Combinative hypergraph learning for semi-supervised image classification

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\textbf{A B S T R A C T}

Recent years have witnessed a surge of interest in hypergraph-based transductive image classification. Hypergraph-based transductive learning models the high-order relationship of samples by using a hyperedge to link multiple samples. In order to extend the high-order relationship of samples, we incorporate linear correlation of sparse representation to hypergraph learning framework to improve learning performance. In this paper, we present a new transductive learning method called combinative hypergraph learning (CHL). CHL captures the similarity between two samples in the same category by adding sparse hypergraph learning to conventional hypergraph learning. And more, we propose two strategies to combine the two hypergraph learning methods. Experimental results on two image datasets have demonstrated the effectiveness of CHL in comparison to the state-of-the-art methods and shown that our proposed method is promising.

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1. Introduction

Currently, there is widespread interest in the development of image classification using transductive learning. Because transductive learning explores not only labeled data but also unlabeled data, it achieves a performance better than the methods that learn about classifiers based only on labeled data. Its success is based on one of the following two assumptions: cluster and manifold assumptions. The cluster assumption supposes that the decision boundary should not cross high-density regions, whereas the manifold assumption means that each class lies on an independent manifold. Therefore, more and more researchers have been devoted to transductive learning based on these assumptions in recent years [1–36]. Rosenberg et al. proposed self-training to train object detection systems [8]. Blum and Mitchell used co-training to classify web pages and provided empirical results which achieved significant improvement of hypotheses on real web-page data in practice [9]. Joachims introduced transductive support vector machines (TSVMs) for text classification [10]. Another important transductive learning method is graph-based learning, which is the derived form of hypergraph-based learning on which we have focused in this paper.

The graph-based learning [15–41] achieves a promising performance between the existing transductive learning methods. Its development goes through two stages: simple-graph learning and hypergraph learning. This type of learning is built on a graph, in which vertices are samples and edge weights indicate the similarity between two samples. However, the simple-graph learning methods consider only the pairwise relationship between two samples, and they ignore the relationship in a higher order. Hypergraph learning aims to get the relationship between several samples in a higher order. Unlike a simple graph that has an edge between two vertices, a set of vertices is connected by a hyperedge in a hypergraph, and each hyperedge is assigned a weight. Hypergraph learning derives from simple-graph learning, and thus it achieves a promising performance in many applications. For example, Agarwal et al. utilized hypergraph to clustering by using clique average [14]. Zass and Shashua adopted hypergraph in image matching by using convex optimization [15]. Sun et al. utilized hypergraph to problems of multi-label learning [16]. Huang et al. applied hypergraph cut to video segmentation [17]. Tian et al. proposed hypergraph-based learning algorithm to classify gene expression data by using biological knowledge as a constraint [18]. Huang et al. formulated the task of image clustering as a problem of hypergraph partition [19]. A hypergraph-based image retrieval approach is proposed in [19]. This approach constructs a hypergraph by generating a hyperedge from each sample and its neighbors, and hypergraph-based ranking is then performed. Wong and Lu proposed hypergraph-based 3-D

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object recognition [20]. Bu et al. [21] developed music recommendation by modeling the relationship of different entities through a hypergraph to include music, tag, and users.

In hypergraph learning, the weights of the hyperedges are empirically set according to certain rules. Zhou et al. connected k neighbor points of a given point as a hyperedge [37]. Yu et al. proposed a hypergraph learning method by adaptively coordinating the weights of the hyperedge [38]. In essence, they chose multiple neighborhoods to construct multiple hyperedges for a given point. By this way, how to define a hyperedge and the weight of a hyperedge is the succeeding problem we should take into account. There are mainly two methods to select the neighborhood of a given point, one is to define a hyperedge with fixed number neighbor points, and the other one is to define a ball whose radius is less than some threshold. However, these hypergraph-based methods focus only on proximity relation for distance. There is some other high-order relationship, such as linear relationship. For example, given two vectors a and b for existing a = k*b, the distance between a and b may be very large, but they are similar for one factor difference based on linear relationship. Therefore, the linear relationship is another one we should take into account.

