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Estimation of Maximum Muscle Contraction Frequency in a Finger Tapping Motion Using Forward Musculoskeletal Dynamic Simulations

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ABSTRACT — *A model for forward dynamic simulation of the rapid tapping motion of an index finger is presented in this paper. The one-degree-of-freedom finger model is actuated by two muscle groups (one as a flexor and the other as an extensor). The goal of this analysis is to investigate the maximum tapping motion frequency that the modelled muscles can achieve. To solve the muscle force-sharing problem, each muscle excitation signal is parameterized by a sixth-order polynomial function of time. The results show that the model can match the maximum achievable tapping frequency observed in experimental studies. Using this modelling technique, we can postulate on the mechanisms that limit human capabilities in producing motion.*

1 Introduction

Human motions are produced by muscle contractions. To study the human motions, one should solve a problem where the number of unknowns (muscle contraction levels) is much greater than the number of degrees-of-freedom (DoF). This problem is usually referred to as the muscle force-sharing problem in musculoskeletal system modelling, for which there is no unique solution. To pick one solution out of the many possible solutions, extra criteria should be considered. A common practice is to search for a solution that minimizes some physiological index through an optimization process.

When the goal is to find the optimal time-history of a signal (e.g. muscle forces or activations through the course of an action) one must solve an Optimal Control Problem (OCP). In the books by [1, 2, 3], several approaches for solving a general optimal control problem are presented, including linear quadratic regulator (LQR) control, linear quadratic Gaussian (LQG) control, variational approaches such as direct collocation (DC), model predictive control (MPC), and parameterization. For all techniques, pros and cons are involved. It should be noted that not all of those approaches are applicable to musculoskeletal modelling; for instance, LQR and LQG are for linear systems only, MPC normally works in linear or linearized systems with quadratic optimization form only, and DC requires a complicated implementation, and has been scarcely applied recently [4].

Dynamic Optimization (DO), in spite of high computation cost, results in more realistic results as it considers all the time-course in the optimization procedure, and solves for the time-history of the decision signals. Therefore, in contrast to Static Optimization (SO), DO takes into account the effect of previous time instants on the current instant of simulation.

Local parameterization has been used by a few researchers, e.g., [5, 6]. Locally parameterizing the control signals or state variables sounds like a promising approach as it captures the local dynamics of the system as long as the local considered windows are small enough. However, by increasing the number of parameterization windows, the scale of the optimization problem, and therefore CPU time, increases significantly.

Using a control signal parameterization method, the OCP is converted to a nonlinear optimization problem by using parametric pattern functions as the control inputs. Different parameterization functions might be used, based on the information of the system, degree of nonlinearity, and a priori data. Global and local parameterization might

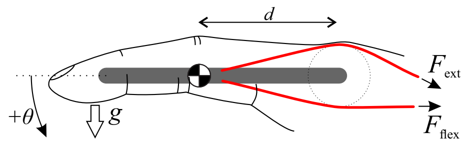


Fig. 1: The schematic of the musculoskeletal finger model

be utilized. For instance, different types of functions can be used for the global control parameterization, such as Fourier series [7, 8, 9], or local functions within finite windows of the simulation using splines [6]. Although global parameterizing, compared to local parameterization, seems to be possibly missing some local dynamics of the system, this approach will provide good sub-optimal results in general. Also, for applications with no drastic changes in the control signals (a priori knowledge of the system behaviour is required), global parameterization will output reasonable results. In addition, global parameterization will reduce the number of decision variables considerably, which results in significant reduction of the CPU time.

The goal of this study is to develop and present a musculoskeletal model for the index finger, which can be used to investigate the dynamics of a fast tapping movement, and to estimate the maximum tapping frequency. The forward dynamics nature of the simulations enables us to study what-if scenarios that are necessary in design optimization and ergonomics. For this purpose, we have used dynamic optimization using a global parameterization to solve the optimal control problem.

