Constrained dynamic optimization of sit-to-stand motion driven by Bézier curves

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ABSTRACT

The purpose of this work is two-fold: first, to synthesize a motion pattern imitating sit-to-stand and second, to compare the kinematics and dynamics of the resulting motion to healthy sit-to-stand. Predicting sit-to-stand in simulation inspired the creation of three models: a biomechanical model, a motion model, and performance criteria as a model of preference. First, the human is represented as three rigid links in the sagittal plane. This model captures aspects of joint, foot, and buttocks physiology, which makes it the most comprehensive planar model for predicting sit-to-stand to date. Second, candidate sit-to-stand trajectories are described geometrically by a set of Bézier curves which seem well suited to predictive biomechanical simulations. Third, with the assumption that healthy people naturally prioritize mechanical efficiency, disinclination to a motion is described as a cost function of joint torques, and for the first time, physical infeasibility including slipping and falling. This new dynamic optimization routine allows for motions of gradually increasing complexity while the model's performance is improving. Using these models and optimal control strategy together has produced gross motion patterns characteristic of healthy sit-to-stand when compared with normative data from the literature.

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INTRODUCTION

Sit-to-stand (STS), a skill with a direct impact on quality of life as an aspect of functional mobility [1], is required to perform actions of daily living [2]. Researchers have been studying sit-to-stand prediction for over 25 years. Initially, time histories of neural signals to lower extremity muscles were optimized. These signals were modelled by linearly interpolated nodes, and motions were determined by minimization of functions of muscle stresses and peak forces [3] or movement time [4]. The effects of varying muscle strength [4] and seat height [5] were sought. The interest in predictive sit-to-stand was later control-oriented, and triple inverted pendulum [6] and trajectory tracking [7-9] problems were investigated. Most recently, research motivations in this area have swung back toward a biomechanical focus. Motions were again predicted using dynamic optimization, but bilateral joint angle profiles [10] were modelled as controls. Each study has claimed to accurately predict sit-to-stand; however, attempts at validation have been inadequate. For example, no study has compared time histories of predicted STS kinematics or dynamics to normative experimental STS data.

The work presented here demonstrates the utility of a three-link, sagittal plane model with deformable buttocks in predictive sit-to-stand. With a future goal of predicting pathological STS, the model created is, for the first time, based on female anthropometrics, as women have proportionally more difficulty performing STS according to self-reporting studies [11].

For this three-degree of freedom model, the location of the hip joint centre and inclination of the upper body are chosen as the controls. Their locations in time are modelled using composite Bézier curves so that a control point has global rather than local influence on curve shape [12], and not only is it unnecessary to bound them in value when optimizing, but the solution space is also smoother. For the first time, the number of control points defining the optimal controls is not predetermined. This avoids either prematurely restricting the solution space or over-doing it and having a larger-than-necessary and potentially unwieldy optimization problem from the beginning.

Defining a cost function for the optimizer is an attempt at modelling preferences in motion. The cost function created here penalizes mechanical effort with respect to the foundational theory that people move in ways that are energy-efficient [13, 14] and, for the first time, motions contrary to standing, (i.e. slipping and falling), which an individual would avoid when getting up from a chair. All previous models have an associated fall-risk, either from neglecting implications of fixing their model's ankle at the ground or ignoring contact mechanics between the buttocks and chair, which this model will not suffer.

The optimal control strategy, dynamic optimization of time histories of the generalized coordinates, allows increasing motion complexity through an iterative technique. It is in harmony with the foundational belief that natural, practiced motions are optimal and learned. The complexity of the optimization problem is increased by performing degree elevation, but by seeding the solver with the solution of the problem with fewer control points, it remains manageable.

The first aim of this work is to model how a healthy individual rises from a seated position. After this aim is accomplished it is a natural progression to seek to predict pathological STS, the reality for an increasing proportion of the population struggling with this motion, from a model with modeled pathology. Once healthy and pathological STS can be predicted in simulation, the potential exists for testing the effects of changes to the patient or environment in simulation. This is a key motivation because, while it is possible to perform clinical studies to examine how people stand from seated, it is more economical and, in a patient population, more compassionate to perform preliminary testing in simulation.

