	ratio of minor to major axes of spheroids

Greek Letters

α	thermal diffusivity
β	thermal compressibility coefficient
ν	kinematic viscosity
θ	angle between outward body normal and gravity vector
ρ	mass density

Introduction

The current state of knowledge about free convection heat and mass transfer from three-dimensional bodies of arbitrary shape is somewhat incomplete. A variety of theoretical expressions, graphical correlations and empirical equations have been developed to represent the coefficients for heat and mass transfer. However, the discrepancies between the expressions proposed for correlations and the different sets of experimental data have still not been completely resolved or explained. The theoretical results are mostly limited to the range of Rayleigh number for which the postulates of laminar boundary-layer theory are applicable, i.e. $(10^4 \le Ra \le 10^8)$. A satisfactory theory [1] has now been developed for the diffusive regime $(Ra \le 10^{-4})$. But a completely satisfactory theory for the transition from the diffusive regime to the laminar regime $(10^{-4} \le Ra \le 10^4)$, the laminar regime $(10^4 \le Ra \le 10^8)$ or the turbulent regime $(Ra \ge 10^9)$, is presently unavailable for bodies of arbitrary shape and aspect ratio.

[1]Professor, Associate Fellow, AIAA

The primary shortcoming of the empirical correlations is their failure to properly take into account the shape, aspect ratio and orientation of the bodies as well as using the most physically correct characteristic length in the Nusselt and Rayleigh numbers. There is a great need for empirical data and correlations of natural convection from isothermal bodies of arbitrary shape in the Rayleigh number range: $10^{-8} \le Ra \le 10^{8}$. With the exception of the sphere and the circular cylinder, very little or no information is available for other geometries in the Rayleigh number range of interest. Raithby, Pollard, Hollands and Yovanovich [2] reported free convection data for two oblate spheroids and a single prolate spheroid for the Rayleigh number ranges: $10^4 \le Ra \le 2.7 \times 10^6$ (oblate of aspect ratio $AR = 0.5$), $2 \times 10^4 \le Ra \le 2.8 \times 10^6$ (oblate of aspect ratio $AR = 0.1$) and $4 \times 10^3 \le Ra \le 3.3 \times 10^6$ (prolate of aspect ratio $AR = 1.93$). The accuracy of the spheroidal test data and the ranges of the Rayleigh number have been increased by the recent work of Hassani [19] and they have been presented in a recent paper [16]. The oblate data for $AR = 0.5$ are now valid for $33 \le Ra \le 2.1 \times 10^7$ and the oblate data for $AR = 0.1$ are valid over the range $177 \le Ra \le 2 \times 10^7$, while the prolate data for $AR = 1.93$ are now valid over the range $65 \le Ra \le 2.88 \times 10^7$. Chamberlain, Hollands and Raithby [6,7] reported results for natural convection into air from small and large spheres in the range: $50 \le Ra \le 10^7$. Yuge [4] and Amato and Tien [5] have reported data for natural convection from isothermal spheres into air and water respectively. Their results fall in the Rayleigh number ranges discussed above.

Most authors use the diameter for the sphere and aligned bi-sphere; the side of the cube, or the major axis for the oblate and prolate spheroids. When these lengths are employed in the Nusselt and Rayleigh numbers, the data plot as separate curves. The oblate and prolate data [2] are shown plotted in Figures 2-5. In each case the same characteristic length was used in both Nu and Ra numbers. It can be seen in Figure 2 that King's length [11] places the data in the Rayleigh number range: $10 \le Ra \le 10^5$, and in Figure 3 it can be seen that the Sparrow-Stretton length [8,9] places the same data in the Rayleigh number range: $10^4 \le Ra \le 10^6$. These lengths appear to give the extreme ranges on the Rayleigh number. On the other hand the classical characteristic length, the major axis, places the data in the Rayleigh number range: $4 \times 10^3 \le Ra \le 6 \times 10^6$ as seen in Figure 4. While this length appears to be more appropriate than the other two lengths because the plotted results are closer together, the data still appear as three separate groups. From these plots, the authors have concluded that each shape is different and, therefore, each will require a separate correlation equation. In fact, in a recent heat transfer text, Kreith and Bohn [13] state *"It is likely that no such simple characteristic length will collapse data for a wide range of geometric shapes and that a separate correlation equation may be required for each shape"*.

