THERMAL SPREADING RESISTANCES IN RECTANGULAR FLUX CHANNELS PART II - EDGE COOLING

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ABSTRACT

The effect of edge cooling is addressed in flux tubes and flux channels. A new analytical solution is obtained for thermal spreading resistance in a rectangular flux channel with edge cooling. This solution contains many limiting cases, including a previously published solution for adiabatic edges. Comparisons are made with the circular flux tube with edge cooling and with adiabatic edges. Simple relationships are developed for edge cooled systems to assess the important of edge cooling. This alleviates the issue of computing or recomputing eigenvalues when the edge cooling conditions change or have no impact. It is shown that this simple approach provides good results for a wide range of dimensionless parameters.

Keywords: Conduction, Spreading Resistance, Heat Spreaders, Contact Heat Transfer, Electronics Packaging

NOMENCLATURE

a, b	=	radial dimensions, m
a, b, c, d	=	linear dimensions, m
A_b	=	substrate area, m^2
A_s	=	heat source area, m^2

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A_{mn}, B_{mn}	=	Fourier coefficients
Bi	=	Biot number, $h\mathcal{L}/k$
Bi_e	=	Biot number, $h_e\mathcal{L}/k$
Bi_{eff}	=	Biot number, $h_{eff}\mathcal{L}/k$
h	=	contact conductance or film coefficient, $W/m^2 \cdot K$
$J_0(\cdot), J_1(\cdot)$	=	Bessel function of first kind
k	=	thermal conductivity, $W/m\cdot K$
$\mathcal{L} = \sqrt{A_s}$	=	arbitrary length scale, m
m, n	=	indices for summations
Q	=	heat flow rate, $\equiv q_s A_s$, W
q_s	=	heat flux, W/m^2
R	=	thermal resistance, K/W
R_{1D}	=	one-dimensional resistance, K/W
R_s	=	spreading resistance, K/W
R_t	=	total resistance, K/W
$R^\star$	=	dimensionless resistance, $\equiv Rk\mathcal{L}$
t	=	substrate thickness, m
T	=	temperature, K
$\overline{T}_s$	=	mean source temperature, K
T_f	=	sink temperature, K

Greek Symbols

β_{mn}	=	eigenvalues, $\equiv \sqrt{\lambda_{xm}^2 + \lambda_{yn}^2}$
$\delta_{xm}, \delta_{yn}, \delta_n$	=	eigenvalues
ϵ	=	source aspect ratio, $\equiv a/b$
ϵ_x	=	source aspect ratio, $\equiv a/c$
ϵ_y	=	source aspect ratio, $\equiv b/d$
ϵ_b	=	baseplate aspect ratio, $\equiv c/d$
θ	=	temperature excess, $\equiv T - T_f$, K
$\overline{\theta}$	=	mean temperature excess, $\equiv \overline{T} - T_f$, K

$$\lambda_{xm}, \lambda_{yn} \quad = \quad \text{eigenvalues}$$
$$\phi \quad\quad = \quad \text{spreading resistance functions}$$
$$\tau \quad\quad = \quad \text{relative thickness}, \equiv t/\mathcal{L}$$

Subscripts

$b \quad\quad = \quad \text{base}$

$e \quad\quad = \quad \text{edge}$

$eff \quad\quad = \quad \text{effective}$

$m, n \quad\quad = \quad \text{m}^{th} \text{ and n}^{th} \text{ terms}$

$s \quad\quad = \quad \text{source}$

$t \quad\quad = \quad \text{total}$

$x \quad\quad = \quad \text{x-dir}$

$y \quad\quad = \quad \text{y-dir}$

INTRODUCTION

In the second part of this paper, the effect of edge cooling is examined. A review of the literature shows that a number of useful solutions for rectangular flux channels have been obtained for a variety of configurations including: compound and isotropic flux channels, single and multiple eccentric heat sources, and orthotropic spreaders.[1−5] One issue not yet examined, is the effect of edge cooling. This issue has recently been addressed for circular disks[6]. This paper addresses the issue of edge cooling in rectangular flux channels by presenting a new solution. Further, simple expressions are established to show the relative importance of edge cooling in thermal resistance calculations. This is done for both the circular disk and rectangular flux channel. The need for a simple predictive approach for edge cooled systems is a result of the fact that for each unique value of edge heat transfer coefficient, a unique set of eigenvalues must be tabulated, making computations more tedious. However, this is not the case for systems with adiabatic edges. Theoretical results will be presented for a range of parameters.