The second aim is to compare the resulting motion to healthy sit-to-stand in a meaningful way, by discussing the physical feasibility of the resulting motion and comparing it with normative data in the literature, and thereby establishing a benchmark for future work in STS prediction.

METHODS

Predicting sit-to-stand requires a biomechanical model and theory of how STS is realized. The model proposed here has fixed feet, and dynamic legs, thighs, and head-arms-torso (HAT) with deformable buttocks. It is constrained to move in the sagittal plane, a simplification made on the basis that healthy people demonstrate sagittal symmetry in STS [15-18]. Body segment parameters of the model are based primarily on the description of a female [19-21] in anticipation of a long-term goal of predicting pathological sit-to-stand of female patient populations. The optimal control problem is

framed as a parameter optimization problem with the goal of minimizing the effort of completing STS, as performed in previous studies [3-5, 10, 22], but this time by modelling the time histories of generalized coordinates with composite Bézier curves.

Biomechanical model construction

A schematic of the human model, created in MapleSim [23], is seen in Figure 1. The arms of the model are crossed in front of the chest as is required of patients performing a clinical STS test [24]. Joint damping [25] and passive elastic moments [26] are included in the model. Knees are constrained from extending beyond straight to avoid bifurcation. Buttocks are added to the HAT as one-dimensional Kelvin-Voigt elements [27, 28] of representative female dimension [29] with a gap [30]. This Modelica model ensures continuity in the force profile and disallows pulling between the buttocks and chair [31].

The chair is assumed to be of steel construction, rigid, backless, armless, and of standard adjustable height [32]. A hyperbolic tangent regularized friction model, previously used for feet [33], is included between the buttocks and chair with coefficient of friction of canvas on steel [34].

Motion model – the controls

Based on previous work in predictive sit-to-stand, the optimal control problem is framed as a parameter optimization problem. The global (X, Y) position of the hip and angle of inclination of the HAT are characterized for the complete motion to fully control this three degree of freedom system. The choice of kinematics as controls is advantageous in

directly describing quiet sitting and standing poses. Moreover, the ability to define the hip position is useful in exploring the effects of seat height and relative anterior-posterior (AP) foot position, two of the three primary determinants of healthy STS [11], on the predicted motion. In this paper, the influence of seat height on STS is investigated while the foot position is held constant.

The controls are represented as three composite Bézier curves, or paths, in the time domain. Bézier curves are smooth parametric curves with points defined by a function of the form

$$p(u) = \sum_{i=0}^{n} p_i \binom{n}{i} u_i (1-u)^{n-i} \qquad u \in [0, 1] \qquad [12]\ (1)$$

where the term $\binom{n}{i}$ is the binomial coefficients. In matrix form this is,

$$p(u) = U\ M_B\ P \qquad\qquad (2)$$

where U is a 1 x n+1 row vector, M_B is the n+1 x n+1 Bézier basis transformation matrix, and P is the n+1 x 1 column vector of two-dimensional control points, p. To transform between u and time, u is multiplied by the final time.

The initial paths are shown in Figure 2. The sitting component is the first part of each path and describes the coordinate in quiet sitting as two sitting control points of equal value, as seen in the figure. The motion component is the middle part of each path and begins and ends with two control points of sitting and standing values, respectively, to enforce first derivative continuity. The standing component is the final part of each path and again has two control points to parameterize quiet standing.

Coordinate values for sitting and standing are determined primarily from experimental joint angles reported in the literature [35]. It was necessary to the purpose of this study that the model contact the chair in sitting; therefore, freedom was given to the sitting hip height coordinate to meet this requirement while conserving the relative AP position of the hip from the ankle described in the literature. Freedom was also granted to the standing hip AP position to allow for natural patterns of balance in quiet stance [36]. In this parameterization of a human motion, there are bounds that must be observed. The hip height is bounded to enforce contact with the chair during sitting, and all points must maintain their sequence in time to prevent the model from being directed to two places at once. That is,

$$p_{i,1} > p_{i-1,1} \qquad i = 1,...,n+1 \qquad (3)$$

Beyond this, only the number and value of control points restricts possible motions. A five second window is prescribed for the path, including a half second for each of the sitting and standing components.