In two recent papers Sparrow and Ansari [10] and Sparrow and Stretton [9] demonstrated that King's rule is incorrect and should not be used. They found Lienhard's length [12] to be better than King's length, but it also gave separate plots for the various shapes. Sparrow and Stretton proposed a length which worked very well with the unity aspect ratio results: sphere, cube and short cylinder data; but failed when it was compared with the non-unity aspect ratio results: oblate, prolate and aligned bi-sphere data.

This paper presents a simple, correlation equation for the space-mean value of the Nusselt number for free convection from isothermal spheroids in the diffusive to laminar flow regimes ($0 < Ra \le 10^8$). The correlation equation is shown to be related to the general solutions developed by similarity methods for $Pr \to \infty$ and the approximate method developed by Raithby and Hollands [14] which is valid for all values of Pr.

Correlation Equations

Natural convection heat transfer correlation equations have been developed for isothermal bodies losing heat to an extensive, stagnant fluid for Rayleigh numbers of the order of 10^5 to 10^8. In this range of Rayleigh number, correlation equations adequately predicted the data when they are expressed in the simple form:

$$Nu = C Ra^m \tag{1}$$

where the correlation coefficients C and m are observed to be approximately 0.45 and 1/4, respectively, for a sphere in air [3,4,7] when the sphere diameter is used as the characteristic length in the Nusselt and Rayleigh numbers.

The inadequacy of the simple form to predict data in the extended range $10 < Ra < 10^8$ has lead researchers to consider the usefulness of the following correlation equations:

$$Nu = B + C Ra^m \tag{2}$$

and

$$Nu = Nu^\infty + C Ra^m \tag{3}$$

The parameters B, C and Nu^∞ in the above equations are dependent on the Rayleigh number range, the body length used in both Nusselt and Rayleigh numbers, and whether the parameter m has been set to the theoretical boundary layer value of 1/4. The correlation coefficient, B, in Eq. (2) represents the intercept value resulting from linear least-squares data fitting, whereas the Nu^∞ in Eq. (3) represents the contribution of molecular diffusion into an infinite, stagnant fluid which corresponds to Ra approaching zero; this is called the diffusive limit.

Amato and Tien [5] have reported correlation equations developed for both heat and mass transfer from spheres into a variety of fluids. One set of correlation equations is based on $\mathcal{L} = D$, $Nu^\infty = 2$, $m = 1/4$, and the fitted coefficient, C, was found to lie in the range, $0.399 \le C \le 0.59$. Two correlation equations were based on $\mathcal{L} = D$, $m = 1/4$, and $B = 5.4$ or 2.3, with $C = 0.44$ or 0.585. The third set of correlation equations were based on $\mathcal{L} = D$, $m = 1/4$, $B = 0$, and the fitted correlation coefficient, C, was found to lie in the range, $0.51 \le C \le 0.56$. Amato and Tien [5] used Eq. (3) with $\mathcal{L} = D$, $Nu^\infty = 2$, $m = 1/4$, and determined $C = 0.500$ for heat transfer into water. They reported a mean deviation of less than 11% provided $3 \times 10^5 \le Ra_D \le 8 \times 10^8$.

Shell [17] calculated the mean value of Nu for a sphere in air and found that

$$Nu_D = 0.429 Gr_D^{1/4} \tag{4}$$

where Gr_D is the Grashof number. This expression was confirmed by measurements in air. Assuming a value of 0.71 for the Prandtl number, the correlation coefficient in Eq. (4) becomes 0.467.

Churchill and Chu [20] have used the diffusive limit and the boundary-layer asymptote in the following *blended* form to increase the range and accuracy of the correlation equation:

$$Nu = [B^n + (CRa^{1/4})^n]^{1/n} \tag{5}$$

The parameter m was set to the boundary-layer value of 1/4. They found that flat plate and horizontal circular cylinder data could be predicted accurately with a value of $n = 1$ provided $Ra < 10^8$.