PROBLEM STATEMENT

Thermal spreading resistance arises in many electronic cooling applications where heat enters a domain through a finite area. In typical applications, the system may be idealized as having a central heat source placed on the upper surface of a substrate or baseplate, while the lower surface is cooled with a constant conductance which may represent a heat sink, contact conductance, or convective heat transfer. In many systems edge cooling may be a significant factor. In the present solution all edges are assumed to be cooled with a constant edge heat transfer coefficient h_e in the case of a circular flux tube, or different edge coefficients $h_{e,x}$ and $h_{e,y}$ in the case of a rectangular flux channel. The region outside of the heat source on the source plane is taken to be adiabatic.

In this idealized system, the total thermal resistance of the system may be defined as

$$R_t = \frac{\overline{T}_s - T_f}{Q} = \frac{\overline{\theta}_s}{Q} \qquad (1)$$

where $\overline{\theta}_s$ is the mean source temperature excess and $Q = q_s A_s$ is the total heat input of the device. The mean source temperature is given by

$$\overline{\theta}_s = \frac{1}{A_s} \iint_{A_s} \theta(x, y, 0) \, dA_s \qquad (2)$$

In applications involving adiabatic edges, the total thermal resistance is composed of two terms: a uniform flow or one-dimensional resistance and a spreading or multi-dimensional resistance, which vanishes as the source area approaches the substrate area. These two components are combined as follows:

$$R_t = R_{1D} + R_s \qquad (3)$$

When edge cooling is present, the resistance remains multi-dimensional for all conditions except for the special case of $h_e = 0$. In this limit, Eq. (3) may be used to determine the spreading resistance component. Otherwise for all other values of h_e, the resistance cannot be separated into these individual components.

Circular Flux Tubes

In the case of a circular flux tube, as shown in Fig. 1, Laplace's equation in circular cylinder coordinates

$$\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (4)$$

must be solved in two dimensions. The following boundary conditions are prescribed:

$$r = 0, \quad \frac{\partial \theta}{\partial r} = 0$$

$$r = b, \quad \frac{\partial \theta}{\partial r} + \frac{h_e}{k} \theta = 0$$

$$z = 0, \quad \frac{\partial \theta}{\partial z} = -\frac{q_s}{k}, \qquad A < A_s \qquad (5)$$

$$\frac{\partial \theta}{\partial z} = 0, \qquad A_s < A < A_b$$

$$z = t, \quad \frac{\partial \theta}{\partial z} + \frac{h}{k} \theta = 0$$
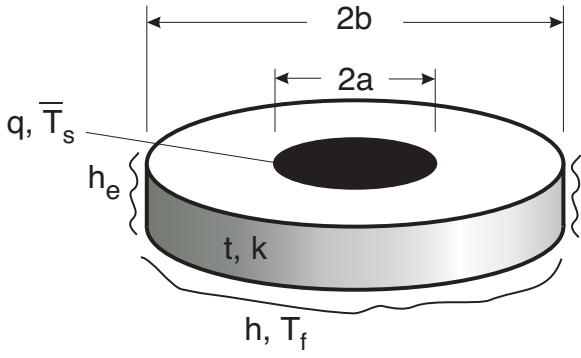
Fig. 1 - Circular Flux Tube with Edge Cooling

The solution to this problem for the total thermal resistance R_t, was recently obtained by one of the authors[6]. It may be written in the following dimensionless form:

$$R_t^\star = \frac{4}{\sqrt{\pi}\epsilon} \sum_{n=1}^{\infty} \frac{J_1^2(\delta_n \epsilon)\phi_n}{\delta_n^3[J_0^2(\delta_n) + J_1^2(\delta_n)]} \qquad (6)$$

and

$$\phi_n = \frac{\sqrt{\pi}\epsilon\delta_n + Bi\tanh(\delta_n\epsilon\sqrt{\pi}\tau)}{Bi + \delta_n\epsilon\sqrt{\pi}\tanh(\delta_n\epsilon\sqrt{\pi}\tau)} \qquad (7)$$

where $R_t^\star = R_t k\sqrt{A_s}$, $\tau = t/\sqrt{A_s}$, $Bi = h\sqrt{A_s}/k$, and $\epsilon = \sqrt{A_s/A_b} = a/b$, and δ_n are the eigenvalues. The eigenvalues are obtained from application of the second boundary condition along the disk edges, and requires numerical solution to the following transcendental equation:

$$\delta_n J_1(\delta_n) = Bi_e J_0(\delta_n) \qquad (8)$$

where $\delta_n = \lambda b$, $Bi_e = h_e b/k$ is the edge Biot number, and $J_0(\cdot)$ and $J_1(\cdot)$ are Bessel functions of the first kind of order zero and one, respectively. A unique set of eigenvalues is computed for each value of Bi_e. Simplified expressions for predicting the eigenvalues were developed by Yovanovich[6] using the Newton-Raphson method.