Motion preference – performance criterion

Candidate motions are evaluated for optimality, including feasibility. Impossible configurations ($error_h$), failed momentum transfer ($error_t$), and lifting ($error_l$) or slipping ($error_s$) of the foot are penalized as follows,

$$cost_{error} = w_h \int_0^5 error_h^2 \, dt + w_t \int_0^5 error_t^2 \, dt + w_l \int_0^5 error_l^2 \, dt + w_s \int_0^5 error_s^2 \, dt \qquad (4)$$

where $w_h = 1N^2$, $w_t = 10^3N^2$, $w_l = 1m^2$, and $w_s = 1m^2$ are weighting factors and $error_h$,

$error_t$, $error_l$, and $error_s$ are excursions of the specified hip location beyond the model's

workspace, excursions of the centre of pressure beyond the base of support, pulling

forces at the ankle, and lateral forces at the ankle exceeding stiction, respectively.

Details of how these errors were calculated are provided in the appendix. In a successful

STS transfer, there is zero cost associated with these errors.

Beyond feasibility, optimality is determined in accordance with the theory that

the healthy population performs everyday motions in ways demanding minimal

exertion, which in this model is considered as the time history of active joint torques

[10] determined by inverse dynamics. Therefore, the effort of a candidate STS motion is

evaluated as the sum of required joint torques, squared, as in the equation below.

$$cost_{torque} = \int_0^5 A_M^2 \, dt + \int_0^5 K_M^2 \, dt + \int_0^5 H_M^2 \, dt \qquad (5)$$

where A_M, K_M, and H_M are the net ankle, knee, and hip joint moments, respectively.

The overall cost of a candidate STS motion is

$$cost = cost_{torque} + cost_{error} \qquad (6)$$

Computation of optimal controls

The paths, and therefore the candidate STS motions, are determined by the locations of

the control points. Initially, the possible shapes of these curves are limited as only the

two intermediate points of each path may move in time and only points parameterizing

the hip height while sitting, and hip AP position while standing may change in value. For

greater freedom, two control points, free to move in time and value, are added to the

motion component of each path while preserving the shape of the path by the process

of degree elevation.

By degree elevation, the number of control points is increased while the shape of

the curve is maintained. Because both the previous (p) and new (p') set of control points

must generate the same curve, it is true that

$$p'(u) = p(u)$$

For one degree elevation, this is

$$\sum_{i=0}^{n+1} p'_i \binom{n+1}{i} u_i(1-u)^{n+1-i} = \sum_{i=0}^{n} p_i \binom{n}{i} u_i(1-u)^{n-i}$$

Reference [12] shows this is true when

$$p'_i = \left(\frac{i}{n+1}\right) p_{i-1} + \left(1-\frac{i}{n+1}\right) p_i \qquad i = 0,\ldots,n+1 \qquad (7)$$

In short, a Bézier curve described by a set of control points may be described by a larger

set of control points determined from the original set.

An initial optimization problem is solved to establish a feasible STS starting-point.

The initial motion is passed to the model, and errors in feasibility are calculated. Control

point locations are adjusted by *fmincon* in MATLAB [37] to decrease errors (eq. 4). The

routine exits with the first solution with zero associated error. This STS motion is used to

seed the solver in the iterative dynamic optimization routine.

The iterative dynamic optimization routine is shown in Figure 3. The initial

motion is passed to the model, and errors in feasibility and the joint torques required to

complete the STS are calculated. Control point locations are adjusted by *fmincon* to

decrease cost (eq. 6). The optimal control points are those that minimize cost with zero

associated error. The cost of this candidate motion is recorded when the routine

terminates. This process is repeated after elevating the degree of each path by one,

giving increased freedom to possible solutions. This process is iterated until successive

solutions demonstrate convergence. The range of required joint torques is evaluated

against normative joint torque strengths [18], as in Figure 4, as a final, manual, check of

feasibility and a prediction acceptance criterion.