Raithby and Hollands [14] have used the Churchill-Usagi [21] method of *blending* limiting solutions to correlate free convection from bodies of complex shape. They recommend the equation

$$Nu = (Nu_l^n + Nu_t^n)^{1/n} \tag{6}$$

where Nu_l and Nu_t are the Raithby-Hollands method solutions for the thin laminar and turbulent boundary layers respectively. This method was used with success [7,14] to correlate air data for two spheres, an aligned bi-sphere and a cube in three orientations. As an example of this method, the sphere correlation equation is presented here to illustrate several points discussed above. The sphere correlation equation is

$$Nu = [(2 + 0.452Ra^{1/4})^6 + (0.099Ra^{1/3})^6]^{1/6} \tag{7}$$

where the characteristic length is the sphere diameter, $Nu^\infty = 2$, and the Churchill-Usagi parameter $n = 6$. They reported excellent agreement between these correlation equations and the air data of Chamberlain [7]. The Churchill-Usagi correlation parameter used by Raithby and Hollands must be determined empirically to give the best fit to the data.

In a recent paper Hassani and Hollands [17] have modified and simplified the Raithby-Hollands method by introducing the diffusive limit $Nu_{\sqrt{A}}^\infty$ of Yovanovich [1] and the characteristic length, $\sqrt{A}$, proposed by Yovanovich in the Nusselt numbers which appear in their correlation equation. They also introduced another characteristic length, H, in the laminar and turbulent Rayleigh numbers which are in the correlation equation. This new length is seen to be closely related to the length proposed by Yovanovich for all body shapes where $\sqrt{A}/H = \pm 5\%$, and 9% for the bi-sphere. The largest difference of 37.5% is observed for the horizontal, circular disk. They proposed the correlation equation:

$$Nu_{\sqrt{A}} = \left[\left[[\overline{C_l} Ra_H^{1/4}]^p + [\hat{C}_t Ra_H^{1/3}]^p \right]^{n/p} + [Nu_{\sqrt{A}}]^n \right]^{1/n} \tag{8}$$

where n and p are a new set of Churchill-Usagi parameters. As anticipated the choice of $\sqrt{A}$ as the characteristic length has reduced the range of the parameter n from 1.01 to 1.14 for the variety of body shapes considered in their paper. The other parameter p is determined by means of another correlation equation developed by Hassani and Hollands [16]. The laminar Rayleigh number coefficient, $\overline{C_l}$, is reported to be a constant for all body shapes, and the turbulent Rayleigh number coefficient, $\hat{C}_t = \overline{C_t}\sqrt{A}/H$, has values ranging between 0.090 for the cube in orientation 1 and 0.114 for the bi-sphere.

There is at present no theoretical basis to support the use of one correlation equation over another for a range of body shapes, aspect ratio, orientation and a wide range of the Rayleigh number. It is, therefore, proposed to compare the simplest correlation equation which is based on the linear superposition of the Yovanovich diffusive limit [1] and the laminar boundary layer limit based on the Yovanovich characteristic length with air data for a variety of body shapes, aspect ratio and orientation. The proposed simple correlation equation is

$$Nu_{\sqrt{A}} = Nu_{\sqrt{A}}^\infty + C_{\sqrt{A}} Ra_{\sqrt{A}}^{1/4} \tag{9}$$

where $Nu_{\sqrt{A}}^\infty$ is the diffusive limit and $C_{\sqrt{A}}$ is an empirical correlation coefficient to be determined from the air data for $10^4 \le Ra_{\sqrt{A}} \le 10^8$ or all data.

The simple correlation equation can be modified to account for Prandtl numbers different from the air value by the use of the Prandtl number function $F(Pr)$ defined as [9,14]

$$F(Pr) = \frac{0.670}{[1 + (0.492/Pr)^{9/16}]^{4/9}} \tag{10}$$

which was developed by Churchill and Chu [20] and used effectively by Sparrow and Stretton [9] to correlate their air and water data. This function has the value of 0.514 for $Pr = 0.71$. The laminar Rayleigh number coefficient, $C_{\sqrt{A}}$, [18] is related to the laminar boundary layer body-gravity function $G_{\sqrt{A}}$ which is dimensionless and the Prandtl number function, Eq.(10), [18]:

$$C_{\sqrt{A}} = F(Pr) G_{\sqrt{A}} \tag{11}$$

and

$$G_{\sqrt{A}} = \left[\frac{1}{A} \iint_A \left(\frac{P}{\sqrt{A}} \sin\theta \right)^{1/3} dA \right]^{3/4} \tag{12}$$

can be derived from the boundary layer equations by similarity methods for $Pr \to \infty$ for any arbitrary body shape which does not possess horizontal planes, corners, or surface depressions. This new function can also be derived from the relationship developed by the Raithby-Hollands approximate method [14] by assuming $\mathcal{L} = \sqrt{A}$ is the characteristic body length and performing the integration over the total body surface. The geometric parameter P which appears in the integrand is the local perimeter of the body, and θ is the local angle between the outward normal to the body surface and the direction of the gravity vector. This function can be evaluated analytically for the oblate and prolate spheroids, and some other simple body shapes. It is noted [18] that it is a relatively weak function of the body shape, aspect ratio and orientation for a relatively wide range of these geometric parameters.

Results and Discussion

The air data of Nu and Ra for isothermal spheres [6,7], oblate and prolate spheroids [2,16,19], are converted to $Nu_{\sqrt{A}}$ and $Ra_{\sqrt{A}}$. The sphere data are converted by the conversion factors: $\sqrt{\pi}$ and $\pi^{3/2}$ for Nu and Ra respectively.

The relationships for conversion of the oblate spheroidal data are

$$Nu_{\sqrt{A}} = f_o(u) Nu_B \tag{13}$$

and

$$Ra_{\sqrt{A}} = f_o^3(u) Ra_B \tag{14}$$

where

$$f_o(u) = \sqrt{\frac{\pi}{2}}\left[1 + \frac{(1-e^2)}{2e}\ln\left(\frac{1+e}{1-e}\right)\right]^{1/2} \tag{15}$$

and

$$e = \sqrt{1-u^2} \tag{16}$$

$$u = \frac{C}{B} = AR \tag{17}$$

The relationships for the conversion of the prolate spheroidal data are

$$Nu_{\sqrt{A}} = u f_p(u) Nu_B \tag{18}$$

and

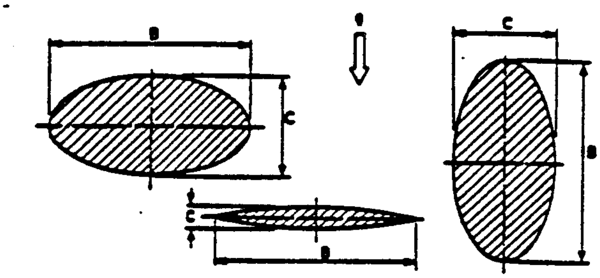
$$Ra_{\sqrt{A}} = u^3 f_p^3(u) Ra_B \tag{19}$$

where

$$f_p = \sqrt{\frac{\pi}{2}}\left[1 + \frac{\sin^{-1} e}{e\sqrt{1-e^2}}\right]^{1/2} \tag{20}$$

and e and u are defined in Eqs. (16) and (17), but for prolate spheroids the aspect ratio, $AR = 1/u$.

Log-log plots of $Nu_{\sqrt{A}}$ versus $Ra_{\sqrt{A}}$ for the various body shapes: prolate spheroid ($AR = 1.93$), spheres, thick oblate spheroid ($AR = 0.50$), and thin oblate spheroid ($AR = 0.1$) shown in Figure 1 are presented in Figures 2-9. Figures 2-5 clearly illustrate the effect of various characteristic body lengths on the plotting of the spheroidal air data [2,19]. The body length proposed by King [11] yields the lowest values of Nu and Ra, and the body length proposed by Sparrow and Stretton [16] yields the maximum values of Nu and Ra. The other two characteristic lengths, major axis of the spheroids and $\mathcal{L} = \sqrt{A}$ produce intermediate values of Nu and Ra as seen in Figs. (4,5). The major axis of the spheroids and the new diffusive length $\mathcal{L} = \sqrt{A}$ are clearly seen to be superior to the other two lengths. The diffusive length was shown to be superior to the major axis [1] or any other length based on a single body dimension.