It is now clear that the dimensionless total resistance depends upon

$$R_t^\star = f(\epsilon, \tau, Bi_e, Bi) \qquad (9)$$

whereas the dimensional total resistance depends upon

$$R_t = f(a, b, t, h_e, h) \qquad (10)$$

Rectangular Flux Channels

In the case of a rectangular flux channel, as shown in Fig. 2, Laplace's equation in cartesian coordinates

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (11)$$

must be solved in three dimensions. The heat source is placed at the centroid of the flux channel which is symmetrically cooled along the edges. The following boundary conditions are prescribed:

$$x = 0, \quad \frac{\partial\theta}{\partial x} = 0$$

$$x = c, \quad \frac{\partial\theta}{\partial x} + \frac{h_{e,x}}{k}\theta = 0$$

$$y = 0, \quad \frac{\partial\theta}{\partial y} = 0$$

$$y = d, \quad \frac{\partial\theta}{\partial y} + \frac{h_{e,y}}{k}\theta = 0 \qquad (12)$$

$$z = 0, \quad \frac{\partial\theta}{\partial z} = -\frac{q_s}{k}, \qquad A < A_s$$

$$\frac{\partial\theta}{\partial z} = 0, \qquad A_s < A < A_b$$

$$z = t, \quad \frac{\partial\theta}{\partial z} + \frac{h}{k}\theta = 0$$

Here $h_{e,x}$ and $h_{e,y}$ denote the values of the edge heat transfer coefficient along the x-edge and y-edge, respectively.
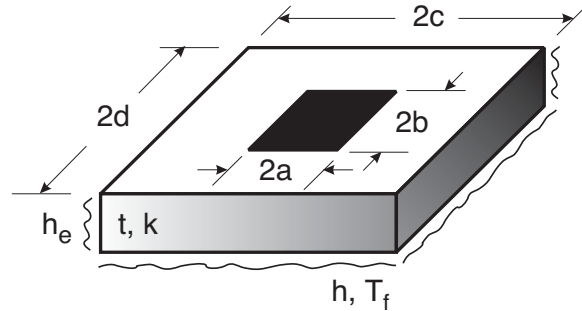

Fig. 2 - Rectangular Flux Channel with Edge Cooling

Presently, no solution exists for this configuration. Thermal spreading resistance in a rectangular flux channel with adiabatic edges was recently obtained by the authors[3]. The solution methodology is the same and the resulting solution is quite similar, with the exception of the definition of the eigenvalues. The solution for an isotropic flux channel with edge cooling may be obtained by means of separation of variables.[9−11]

The solution is assumed to have the form $\theta(x,y,z) = X(x) * Y(y) * Z(z)$, where $\theta(x,y,z) = T(x,y,z) - T_f$. Applying the method of separation of variables yields the following general solution for the temperature excess in the substrate which satisfies the thermal boundary conditions along the two planes of symmetry, $x = 0$ and $y = 0$:

$$\theta(x,y,z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos(\lambda_{xm} x) \cos(\lambda_{yn} y) \times$$
$$[A_{mn} \cosh(\beta_{mn} z) + B_{mn} \sinh(\beta_{mn} z)] \qquad (13)$$

where λ_{xm}, λ_{yn}, and $\beta_{mn} = \sqrt{\lambda_{xm}^2 + \lambda_{yn}^2}$ are the eigenvalues. The eigenvalues are obtained from the following equations:

$$\delta_{xm} \sin(\delta_{xm}) = Bi_{e,x} \cos(\delta_{xm}) \qquad (14)$$

and

$$\delta_{yn} \sin(\delta_{yn}) = Bi_{e,y} \cos(\delta_{yn}) \qquad (15)$$

where $Bi_{e,x} = h_{e,x} c/k$, $\delta_{xm} = \lambda_{xm} c$, $Bi_{e,y} = h_{e,y} d/k$, and $\delta_{yn} = \lambda_{yn} d$. These equations must be solved numerically for a finite number of eigenvalues for each specified value of the edge cooling Biot numbers. The separation constant β_{mn} is now defined as

$$\beta_{mn} = \sqrt{\left(\frac{\delta_{xm}}{c}\right)^2 + \left(\frac{\delta_{yn}}{d}\right)^2} \qquad (16)$$

Application of the lower surface boundary condition yields the following relation

$$A_{mn} = -B_{mn} \cdot \phi_{mn} \qquad (17)$$

where

$$\phi_{mn} = \frac{\beta_{mn} t + \frac{ht}{k} \tanh(\beta_{mn} t)}{\frac{ht}{k} + \beta_{mn} t \tanh(\beta_{mn} t)} \qquad (18)$$