In this paper, first we introduce the details of the musculoskeletal index finger model. Then the optimization problem and our approach to solve it is presented. Selected simulation results and the discussion are presented next, which are followed by the paper conclusions.

2 Finger Tapping Musculoskeletal Model

In this section, the musculoskeletal model for forward dynamic simulation of the rapid finger tapping motion is presented. The model consists of a rigid index finger rotating around the metacarpal-phalangeal joint (e.g. a one-dof pendulum, see Fig. 1) with two muscle groups: one as flexor and the other as extensor.

The muscle model is a three-element Hill model based on [10]. The activation and contraction dynamics expressions employed for this model are presented in the Appendix.

The following assumptions are made for the finger modelling and simulation:

1. The maximum isometric force F^m_{max} is assumed to be 100 N for both the extensor and the flexor muscles. It seems reasonable for the flexor muscle since it is supposed to act as a resultant of all flexor muscles. For the sake of similarity of the flexor and the extensor, the same value is assumed for both.

2. Anthropometric properties of the index finger, including length, mass and moment of inertia are taken from [11]. The composite moment of inertia is calculated given the moments of inertia of the three phalanges of the index finger.

3. Muscle moment arms are assumed to be constant during the motion because of small finger rotation amplitude, and both radii are assumed to be 10 mm, which agrees with the dimensions of metacarpophalangeal joint [12].

The dynamics of the index finger can be summarized as follows:

$$\ddot{\theta} = \frac{1}{I}\left(mgd\cos(\theta) + r_f F_f - r_e F_e\right) \tag{1}$$

where θ is the index finger angle (positive in flexion direction), and m and d are the finger mass and the centre of mass location, respectively. r_f and r_e are the flexor and extensor moment arms, which are multiplied by flexor and

parameter	value
I	2×10^{-5} kg.m^2
m	0.25 kg
d	0.040 m
F_{max}	100 N
L_{ce}^{opt}	0.178 m
r_f, r_e	0.01 m

Tab. 1: List of simulation parameters

extensor muscle forces, F_f and F_e, to produce the joint moments. The list of all the simulation parameters and their numerical values are listed in Tab. 1.

For the tapping motion, the desired joint angle is defined as follows:

$$\theta_d(t) = 0.21 \sin(\omega_d t) \tag{2}$$

where 0.21 rad is the amplitude of the considered motion according to [13], and $\omega_d = 2\pi f_d$, in which f_d is the frequency of the sinusoidal motion.

Forward dynamics is the simulation approach. The inputs to the simulations are the two muscle excitation signals, which result in muscle forces. The equations of motion are then integrated forward in time to find the changes in finger kinematics. The forward dynamics simulation allows us to study the maximum frequency of the tapping motion through mathematical modelling. This is the advantage of forward dynamics over inverse dynamics, since "what-if" questions can be answered.

3 Optimization Problem Description

To define the control signals through the course of the tapping motion, they are globally parametrized by 6th-order polynomials:

$$u = \sum_{i=0}^{6} p_i t^i \tag{3}$$

where p_i is the i^{th} polynomial coefficient. The first reason for assuming such a pattern is that filtered, rectified, and normalized EMG signals are quite smooth and can be curve-fitted by a suitable continuous mathematical function such as a polynomial, and the second is that assuming a continuous and continuously differentiable function like a polynomial will help the optimizer to meet the nonlinear constraints on the excitation signal within the optimization problem definition. Thirdly, assuming a parametric continuous function may possibly lead to symbolic simplifications and analytical solutions.

Therefore, the optimizer job is to look for the optimal coefficients of the two control signals, for a total of 14 variables. A set of nonlinear constraints will be imposed to the problem to meet the bounds on the neural excitations, i.e., $0 \leq u \leq 1$.