	C/B = 0.5 OBLATE SPHEROID	C/B = 0.1 OBLATE SPHEROID	B/C = 1.93 PROLATE SPHEROID
C (mm)	39.2	8.00	40.0
B (mm)	78.6	80.0	77.1

Fig 1 Cross section and dimensions of prolate and oblate spheroids

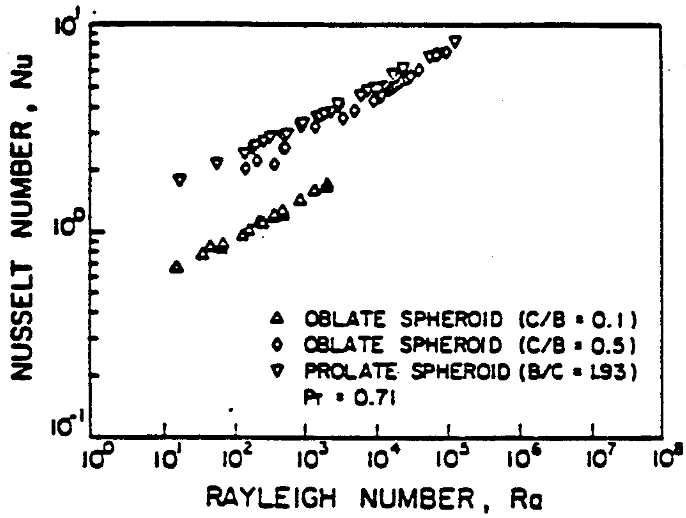


Fig 2 Effect of King's characteristic length on a spheroidal air data [2]

The data points for $10^4 \leq Ra_{\sqrt{A}} \leq 10^8$ were used with the following equation to obtain a set of values of the laminar boundary-layer correlation coefficients:

$$\left(C_{\sqrt{A}}\right)_i = \frac{(Nu_{\sqrt{A}})_i - Nu^{\infty}_{\sqrt{A}}}{\left(Ra^{1/4}_{\sqrt{A}}\right)_i} \tag{21}$$

where i denotes the ith data point and $Nu^{\infty}_{\sqrt{A}}$ is the diffusive limit of Yovanovich [1] reported in Table 2.

It can be seen that the largest difference in the values of $Nu_{\sqrt{A}}$ is between the thin oblate spheroid of aspect ratio $AR = 0.1$ and the prolate spheroid of aspect ratio $AR = 1.93$ and this relative difference is approximately 6.7%. The arithmetic average of the $(C_{\sqrt{A}})_i$ for each body shape as well as correlation coefficients obtained by means of a least-squares method are given in Table 3. A least-squares method applied to all data yielded coefficients which are within ±2% of those determined

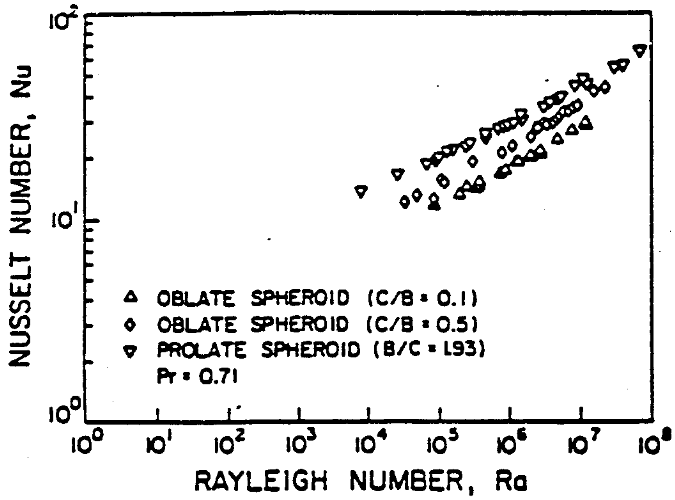


Fig 3 Effect of Sparrow-Stretton length on spheroidal air data [2]