This paper takes into account the clustering assumption that the similar points in feature space more likely belong to the same category. Then, we define the similarity by two assumptions that we call as distance-similarity and linear-similarity. Distance-similarity is that the points derived from a category are located close to each other. The neighborhood size of the hypergraph based on this assumption is chosen as a fixed number. Linear-similarity is that a data point can more likely be represented linearly by the data points nearby which belong to the same category as this data point. It is analogous to manifold assumptions aforementioned. The neighborhood size of the hypergraph based on this assumption is undefined since it is decided by the sparse representation method. Inspired by the two assumptions, we construct the following two kinds of hypergraphs on a data set, one is conventional hypergraph which is based on the distance-similarity, the other one is sparse hypergraph which is based on linear-similarity and derived from the thinking in Refs. [53, 54]. We linearly combine the two hypergraph learning methods by two strategies. The first strategy is to combine the hyperedges of the two hypergraphs to form a new hyperedge set. The second strategy is to linearly combine the confidence of the labeling of the two hypergraph learning methods to form a new confidence of the labeling to define the label of data points.

The contributions of this paper are as follows:

1. A novel algorithm named Combinative Hypergraph Learning (CHL) is proposed for image classification. To consider the high-order information, we incorporate sparse representation into the standard hypergraph learning framework. Hence, our algorithm achieves a better performance than that with only conventional hypergraph learning or with only sparse hypergraph learning.

2. We provide two strategies to combine linearly conventional hypergraph learning and sparse hypergraph learning. The essence is that we combine linearly the results of the two hypergraph learning methods with same weights for one strategy and with different weights for the other strategy.

3. We conduct comprehensive experiments to empirically analyze our algorithm on two image databases. The experimental results demonstrate that our algorithm outperforms other methods including Transductive SVM [10], Simple Graph-Based Learning [50], and Semi-supervised Discriminant Analysis (SDA) [42, 51] classificatons.

The rest of this paper is organized as follows. Section II describes the proposed image classification by combinative hypergraph learning. Section III shows experiments on practical image data-sets. Section IV concludes the paper.

## 2. Hypergraph learning

This section introduces conventional hypergraph learning and sparse hypergraph learning first, and shows combinative hypergraph learning theory with two strategies in the following text. In Table 1, we provide important notations used in the rest of this paper to present the technique details of the proposed method. Fig. 1 illustrates the whole framework of our method.

### 2.1. Conventional hypergraph learning

Given c categories of images including m training data points \((x_1, y_1), \ldots, (x_m, y_m)\), and n testing data points \((x_{m+1}, 0), \ldots, (x_{m+n}, 0)\), where \(x_i \in \mathbb{R}^d\), \(1 \leq i \leq m+n\) is sampled from the input space; \(y_i = [0, \ldots, 1, \ldots, 0]^T \in \mathbb{R}^c\), \(1 \leq i \leq m\), is the label vector of \(x_i\), where the g-th component is 1 if \(x_i\) belongs to the g-th category, otherwise, 0; and \(0\) is a vector with c components of zero.

| Table 1 Important notations used in this paper. |
|-----------------|-------------------------------------------------|
| **Notations**    | **Descriptions**                               |
| \(H\) \= (x, e) | The representation of a hypergraph, where x and e indicate the sets of vertices and hyperedges, respectively |
| A                | The incidence matrix for the hypergraph       |
| dist             | The distance between two samples              |
| \(\varphi\)      | The diagonal matrix of the vertex degrees      |
| \(\gamma\)       | The diagonal matrix of the hyperedge degrees   |
| \(\varepsilon\)  | The diagonal weight matrix and its (i)-th element is the weight of the i-th hyperedge |
| L                | The constructed hypergraph Laplacian matrix   |
| \(y_i\)          | The label vector for i-th class. Its j-th element is 1 if the j-th object belongs to the i-th class, and otherwise it is 0. |
| \(F\) \= \([f_1, f_2, \ldots, f_l]\) | F represents the relevance score matrix for all samples, and \(f_i\) is the to-be-learnt relevance score vector for class i. |
| c                | The number of classes in the dataset          |
| m                | The number of labeled images in the dataset    |
| l                | The number of hyperedges                      |
| \(A^s\), \(dist^s\), \(F^s\) | The superscript "sp" denotes sparse-based which show difference to A, dist and F, respectively |
| \(A^*, F^*\)     | The concatenated results.                     |
| \(\Sigma = [s_1, s_2, \ldots, s_l]\) | The base of sparse representation |
| \(w = [w_1, w_2, \ldots, w_l]\) | The coefficient vector of sparse representation |
| \(W = [w_1, w_2, \ldots, w_l]\) | The coefficient matrix of sparse representation |
The hypergraph, $H = (\mathbf{x}, \varepsilon)$, is formed by the vertex set, $\mathbf{x}$, and the hyperedge set, $\varepsilon$, an incidence matrix, $\mathbf{A}$, whose size is $|\mathbf{x}| \times |\varepsilon|$, denotes the hypergraph with the following elements:

$$A(i, j) = \begin{cases} 1, & \text{if } \mathbf{x}_i \in \varepsilon_j \\ 0, & \text{if } \mathbf{x}_i \notin \varepsilon_j \end{cases}$$

(1)

where $\varepsilon_j$ denotes the $j$-th element of the hyperedge set. The distance between two data points is

$$\text{dist}(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{\sigma^2}\right)$$

(2)

We use $\mathbf{\Phi}$, $\mathbf{\Gamma}$ and $\mathbf{\Omega}$ to denote diagonal matrices of vertex degrees, hyperedge degrees and hyperedge weights, respectively. $\Phi_{ii}$ denotes the entry $(i, i)$ of matrix $\mathbf{\Phi}$. $\Omega_i$ and $\Gamma_i$ have similar meanings. Then, the initial weight, $\omega_i$, is

$$\omega_i = \sum_{\mathbf{x}_j \in \varepsilon_i} \text{dist}(\mathbf{x}_i, \mathbf{x}_j).$$

(3)

Based on $\mathbf{A}$, the $i$-th vertex degree, $\Phi_i$, is

$$\Phi_i = \sum_{\varepsilon_j \ni i} \omega_j A(i, j),$$

(4)

and the $i$-th hyperedge degree, $\gamma_i$, is

$$\gamma_i = \sum_{\mathbf{x}_j \in \varepsilon_i} \Phi_j .$$

(5)

In this letter, we adopt the regularization framework proposed in Yu et al. [38], i.e.,

$$\underset{\mathbf{f}_i}{\text{arg min}} \sum_{j=1}^{c} (f_{ij}^T f_{ij}) + \lambda \|f_{ij} - Y_i\|^2 + \mu \|\text{diag}(\mathbf{\Omega})\|^2$$

s.t. $\sum_{j=1}^{l} \omega_j = 1, \quad \omega_j \geq 0, \quad j = 1, \ldots, l.$

(6)

**Fig. 1.** Workflow of the hypergraph learning for image classification with two combinative strategies. (a) Strategy one and (b) strategy two.
where $L = 1 - \varphi^{-1/2}A Y^{-1} A^\top \varphi^{-1/2}$, $l$ is the number of hyperedges; $\text{diag}(\omega) = (\omega_1, \omega_2, \ldots, \omega_l)$; $c$ is the number of classes; $F$ is a matrix, $F = (f_1, \ldots, f_c) \in \mathbb{R}^{(m+n) \times c}$, $f_c$ is the confidence of the labeling for the $c$th category; $\lambda > 0$ and $\mu > 0$ are two trade-off parameters to balance the empirical loss, the weight and the regularization; $Y = (y_1, y_2, \ldots, y_m, 0, \ldots, 0)^\top$, $y \in \mathbb{R}^{(m+n) \times c}$, $Y_i$ is the $i$th column of $Y$. The third term is introduced to avoid a degenerate solution caused by the former two terms existing only in the regularisation. Here we add two constraints, one to fix the summation of the weights $\sum_{i=1}^n \omega_i = 1$ and the other to avoid negative weight $\omega_i \geq 0$.

Because the classifier function is not jointly convex with respect to $F$ and $\omega$, we solve one variable by fixing another variable.