The objective function for simulating this model is defined as a linear combination of two cost functions $J = \mu J_1 + (1-\mu) J_2$:

$$J = \frac{\mu}{\tau_d} \sum_{j=1}^{2} \int_0^{\tau_d} a_j^2 \, dt + \frac{1-\mu}{\tau_d \, \sigma(\theta_d^2)} \int_0^{\tau_d} (\theta_s - \theta_d)^2 \, dt \tag{4}$$

where the first term in the objective functional describes the physiological effort index based on [14], whereas the second term accounts for the tracking error. In (4), $\tau_d = 1/f_d$ is the desired motion period, j is the muscle index, θ_s and θ_d are the simulated and desired joint angles, respectively, and σ refers to standard deviation. The weight factor μ indicates the relative importance of the physiological term against the tracking error. Since in this

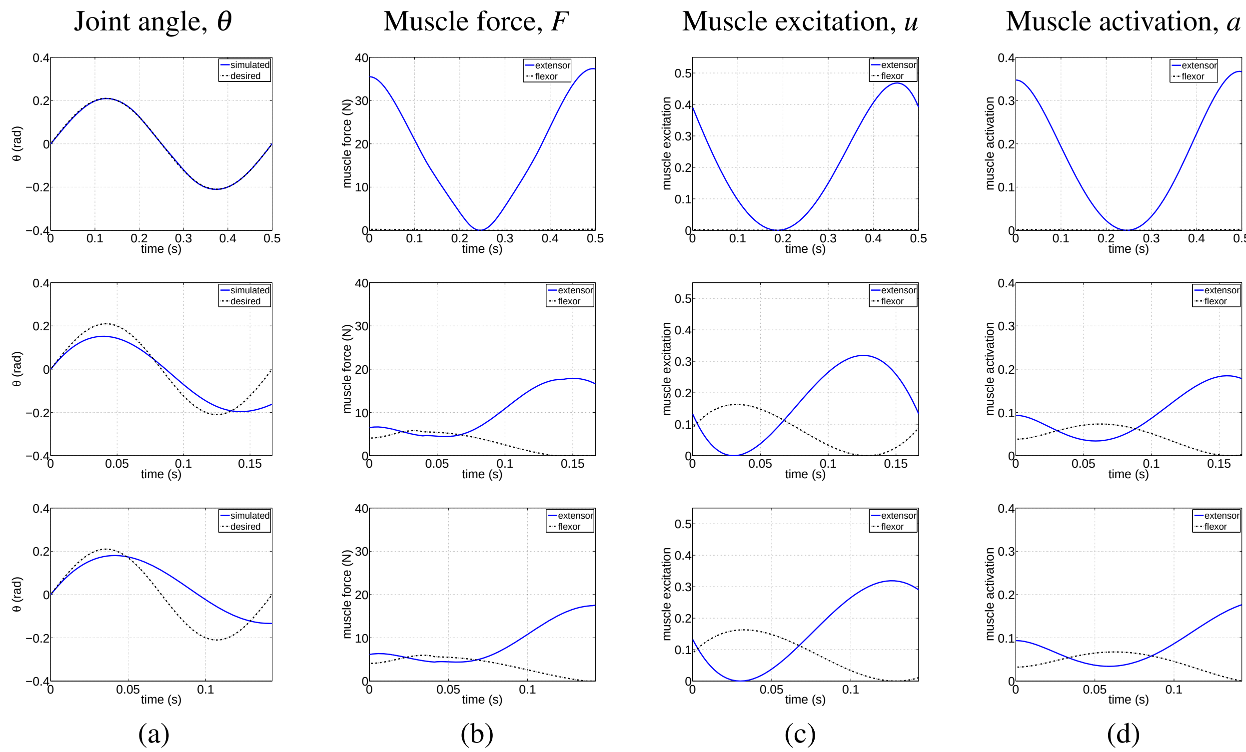


Fig. 2: The simulation results for three different frequencies: 2 Hz (top), 6 Hz (middle), and 7 Hz (bottom). Column (a) shows the tracking performance, and columns (b)-(d) show the muscle force, excitation, and activation respectively.

simulation, tracking of the motion is much more important, the weight factor is assumed to be $\mu = 0.1$. It must be noted that the objective functional is written so that each term is dimensionless.