Comparison with normative sit-to-stand

A predicted STS is evaluated against two sets of normative data: one, from a paper by

Nuzik et al., describing angular positions at evenly-spaced intervals of STS [35] and the

other, from a paper by Kralj et al., defining the timing of kinematic and kinetic events

[15] of STS in a healthy population. As has been mentioned, STS is strongly influenced by

seat height and AP foot location. It is impossible to replicate definitively the sitting pose

in either paper with the limited information given for the stature of our biomechanical

model, so the results of three conditions are examined. The first considers a chair height

of 51cm, the maximum height of a standard adjustable chair [32], which provides best

agreement to sitting joint angles from Nuzik et al. The second considers a chair lowered

to 46cm, the only height common between papers, with foot location kept constant.

The third results are for a chair furthered lowered to 42 cm, a height within range of the

chairs used by Kralj et al., and the minimum height of a standard adjustable chair. All

chair conditions are for the same subject, as if the same person sat in three different

chairs.

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The kinematics and kinetics calculated, for the optimal paths predicted, are next compared to the normative occurrence of events separating the phases of STS: quiet sitting, initiation, seat unloading, ascending with vertical acceleration, deceleration, stabilization, and quiet standing [15].

RESULTS

STS predictions are fully parameterized by the locations of the control points. The initial and optimal coordinate paths of STS from a 46cm chair are shown in Figures 2 and 5, respectively. As can be observed, the prediction reshapes the paths of the generalized coordinates and shortens the duration of STS from 3.96s to 1.24s. The joint torques associated with this motion are shown in Figure 4 and are within joint torque strengths of old females reported in the literature [18].

Paths of optimal control points were input to the model and ground reaction forces were determined through inverse dynamics to define the start and end of the predicted STS [15]. The resulting motion is shown as a series of snapshots in Figure 6. The prediction looks reasonable, as has been said of past work. From sitting, the model flexes the HAT, the buttocks lift from the chair, and the ankles dorsiflex and then return to a neutral posture while the knees and hips extend to standing, as would be expected in healthy STS [16]. To examine how valid is this result, it is compared to normative data from the literature in Figures 7 and 8.

The differences between predicted results and normative data are most pronounced during standing. The STS model chose a standing posture with knees

straighter and HAT more forward than observed in the experiment. However, the predicted angles closely follow the normative trends during STS and, from the 51cm chair, are often within 1 standard deviation of healthy variation at the knee and ankle. Because of the model's stature, initial angles are deviated for the lower chairs. The prediction with 46cm chair faithfully represents the range of ankle dorsiflexion from Nuzik et al. from a chair of equal height as seen in Figure 8. A trend of increasing peak hip flexion with decreasing chair height was noticed in the hip angle curve of Figure 8. And the peak flexion angular velocity of the hip increased as the chair height decreased from 51cm to 46cm and 42cm as seen in Figure 9.

From all seat heights, the final event of STS, standing on, was predicted to occur within 1.5 seconds, much quicker than even the minimum of what was observed in experiment. However, when event occurrences from the lower chairs are considered as a percent of STS and compared to their normative occurrence in STS, all but event 4 are predicted within range and most are within one standard deviation of what is expected from a healthy population.

DISCUSSION

This study has described how a healthy female stands from a seated position using a three-link biomechanical model. The proposed model is the most comprehensive planar model used for predicting sit-to-stand to date and is capable of producing the following gross motions of healthy STS when the chosen controls are driven: HAT flexion, seat off, ankle dorsi- and plantar flexion, knee and hip extension.

Minimizing exertion using a function of joint torques (eq. 5) when evaluating candidate motions is not new; however, minimizing infeasibilities (eq. 4) in the motion is. These performance criteria appear to contain some legitimacy in how healthy people stand from seated position and as a result, it predicts STS with generally good agreement to experimental results in the literature [15, 16, 35].

A future goal of this work is in predicting pathological STS. This goal will require adjusting the model to capture aspects of pathology, and investigating if the predicted results are in harmony with STS as affected by the given pathology. For example, by increasing the stiffness of joints or decreasing the admissible joint torque strengths, arthritis or muscle atrophy may be imitated. Although a more detailed model may be more directly applicable to modelling pathology, it is reasonable to examine what may be predicted using the model in this paper and building from it as appropriate.

This is the first use of Bézier curves in dynamic optimization of STS, an application for which they have proven advantageous. Their shape-conserving properties in degree elevation allow the solution space to start with a small number of control points, or variables, to optimize. The iterative optimization routine maintains a relatively smooth solution space and facilitates the use of a computationally inexpensive, gradient-based solver.