First we initialize $\omega$ with (3), so the solution of $F$ becomes

$$F = \frac{\lambda}{\lambda + 1} \left( I - \varphi^{-1/2} A Y^{-1} \omega A^\top \varphi^{-1/2} \right)^{-1} Y.$$

(7)

Then we update the weights, $\omega$, with an iterative coordinate descent method as follows:

$$\begin{align}
\omega_i^{t+1} &= 0, \quad \omega_i^{0} = \omega_i + \alpha_{0i}, & \text{if } 2 \mu (\omega_i^{0} + \alpha_{0i}) + (s_i - s_j) \leq 0 \\
\omega_i^{t+1} &= \omega_i^{t} + \alpha_i, \quad \omega_i^{t+1} = 0, & \text{if } 2 \mu (\omega_i^{t} + \alpha_{t+1}) + (s_i - s_j) \leq 0 \\
\omega_i^{t+1} &= \omega_i^{t} + \alpha_{t+1}, & \text{otherwise}
\end{align}$$

(8)

where $s_i = \sum_{j=1}^n \left( r_{ij} / (r_{ij} + 1) \right)^{-1}$, $r_{ij} = f^\top_i \varphi^{-1/2} A_i$, $r_i$ is the $i$-th component of $r_i$, $\omega_i$ denotes the $t$-th iteration of $\omega_i$ and the initial iteration $t$ is 0. Based on the coordinate descent method, an iterative process alternately updates the labels and the weights.

In the next iteration, we calculate the new $F$ with the new $\omega$. The iteration ceases at a given state. After obtaining $F$, we set the $g$-th class to the $i$-th data point if the $g$-th component is the maximum in the $i$-th row of $F$. A more detailed solution of (6) appears in Ref. [38].

2.2. Sparse hypergraph learning

Sparse hypergraph learning is similar to conventional hypergraph learning. Therefore, this section will show only the difference between sparse hypergraph learning and spectral hypergraph learning to avoid repetition. It is the hyperedge construction and the hyperedge weight definition which will be introduced after the introduction of sparse representation in this section.

The application of sparse representation to computer vision has attracted a lot of attention in recent years [43–46]. This paper solves the sparse representation problem based on $\ell_1$ minimization.

Given a vector $x \in \mathbb{R}^d$, which can be represented on the basis of $d$ vectors $[c_1 \in \mathbb{R}^d]_{i=1}^d$. By setting a matrix $\Sigma = [c_1, c_2, \ldots, c_d]$ we can rewrite $x$ as

$$x = \sum_{i=1}^d W_{c_i} = \Sigma W,$$

(9)

where $W = [w_1, w_2, \ldots, w_d]^\top$. Both $x$ and $W$ represent the same data point, one in the space domain and the other in the $\Sigma$ domain. Our object is to find a sparse representation of $x$ in a properly chosen basis $\Sigma$, namely, $W$ must have as few nonzero components as possible. According to [48], such a sparse representation can be obtained by solving the optimization problem

$$\min ||W||_0 \quad \text{subject to } x = \Sigma W,$$

(10)

where $||W||_0$ is the $\ell_0$ norm of $W$, i.e., the number of nonzero entries. However, such an optimization problem is in general non-convex and NP-hard. According to Refs. [47,48], we can replace the non-convex optimization in (10) by the following convex $\ell_1$ optimization formulation:

$$\min ||W||_1 \quad \text{subject to } x = \Sigma W.$$  \hspace{1cm} (11)

Now, consider all data points in a dataset, $x = [x_1, x_2, \ldots, x_n]$. Each data point $x_i$, has a sparse representation $w_i$. By setting the $i$-th element of $w_i$, $(w_i)_j = 0$, the optimization formulation can be rewritten as

$$\min \sum_{i=1}^n ||W_i||_1 \quad \text{subject to } x_i = \Sigma w_i.$$  \hspace{1cm} (12)

Assume that the data set is drawn from a union of $c$ independent linear subspaces, namely, that the data set include $c$ categories of the object. According to Ref. [54], we can obtain block sparse solutions with the nonzero block corresponding to points in the same subspace if the aforementioned assumption holds. We can recover a block sparse representation of a new data point as a linear combination of the points in the same subspace. This means that a data point $x_i$ and the data points whose index corresponds to nonzero entry of $W_i$ are derived from the same category.

We now show how to define a hyperedge and its weight, which are different from spectral hypergraph construction. For discrimination, we use $A^{sp}$, $\text{dist}^{sp}$ and $F^{sp}$ to replace the incidence matrix $A$, distance between two data points dist and classifier function $F$ shown in the above section, respectively. We define $A^{sp}$ as

$$A^{sp}(i, j) = \begin{cases} 1, & \text{if } i = j \\
1, & \text{if } ||(w_i)_j|| > 0, \\
0, & \text{otherwise}
\end{cases}$$

where $||(w_i)_j||$ denotes the absolute of $j$-th entry of $w_i$, and dist$^{sp}$ as

$$\text{dist}^{sp}(x_i, x_j) = ||(w_i)_j||.$$  \hspace{1cm} (14)

The following processing is the same as conventional hypergraph learning. Here, we take $F^{sp}$ as the solution of the sparse hypergraph learning.