The optimization procedure was similar to that used in [7, 8, 9]. In brief, Sequential Quadratic Programming (SQP) as implemented in the `fmincon` function in the Optimization Toolbox of Matlab® is used as the optimizer. For the initial guess needed in SQP, results of the same case using a Genetic Algorithm as the optimizer were used. For more details on the optimization set up and the convergence criteria, refer to [7, 8].

4 Results

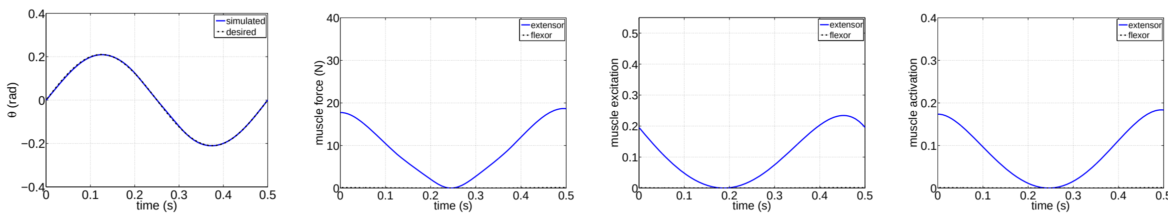
Different sets of simulations were run. In the first set of simulations, the major focus has been on motion frequency variation. A separate set of simulations was also done to see the finger mass effect. Also, it was investigated how the optimization weight factor would affects the results.

The first set of results is presented in Fig. 2. In this investigation, the focus has been on how increasing the motion frequency affects the results. The purpose was to find the maximum frequency that this biomechanical system could follow. Motion frequency started from 2 Hz (top row in Fig. 2) and was increased to 3, 4, 5, 6 (middle row), and 7 Hz (bottom row), where it was observed that the system was not able to produce the desired motion any more. The plots show θ_d and θ_s (desired and simulated motions), muscle forces, excitations, and activations (respectively, from left to right). When the system fails to follow the desired motion, the value of the total cost function increases significantly (see Tab. 2), and the created motion differs from the desired motion.

A separate study was also done to investigate the sensitivity of the simulation results to the finger mass. To this goal, finger mass and moment of inertia are reduced to 50%, and the optimal control problem is resolved for this case at f_d=2 Hz. Optimal muscle excitations, activations, and forces of this case are shown in Fig. 3. Comparing these results with the same case in Fig. 2 (top row, f_d=2 Hz) shows that the quality of the motion tracking is the same, but the excitation values in the case with 50% mass is roughly half the original values. This is reasonable since the dominant term in the dynamics of the finger is the inertia. Moreover, the motion velocity is not high

Frequency (Hz)	J	J_1	J_2
2	0.041	0.101	0.035
4	0.196	0.145	0.201
5	0.211	0.176	0.215
6	0.898	0.109	0.986
7	1.553	0.097	1.714

Tab. 2: Variation of motion frequency and cost function values



(a) (b) (c) (d)

Fig. 3: Optimal results for f_d=2 Hz and 50% of index finger mass: (a) motion tracking, (b) forces, (c) activations, and (d) excitations

enough to significantly affect the force-velocity relation.

5 DISCUSSION

There are a number of studies in the literature on finding the maximal frequency or speed at which a finger can move. Kuboyama et al. [15] mentioned 6.46 Hz while Morrison and Newell [16] reported 6.92±0.56 Hz. The results of this study imply that this maximal frequency is around 6 Hz which is close to the available values in the literature. These mentioned references have measured the desired value experimentally, so the maximal motion frequency extracted from the results of this study predicts the experiments reasonably well.

When f_d=2 Hz, the simulated and the desired motions are identical, resulting in a small value for J_2 (see Tab. 2). Furthermore in this small frequency, the extensor activity is much more than the flexor. This is due to the fact that the muscles are uni-articular (they span only one joint), and theoretically no co-activation should occur in the optimal results [17]. The results imply that at a low frequency, the gravity can produce enough flexion acceleration; thus, the flexor muscle has negligible activity.