The final Fourier coefficients may be obtained by taking a double Fourier expansion of the upper surface condition. This yields the following expression:

$$B_{mn} = \frac{-(Q/k) \int_0^a \cos(\lambda_{xm} x) dx \int_0^b \cos(\lambda_{yn} y) dy}{4ab\beta_{mn} \int_0^c \cos^2(\lambda_{xm} x) dx \int_0^d \cos^2(\lambda_{yn} y) dy} \qquad (19)$$

Upon evaluation of the integrals one obtains

$$B_{mn} = \frac{-(Q/k) \sin(\delta_{xm} a/c) \sin(\delta_{yn} b/d)}{ab\beta_{mn}[\sin(2\delta_{xm})/2 + \delta_{xm}][\sin(2\delta_{yn})/2 + \delta_{yn}]} \qquad (20)$$

With both Fourier coefficients now known, the mean surface temperature excess may be found from Eq. (2). Using this result and Eq. (1), the total resistance becomes:

$$R_t = \frac{cd}{ka^2 b^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty}$$

$$\frac{\sin^2(\delta_{xm} a/c) \sin^2(\delta_{yn} b/d) \phi_{mn}}{\delta_{xm} \delta_{yn} \beta_{mn}[\sin(2\delta_{xm})/2 + \delta_{xm}][\sin(2\delta_{yn})/2 + \delta_{yn}]} \qquad (21)$$

The total resistance now depends on

$$R_t = f(a, b, c, d, t, k, h, h_{e,x}, h_{e,y}) \qquad (22)$$

The total resistance may be non-dimensionalized using $\mathcal{L} = \sqrt{A_s} = 2\sqrt{ab}$ to give:

$$R_t^\star = \frac{2\sqrt{\epsilon_x \epsilon_y \epsilon_b}}{\epsilon_b \epsilon_x^2 \epsilon_y^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\delta_{xm} \delta_{yn} \sqrt{\delta_{xm}^2/\epsilon_b^2 + \delta_{yn}^2}} \cdot$$

$$\frac{\sin^2(\delta_{xm} \epsilon_x) \sin^2(\delta_{yn} \epsilon_y) \phi_{mn}}{\left[\frac{\sin(2\delta_{xm})}{2} + \delta_{xm}\right]\left[\frac{\sin(2\delta_{yn})}{2} + \delta_{yn}\right]} \qquad (23)$$

where

$$\phi_{mn} = \frac{\xi + Bi \tanh(\xi\tau)}{Bi + \xi \tanh(\xi\tau)} \qquad (24)$$

and

$$\xi = 2\sqrt{\epsilon_x \epsilon_y \epsilon_b} \sqrt{\frac{\delta_{xm}^2}{\epsilon_b^2} + \delta_{yn}^2} \qquad (25)$$

Thus, the dimensionless total resistance depends upon

$$R_t^\star = f(\epsilon_x, \epsilon_y, \epsilon_b, \tau, Bi, Bi_{e,x}, Bi_{e,y}) \qquad (26)$$

where $\epsilon_x = a/c$, $\epsilon_y = b/d$, $\epsilon_b = c/d$, $\tau = t/\sqrt{A_s}$, $Bi = h\sqrt{A_s}/k$, $Bi_{e,x} = h_{e,x} c/k$, $Bi_{e,y} = h_{e,y} d/k$, and $R_t^\star = k\sqrt{A_s} R_t$.

This general result has many geometric special cases. These include: semi-infinite flux tubes $t \to \infty$, infinite plate $c, d \to \infty$, half-space $t, c, d \to \infty$, three dimensional strips $b = d$ or $a = c$, and adiabatic edges $h_{ex} \to 0$ and $h_{e,y} \to 0$. A particular interesting property is the case when adiabtic edges are present. It can be shown in this case, that when $Bi_{e,x} \to 0$ and $Bi_{e,y} \to 0$, the double summation which represents the total resistance, consists of the one dimensional resistance when $m = n = 1$ and the spreading resistance when the remaining terms are summed.

4

In the next section these two solutions are examined, and it is shown that considerable computational effort is saved by modelling the rectangular system as an equivalent circular disk for a wide range of channel aspect ratios.