It was noticed that predictions of STS from decreasing chair height produce increasing angular displacements. This is a phenomenon observed in a healthy population [11] and the first serendipitous result of this work. A second is the accurately predicted trend of increased hip flexion angular velocity with decreased chair height,

from approximately 100% knee height to approximately 80% knee height [11, 38], at STS initiation. Finally, the observation that the demand of predicted STS, defined in terms of cost (eq. 6), increases with decreasing chair height as expected [11].

The consistencies between STS predicted in this theoretical work and STS reported from experiments speak to the quality of this model and the motion optimization approach. Predicting the over-all motion patterns observed in the STS experiments of Schenkman et al. [16] and Nuzik et al. [35], and the characteristic events of STS defined from the experiments of Kralj et al. [15] is unprecedented in predictive STS research. These results, for a healthy model, give confidence that the model and STS prediction strategy are a credible starting point to predicting pathological STS.

However, there are some places where the prediction fell short. The model did not faithfully predict the standing posture reported in the literature and assumes an end posture with the ankles less flexed and the HAT more inclined than expected. From Figure 4 it is clear that the predicted standing posture is more cost effective (eq. 5) than the prescribed standing posture. This difference is indicative that people prioritize more than mechanical efficiency when standing, such as spatial awareness and the ability to reject an environmental disturbance, and this model does not address these. Regardless, this predicted end-posture describes statically stable standing and deemed acceptable for this exercise.

It is difficult to put these results into context in the greater research community because of the minimal comparison of existing studies to other studies or experimental findings. It is possible to say that this model is superior to others in that it is physically

plausible where others are not, in terms of attention to torque limits, for example. It is more versatile than existing models that use, at best, a pre-set numbers of nodes, evenly spaced in time. Possibly the greatest boon of this purely predictive work to another researcher is the unprecedented comparison of both kinematic and kinetic results to normative data that should serve as a benchmark for future work in STS prediction.

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APPENDIX

The position of the foot is predetermined and fixed. This introduces the possibility for the model to respond unnaturally in situations where a foot ought to lift from or slip relative to the ground. These cases are determined considering the foot as in the free body diagram (Figure 10) below. The foot of the model is of known dimension (d) [20] and weight (W) [19]. The coefficient of static friction (μ) between skin and metal [34] is assumed between the foot and ground. For forces, A_Y, A_X, and moment A_M, the system is determined and it is possible to solve the static equilibrium equations for r_N, F_f and N to establish if the conditions for static equilibrium are in violation.

The modelled system is in error when conditions are incongruent with static equilibrium of the foot. The following equations quantify these errors. First, when the modelled foot, were it not fixed to the ground, ought to lift,

$$error_l = \begin{cases} |N| & N < 0 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

When it should slip,

$$error_s = \begin{cases} F_f - \mu N & F_f > \mu N \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

And when it should tip,

$$error_t = \begin{cases} r_N - d_{toe} & r_N > d_{toe} \\ r_N + d_{heel} & r_N < -d_{heel} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

A last implication of fixing the feet is defining the model's workspace. The chosen

controls, described in the following section, may specify an (X, Y) hip location outside of

its workspace, so the following equation is added

$$
error_h = \begin{cases} (X^2 + Y^2)^{\frac{1}{2}} - (l_t + l_l) & (X^2 + Y^2)^{\frac{1}{2}} > l_t + l_l \\ 0 & \text{otherwise} \end{cases} \tag{11}
$$

where l_t and l_l are the length of the thigh and leg, respectively.

Figure Captions List

Fig. 1   A schematic of the three-link sagittal plane model while seated

Fig. 2   Three Bézier curves describing a STS motion with sitting, motion and

standing components

Fig. 3   Iterative routine to determine an optimal STS

Fig. 4   Joint torques for optimal STS from a 46cm chair compared to joint torque

strengths [18]

Fig. 5   Optimal STS from a 46cm chair

Fig. 6   Evenly spaced snapshots of predicted STS

Fig. 7   Predicted STS event timing from three chairs compared to experimental

means, standard deviations, and ranges [15]

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Fig. 9   Maximum flexion angular velocity of the hip

Fig. 10   Free-body diagram of the foot