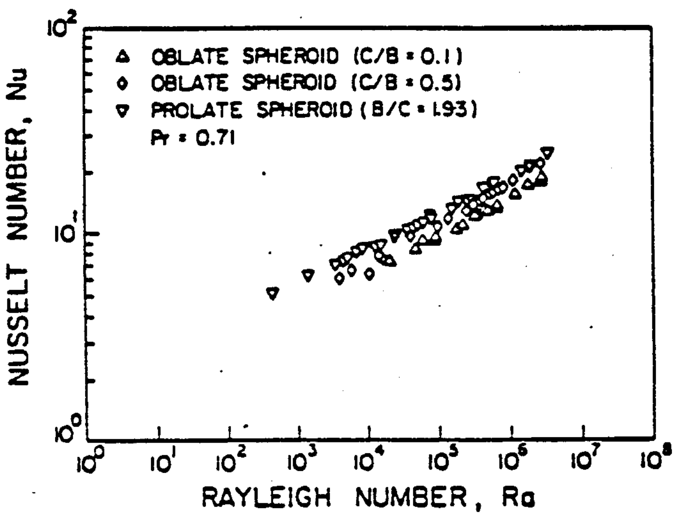


Fig 4 Effect of major axis on spheroidal air data [2]

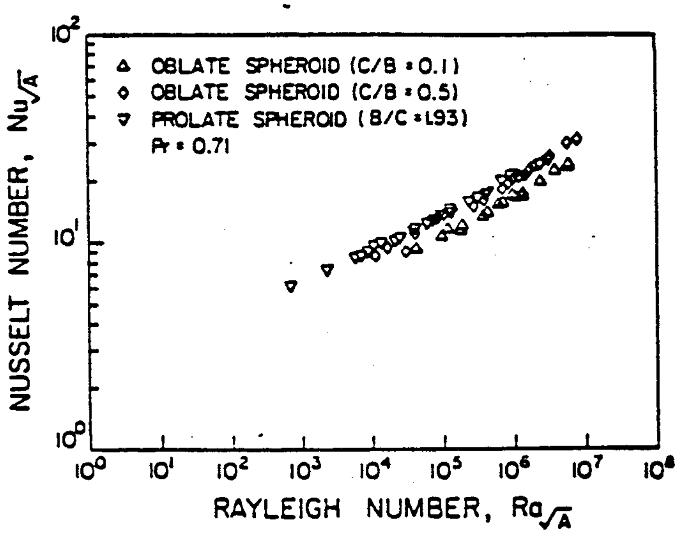


Fig 5 Effect of diffusive body length on spheroidal air data [2]

Table 1: Nusselt Numbers at Zero Rayleigh Number for Various Body Shapes [1]

Body Shape	$Nu^{\infty}_{\sqrt{A}}$
Sphere	3.545
Prolate Spheroid (AR = 1.93)	3.566
Oblate Spheroid (AR = 0.5)	3.529
Oblate Spheroid (AR = 0.1)	3.342

Table 2: Correlation Coefficients for the Spheroids

Body Shape	$C_{\sqrt{A}}$
Sphere	0.526
Prolate Spheroid (AR = 1.93)	0.520
Oblate Spheroid (AR = 0.5)	0.500
Oblate Spheroid (AR = 0.1)	0.395

by the method described above. The average method is recommended because the coefficients give more accurate results.

These empirically determined coefficients are remarkably similar considering the variety of body shapes, their aspect ratios and orientations. As anticipated the largest value is associated with the prolate spheroid which is a streamlined body and the smallest value is associated with the thin oblate spheroid which is a blunt body. The prolate coefficient is approximately 32% larger than the thin oblate coefficient. Excluding the thin oblate spheroid, the coefficients lie in the range $0.500 \le C_{\sqrt{A}} \le 0.520$ where the lower bound corresponds to the thick oblate spheroid and the upper bound to the prolate spheroids.