ANALYSIS

Given the solutions for the circular disk and rectangular flux channel with edge cooling, it is now possible to show that there exists a physical equivalence between these systems and systems with adiabatic edges, Figs. 3 and 4. In determining this equivalence, we choose to maintain the total convective heat transfer rate at the edges and bottom surface. This is achieved by means of the following energy balance

$$Q_{total} = Q_{base} + Q_{edges} \tag{27}$$

or

$$h_{eff} A_b (T_b - T_f) = h_b A_b (T_b - T_f) + h_e A_e (T_e - T_f) \tag{28}$$

For small aspect ratios ϵ and thin substrates τ, $T_e \sim T_b$, which leads to the following condition:

$$h_{eff} A_b = h A_b + h_e A_e \tag{29}$$

where h_{eff} is an effective bottom surface heat transfer coefficient.
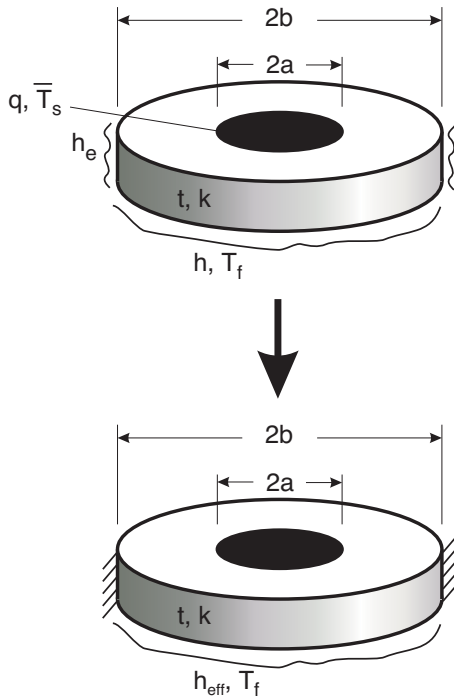
Fig. 3 - Circular Flux Tube with Edge Cooling Transferred to Bottom Surface
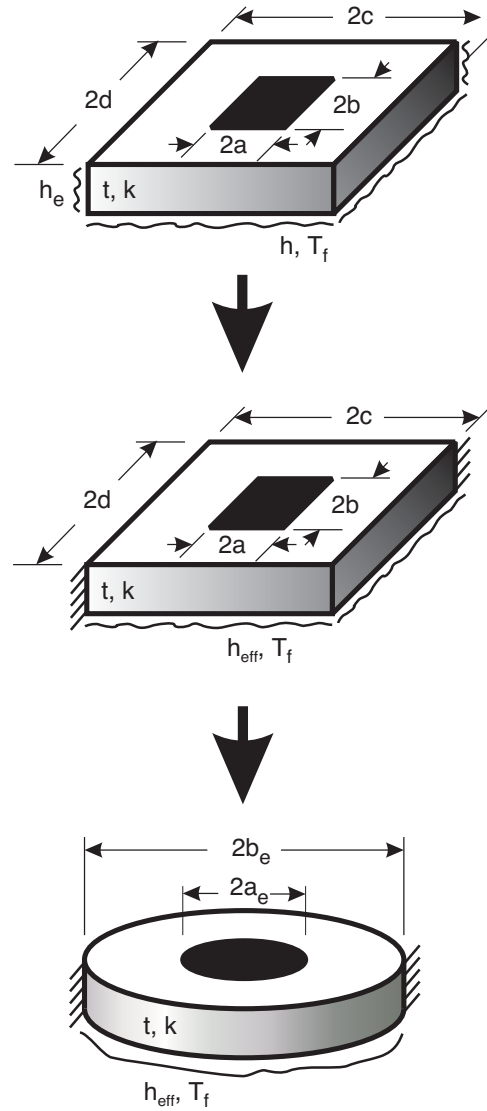
Fig. 4 - Rectangular Flux Channel with Edge Cooling Transferred to Bottom Surface

Using the edge and lower surface areas of the disk it may be shown that

$$Bi_{eff} = Bi \left(1 + \frac{Bi_e}{Bi} 2\pi\tau\epsilon^2 \right) \tag{30}$$

where $Bi_{eff} = h_{eff} \sqrt{A_s}/k$ is the effective Biot number for the lower surface of a flux tube with adiabatic edges. The edge cooling Biot number is $Bi_e = h_e b/k$, and $\tau = t/\sqrt{A_s}$, $\epsilon = \sqrt{A_s/A_b}$, and $Bi = h\sqrt{A_s}/k$ are defined as before. It may now be seen from this expression that when the edge cooling Biot number is small and/or the relative thickness and/or relative contact are small, the effect of edge cooling is negligible. In otherwords when,

$$\frac{Bi_e}{Bi} 2\pi\tau\epsilon^2 \to 0 \tag{31}$$

5

edge cooling can be neglected regardless of its magnitude.