2.3. Combinative hypergraph learning

From the above two learning methods, we obtain two hypergraphs and combine them with two strategies.

2.3.1. Strategy one

Assume that we obtain two incidence matrixes $A$ and $A^{sp}$ of the hypergraph aforementioned. We concatenate $A$ and $A^{sp}$ to a new incidence matrix $A^*$ whose columns number is the sum columns of $A$ and $A^{sp}$ as follows:

$$A^* = [A A^{sp}],$$

(15)

where the rows number of $A$ is equal to that of $A^{sp}$. And the corresponding weight of a hyperedge is calculated with Eqs. (2) and (14), respectively. Then, it performs as the conventional hypergraph learning process. We can obtain classification function $F^*$, then the classification of $i$-th sample can be accomplished by assigning it to the $g$-th class that satisfies $g = \arg\max G^{sp}_i$. This method constructs two hyperedge sets and the corresponding weights into a new hypergraph which takes more high-order information into account. In other words, it combines two kinds of similarities: distance-similarity and linear-similarity. We consolidate the similarities between samples in the same category by putting the sparse hyperedges into the conventional hyperedges.

Strategy two

We obtain two confidences of the labeling, $F$ and $F^{sp}$, for conventional hypergraph learning and sparse hypergraph learning. For this strategy, we define $F^*$ as

$$F^* = \alpha^* F + (1 - \alpha^*) F^{sp}$$

where $\alpha^*$ is a parameter.
\[
\alpha^* = \arg \min_{\alpha} \sum_{i=1}^{m} \| a \mathbf{f}_i + (1 - a) \mathbf{F}^0 - \mathbf{y} \|_2^2 \\
\text{subject to} \quad a \in [0, 1]. \tag{16}
\]

It means that, here, we combine linearly only the results of the two hypergraph learnings with the weights \(a \) and \(1 - a\). We can get \(\alpha^* \) by minimizing the loss of classification contribution value to truth classification value with only the labeled data point. The minimized function is easy to be solved since there is only one variable. After obtaining \(\mathbf{P}^*\), we set the \(g\)-th class to the \(i\)-th data point if the \(g\)-th component is the maximum in the \(i\)-th row of \(\mathbf{P}^*\).

For this strategy, we learn the confidence of the labeling independent for the two hypergraph-based learnings. Then, we combine only the two confidences of the labeling linearly with optimal weights.

3. Experiments

We performed experiments for image classification on two datasets: MNIST handwritten digit image data set [49] and ORL face image data set [11]. In order to assess the performance of the proposed method, we compared our method with general classification algorithms including transductive support vector machine (TSVM) [10], Simple-Graph Learning (SGL) [50], Hypergraph Learning (HL) [37], Sparse Hypergraph Learning (SHL) [53] and Semi-supervised Discriminant Analysis (SDA) [51] classifications. And more, we analyzed our method in two strategies.

3.1. Datasets and configurations

The MNIST handwritten digit image data set contains 10,000 images. There are ten classes (digit “0” to “9”) of images, each class has about 1000 images, and each image is 28 × 28 in resolution, which results in a 784-D feature vector. For this dataset, in experiments, we randomly select 400 labeled samples as training samples and the remaining samples as testing samples. The ORL face image data set contains 64 different images for each of the 38 subjects. Each image is 48 × 42 in resolution, which results in a 2016-D feature vector. Images are taken at different times, with varied lighting, facial expressions, and facial details. In experiments of this dataset, we randomly select 20 labeled samples of each category as training samples and the remaining samples as testing samples.

For all the classification methods, we independently repeat the experiments twenty times with randomly selected training samples and show the averaged results by Mean Average Precision (MAP).

1) Simple-graph Learning addresses the pair-wise relationships between any vertices in the correlated Laplacian graph. We use the regulation, which is similar to the first two items of Eq. (6). However, simple graph learning has difficulty tuning the parameters of \(\lambda\) and \(\mu\). Here, we set \(\lambda\) to be the norm variance of the data points, and \(\mu\) to be \(\lambda/4\).