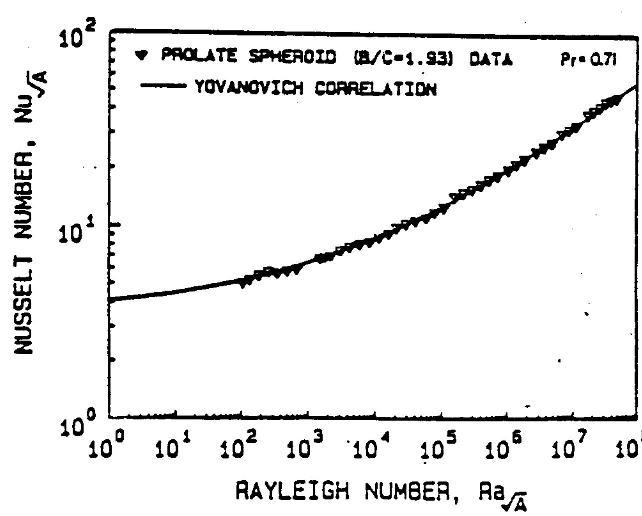


Figure 6 Comparison of simple correlation equation and prolate ($AR = 1.93$) air data [19]

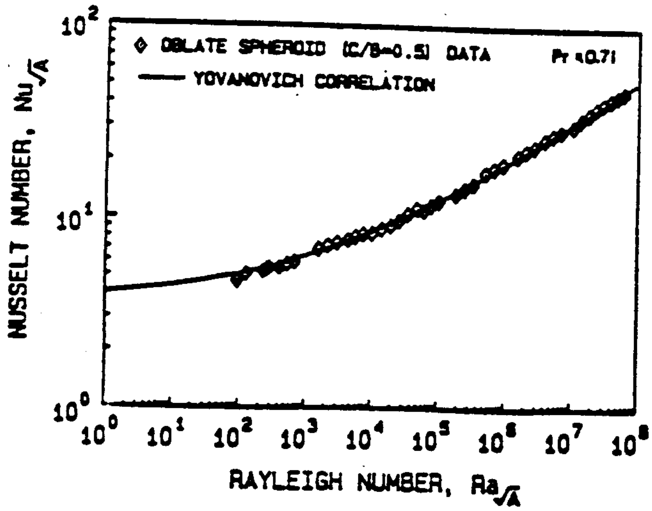


Figure 7 Comparison of simple correlation equation and oblate ($AR = 0.5$) air data [19]

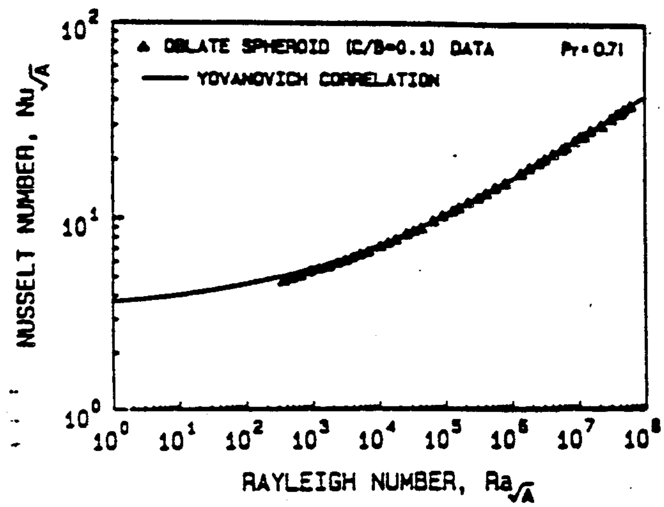


Figure 8 Comparison of simple correlation equation and oblate ($AR = 0.1$) air data [19]

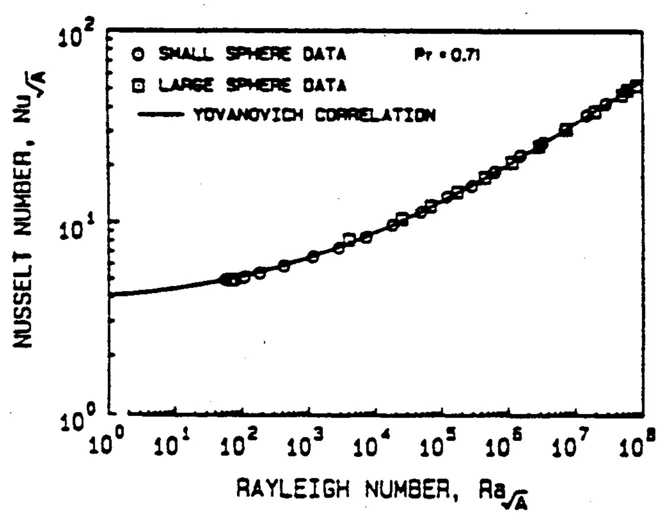


Figure 9 Comparison of simple correlation equation and sphere ($AR = 1.0$) air data [7]