Similarly, for the rectangular flux channel, we may obtain a similar result which now contains the two edge cooling Biot numbers $Bi_{e,x}$ and $Bi_{e,y}$ in addition to the relative contact parameters $\epsilon_x = a/c$ and $\epsilon_y = b/d$, such that

$$Bi_{eff} = Bi \left[1 + \left(\frac{Bi_{e,x} + Bi_{e,y}}{Bi} \right) 4\tau \epsilon_x \epsilon_y \right] \qquad (32)$$

where $Bi_{eff} = h_{eff}\sqrt{A_s}/k$, $Bi_{e,x} = h_{e,x}c/k$, $Bi_{e,y} = h_{e,y}d/k$, and $Bi = h\sqrt{A_s}/k$.

Once again, similar behavior is seen in terms of the relative significance of edge cooling. In otherwords when,

$$\left(\frac{Bi_{e,x} + Bi_{e,y}}{Bi} \right) 4\tau \epsilon_x \epsilon_y \to 0 \qquad (33)$$

edge cooling in the rectangular flux channel may be neglected regardless of its magnitude.

Finally, it can also be shown that the equivalent rectangular flux channel with edge cooling can be modelled as an equivalent circular disk with edge cooling using the results of Part I of this paper provided that

$$h_e = \frac{h_{e,x}c + h_{e,y}d}{c + d} \qquad (34)$$

in addition to

$$\left. \begin{array}{c} a_e = \sqrt{A_s/\pi} \\ b_e = \sqrt{A_b/\pi} \\ t = t \end{array} \right\} \qquad (35)$$

RESULTS AND DISCUSSION

We may now examine the influence of edge cooling in the flux tube and/or channel. For simplicity, only the flux tube solution is considered, since it only depends upon four variables whereas the rectangular flux channel depends upon seven variables. In the computations 200 terms were used in Eq. (6) to provide four decimal place accuracy. The solution for the circular flux tube with edge cooling is plotted in Figs. 5-13 for a range of dimensionless thicknesses and source aspect ratio. Nine combinations of lower surface Bi number and edge Bi_e number are considered. These are obtained by choosing $Bi = 1, 10, 100$ and $Bi_e = 1, 10, 100$. It is clear from the figures that the total resistance for adiabatic and edge cooled systems are equivalent, when source aspect ratio is small and relative thickness is small. However, for larger values of edge Biot number and in systems with large source aspect ratios and/or large relative thickness, the effect of edge cooling becomes important and cannot be neglected.

SUMMARY AND CONCLUSIONS

This paper examined the issue of edge cooling in circular flux tubes and rectangular flux channels. A new solution was obtained for the rectangular flux channel with edge cooling. This solution was shown to have many special limiting cases. Simple expressions were developed for predicting the impact of edge cooling. Comparisons were made with the exact solution for edge cooling and the solution for flux tubes with adiabatic edges. This simplified approach allows for efficient computation as the eignevalues are only computed once in systems with adiabatic edges. Finally, it can be shown using the results of Part I of this paper that, that rectangular edge cooled systems can be modelled as an equivalent circular disk with edge cooling for a wide range of dimensionless parameters.

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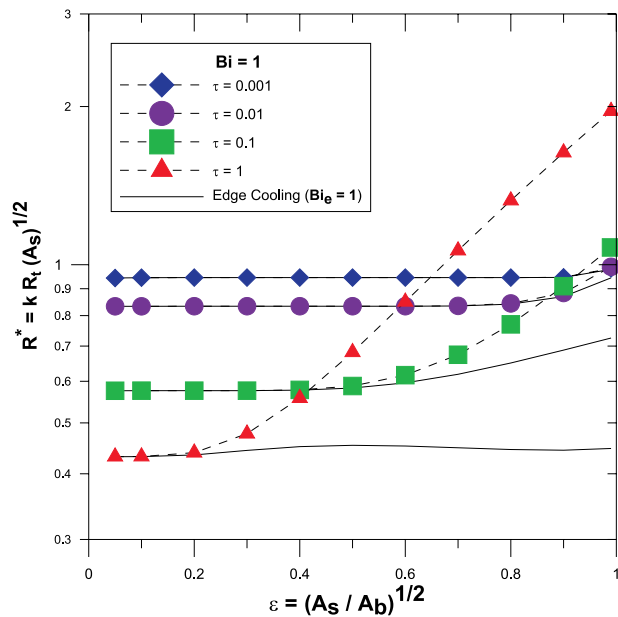
Fig. 5 - Comparison of Edge Cooled Disk for $Bi = 1, Bi_e = 1$.
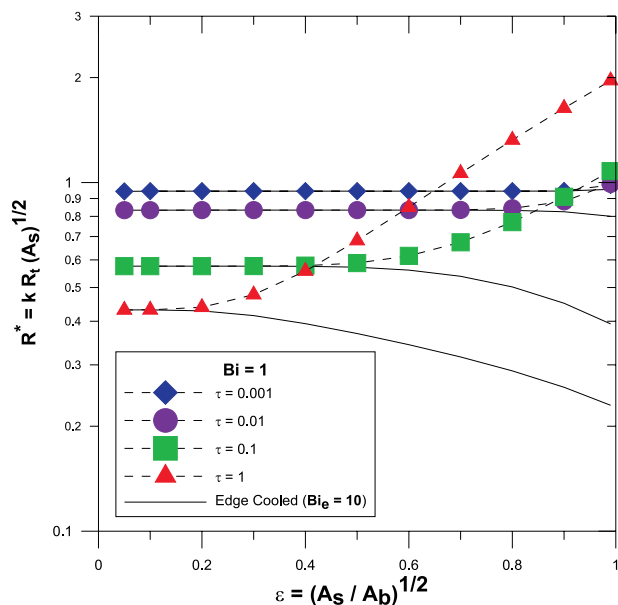
Fig. 6 - Comparison of Edge Cooled Disk for $Bi = 1, Bi_e = 10$.