2) Semi-supervised Discriminated Analysis aims to find a projection which is in respect to the discriminated structure inferred from the training data. The training data, combined with the unlabeled data, are used to build a Laplacian graph which provides a discrete approximation to the local geometry of the data manifold and can be incorporated into objective functions. It can preserve the manifold structure. We set the weight, \(w_{ij} = \exp (-\|\mathbf{n}_i - \mathbf{n}_j\|/2\sigma^2)\), to an edge if nodes \(i\) and \(j\) are connected. And we set neighborhood to be 5 on the \(k\)-Nearest Neighbor classification method.

3) For the method of Transductive Support Vector Machine with RBF kernel, we use the one-versus-all strategy to solve the multiclass problems. The optimal values of the radius parameter and the weighting parameter in regularization are used. As we chose C-SVC as the type of SVM, we set the parameter \(C\) to 20. We also set gamma and coefficient 0 in kernel function to 1 and set the degree to 4. And Unlike SVM, Transductive SVM has an additional parameter that modulates the effect which we set to 100.

4) Hypergraph-based learning. The neighborhood size is set to 5. And we set the iteration time in the alternating optimization process to 10. For Combinative Hypergraph Learning, we initialize \(a\) to 0.01 and iteration step by 0.01.

3.2. Experimental results

Fig. 3 shows the Mean Average Precision (MAP) with several methods on ORL and MNIST datasets. It shows that CHL achieves the best results in all the methods. And the results demonstrate the effectiveness of the proposed method. Compared with SGL, hypergraph-based learning methods achieve better results, and

![Fig. 2](image1.png)

**Fig. 2.** Sample images from (a) MNIST and (b) ORL data sets used in the experiment.

![Fig. 3](image2.png)

**Fig. 3.** Mean Average Precision (MAP) of methods for SGL, SDA, TSVM, SHL, HL, CHL_s1 and CHL_s2 on two datasets.
it demonstrates that hypergraph-based learning considering high-order information is effective. SDA shows better results than T SVM and SHL. For MNIST dataset, CHL_s1 achieves better results by comparing with SDA and HL, while it shows inferior performance for ORL dataset. Both CHL_s1 and CHL_s2 show the best performance for MNIST dataset in all the methods. We affirm that CHL_s2 shows the best performance over all methods for two datasets. For CHL_s1, we concatenate the hypergraph edge sets of hypergraph learning and sparse hypergraph learning with the same weight. However, the experimental results showed that this process is not effective for ORL dataset, which demonstrates that the two hypergraph learning methods are not the same contribution for classification.

For further illustration of our proposed methods, we show the correction ratio in detail for each trial on hypergraph-based methods in Table 2. The correct ratio of HL is better than that of SHL significantly with gaining about 0.24 for ORL dataset, but, they show similar performance for MNIST dataset. We can conclude that the performance of HL is better than that of SHL for the two datasets. By combining the two hypergraph-based learnings with two strategies, the learning performance gets an overall better level than that with conventional hypergraph learning or sparse hypergraph learning independently. However, the performance of the two strategies shows different results. Both CHL_s1 and CHL_s2 achieve better results than HL and SHL for MNIST dataset, while CHL_s1 achieves worse results than that of HL for ORL dataset. For each trial on the ORL dataset, the correct ratio of CHL_s2 is a little higher than that of HL all the time, and it is large when the correct ratio of HL is low. However, the case on MNIST dataset is not the same as the ORL dataset that sometimes the correct ratio of CHL_s2 is higher than that of CHL_s1, sometimes it is inverse. Overall, they show a similar performance. It is definite that at least one strategy achieves better results than that of HL and SHL.

Furthermore, we observe the variable \( \alpha \) in Eq. (15) for each trial. It is local at 0.84 for large probability for ORL dataset and at 0.9 for MNIST dataset. This demonstrates that the weight of distance-similarity is larger than that of linear-similarity, namely, that the contribution of distance-similarity on image classification is larger than that of linear-similarity in our proposed methods.

### Table 2
Correct ratio of hypergraph-based learning for two datasets.

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<th>Serial number</th>
<th>Correct ratio for ORL dataset</th>
<th>Correct ratio for MNIST dataset</th>
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<td>HL</td>
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<td>Average</td>
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<td>0.882</td>
</tr>
</tbody>
</table>
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