Table 3: Comparison of Simple Correlation with Air Data and Other Correlations

		RMS Percent Difference		
Body Shape	AR	R-H	H-H	MMY
Spheres	1	0.84	3.67	0.92
Prolate Spheroid	1.93	1.90	3.56	1.83
Oblate Spheroid	0.50	3.71	4.09	2.60
Oblate Spheroid	0.10	9.38	-	2.34

The laminar boundary layer coefficients were used in the correlation equation and the predictions are compared with the data in the Figs. (6-9) where it can be seen that agreement is excellent for all body shapes and aspect ratios over the full range of *Ra*.

The RMS percent difference between the data and the corresponding predictions for all body shapes are given in Table 3.

The best agreement between the data and the correlation equation occurs with the sphere where the RMS percent difference is less than 1%. For all other body shapes the RMS percent difference is less than 3%. The RMS percent differences between the data and the predictions of the more complex correlation equations of Raithby-Hollands [14] and Hassani-Hollands [18] are also given in Table 4. Overall the RMS percent differences of the simple correlation equation are as small or smaller than those of the more complex correlation equations of Raithby-Hollands [14] and Hassani-Hollands [18] which purportedly accounts for turbulence effects at high Rayleigh number.

Summary and Conclusions

A simple, but accurate, correlation equation is developed for isothermal prolate and oblate spheroids and spheres. The linear superposition of the diffusive and the boundary-layer limits for the the prediction of the Nusselt number is supported by the very good agreement between the theory and the data over the full range of Rayleigh number, body shapes and aspect ratios.

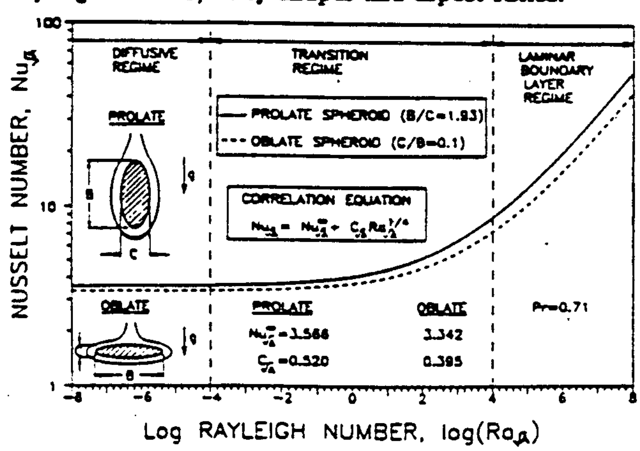


Figure 10 Upper and lower bounds on free convection from isothermal spheroids ($0.1 \leq AR \leq 1.93$)

Figure 10 shows the upper and lower bounds on the Nusselt number in free convection from isothermal spheroids ($0.1 \le AR \le 1.93$) for $10^{-8} \le Ra_{\sqrt{A}} \le 10^{8}$. In the diffusive limit, $R_{\sqrt{A}} \to 0$, the differences between the upper and lower bounds is approximately 7% and in the the laminar flow regime the difference approaches 33%. The difference between the prolate spheroid ($AR = 1.93$) and the oblate spheroid ($AR = 0.50$) is only 1% over the entire range of $Ra_{\sqrt{A}}$.

The contribution of the diffusive limit to the total Nusselt number is significant in the laminar boundary layer regime. At the lower limit where $Ra_{\sqrt{A}} = 10^{4}$, the diffusive limit contributes 40.7% and 45.8% to the total for the prolate and thin oblate spheroids respectively. At $Ra_{\sqrt{A}} = 10^{8}$, the percentages reduce to 6.4% and 7.8% respectively.

The characteristic length based upon the square root of the total surface area appears to be an appropriate length for correlating natural convection from isothermal spheroids.

Simple equations are presented for estimating the single semi-empirical correlation coefficient with acceptable accuracy.

Acknowledgements

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