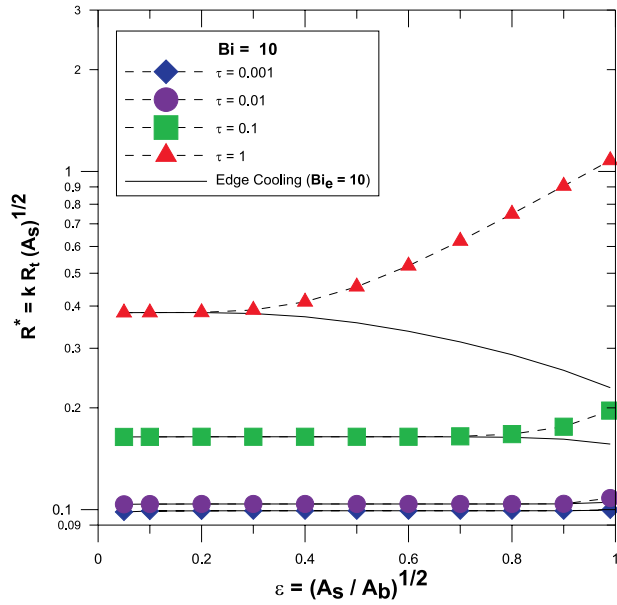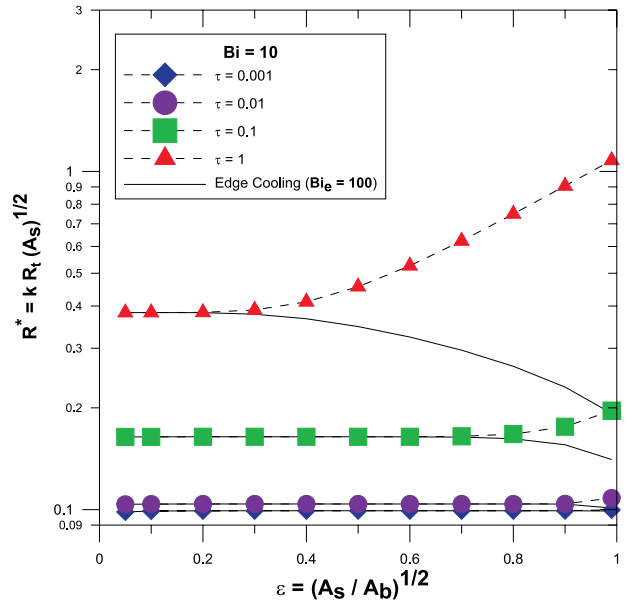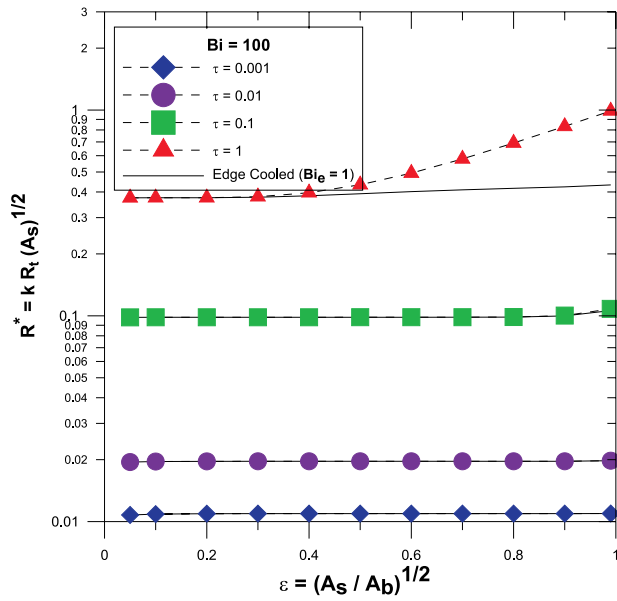**Fig. 7 - Comparison of Edge Cooled Disk for $Bi = 1, Bi_e = 100$.**
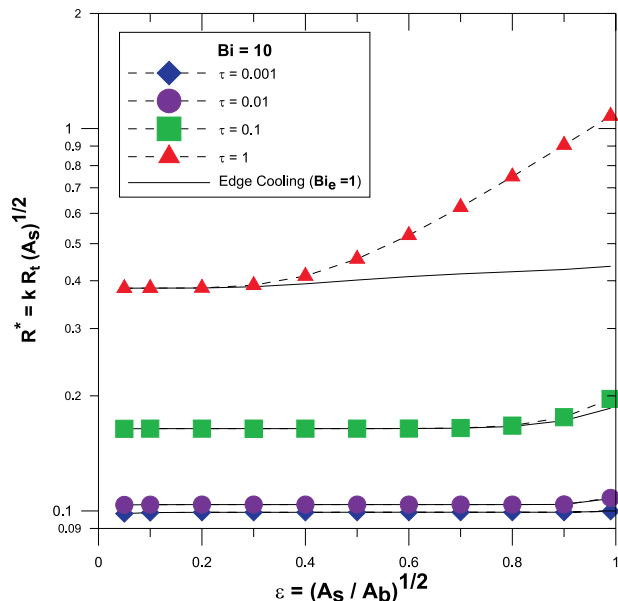
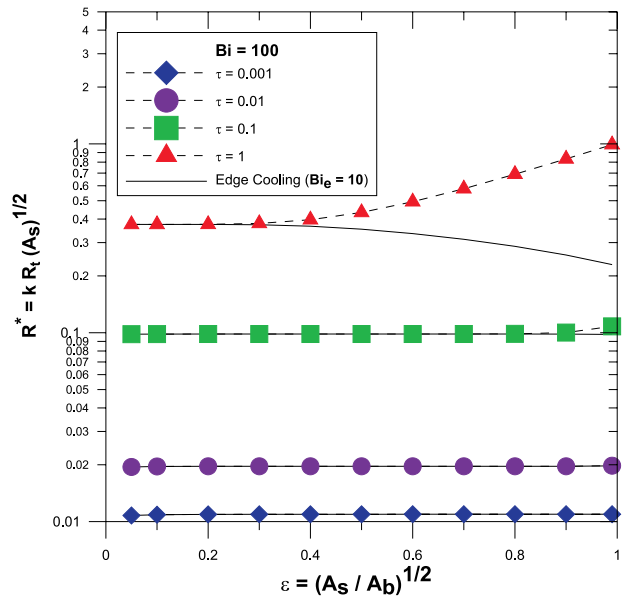**Fig. 8 - Comparison of Edge Cooled Disk for $Bi = 10, Bi_e = 1$.**