As the motion frequency increases, the ability of the finger to follow the desired motion decreases, which can be observed from the increased J_2 values in Tab. 2. Also the physiological effort to track the desired motion is increased (greater J_1 value). From 6 Hz on, it is observed that, although the total cost function increases, the physiological term, J_1, decreases; it means that in high frequencies, increasing the muscle activations does not help to better follow the desired trajectory. By looking at Fig. 2, it is seen that the optimization process has found the simulated trajectories at the least activation efforts, although those trajectories do not look similar to the desired ones.

Multiple phenomena may contribute to the inability of the finger to follow the desired trajectory. The excitation/activation coupling model causes a time delay between neural excitation and activation signals, as well as a small scaling between these two signals [18]. These dynamics are an important cause of the inability to follow fast oscillations. The excitation/activation dynamics essentially performs as a low-pass filter with bandwidth of ∼4.5 Hz (defined as 3 dB amplitude attenuation). At 6 Hz, the amplitude attenuation is 4 dB; i.e., the amplitude of the activation signal is 63% of that of the excitation.

The attenuation of the activation signal amplitude is exacerbated by the muscle properties. A muscle's force production capacity drops as the muscle contraction velocity increases (see Fig. 4). At high motion frequencies, the required contraction velocity is more than the velocity at which the muscle can produce the force to satisfy the

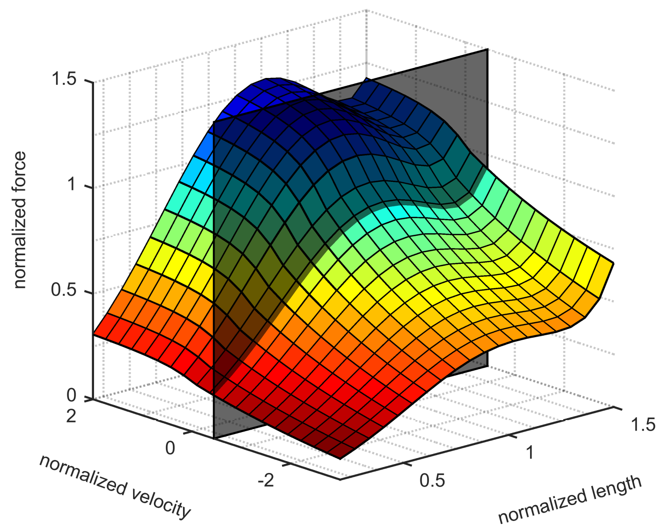


Fig. 4: The force-length-velocity relation in the muscle model

equations of motion. Figure 4, which shows the force-length-velocity relationship, implies that when the concentric contraction velocity increases, the force production ability decreases. Therefore, the muscle can move faster only if it can produce enough force to satisfy the equations of motion. At around 6 Hz (the maximal frequency), the muscle must contract with the maximum velocity of 79.2 mm/s, which along with the filtering effect, will lead to small force generation ability. Since the muscle cannot create enough force at such a velocity in order to satisfy the equations of motion, it is not able to move at this velocity, and can not track the desired motion.

The simulation with altered finger mass showed that the modelling framework is able to simulate the system response even with a large change in model mass. This is a necessary feature for subject-specific simulations. With a more detailed sensitivity analysis, we can investigate the effects of system parameters on the system dynamics, as well as the intended outcomes. For example, including contact dynamics in the musculoskeletal models can allow for design optimization of musical instruments, such as the piano keys, or enhance the ergonomics of computer keyboards.

6 CONCLUSIONS

In this paper we presented a musculoskeletal modelling framework to study the fast finger tapping motion. The forward dynamics simulations show that maximum achievable motion frequency is roughly 6 Hz, matching the experimental observations. We have made arguments that the limiting factor in this case is the excitation/activation filtering effect, as well as the inability of muscles to produce enough force at high contractile velocities. Further studies using non-linear system analysis tools can provide more insight about the limits and requirements of musculoskeletal systems during task executions.