Fig. 9 - Comparison of Edge Cooled Disk for $Bi = 10, Bi_e = 10$**.**

Fig. 10 - Comparison of Edge Cooled Disk for $Bi = 10, Bi_e = 100$**.**

Fig. 11 - Comparison of Edge Cooled Disk for $Bi = 100, Bi_e = 1$**.**

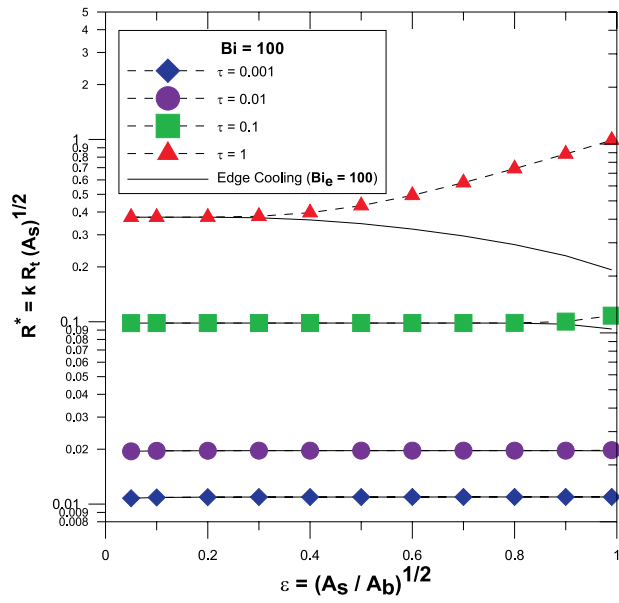**Fig. 12 - Comparison of Edge Cooled Disk for** $Bi = 100, Bi_e = 10$**.**

Fig. 13 - Comparison of Edge Cooled Disk for $Bi = 100, Bi_e = 100.$