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References

[1] D. E. Kirk, *Optimal Control Theory: An Introduction*. Mineola, New York: Dover Publications Inc., 2004.

[2] J. Betts, *Practical Methods for Optimal Control Using Nonlinear Programming*. Philadelphia: SIAM, 2001.

[3] B. C. Chachuat, *Nonlinear and Dynamic Optimization: From Theory to Practice.* Switzerland: Automatic Control Laboratory, EPFL, 2007.

[4] M. Ackermann and A. J. van den Bogert, "Optimality principles for model-based prediction of human gait," *Journal of Biomechanics*, vol. 43, no. 6, pp. 1055–1060, 2010.

[5] M. Ackermann, *Dynamics and Energetics of Walking with Prostheses*. PhD Thesis, University of Stuttgart, Germany, 2007.

[6] D. Garcia-Vallejo and W. Schiehlen, "3d-simulation of human walking by parameter optimization," *Archive of Applied Mechanics*, vol. 82, no. 4, pp. 533–556, 2012.

[7] M. S. Shourijeh and J. McPhee, "Optimal control and forward dynamics of human periodic motions using Fourier series for muscle excitation patterns," *Journal of Computational and Nonlinear Dynamics*, vol. 9, no. 2, p. 021005, 2014.

[8] M. S. Shourijeh and J. McPhee, "Forward dynamic optimization of human gait simulations: A global parameterization approach," *ASME Journal of Computational and Nonlinear Dynamics*, vol. 9, no. 3, p. 031018, 2014.

[9] R. Sharif Razavian, N. Mehrabi, and J. McPhee, "A Neuronal Model of Central Pattern Generator to Account for Natural Motion Variation," *Journal of Computational and Nonlinear Dynamics*, vol. 11, p. 021007, aug 2015.

[10] D. G. Thelen, "Adjustment of muscle mechanics model parameters to simulate dynamic contractions in older adults," *Journal of Biomechanical Engineering*, vol. 125, no. 1, pp. 70–77, 2003.

[11] H. M. Buchner, H. and H. Hemami, "A dynamic model for finger interphalangeal coordination," *Journal of Biomechanics*, vol. 21, no. 6, pp. 459–468, 1988.

[12] A. Unsworth and W. J. Alexander, "Dimensions of the metacarpophalangeal joint with particular reference to joint prostheses," *Engineering in Medicine*, vol. 8, no. 2, pp. 75–80, 1979.

[13] P. L. Kuo, D. L. Lee, D. L. Jindrich, and J. T. Dennerlein, "Finger joint coordination during tapping," *Journal of Biomechanics*, vol. 39, no. 16, pp. 2934–2942, 2006.

[14] R. Happee, "Inverse dynamic optimization including muscular dynamics, a new simulation method applied to goal directed movements," *Journal of Biomechanics*, vol. 27, no. 7, pp. 953–960, 1994.

[15] N. Kuboyama, N. Nabetani, K. Shibuya, K. Machida, and T. Ogaki, "The effect of maximal finger tapping on cerebral activation," *Journal of Physiological Anthropology and Applied Human Science*, vol. 23, no. 4, pp. 105–110, 2004.

[16] H. S. Morrison, S. and K. Newell, "Upper frequency limits of bilateral coordination patterns," *Neuroscience Letters*, vol. 454, pp. 233–238, 2009.

[17] J. Kerwin and D. Challis, "An analytical examination of muscle force estimations using optimization techniques," *Journal of Engineering in Medicine*, vol. 207, pp. 139–148, 1993.

[18] J. He, W. S. Levine, and G. E. Loeb, "Feedback gains for correcting small perturbations to standing posture," *IEEE Transactions on Autonomic Control*, vol. 36, pp. 322–332, 1991.

Appendix: Thelen's Muscle Model Formulation [10]

Excitation/Activation Dynamics

The excitation/activation dynamics is described as:

$$\dot{a}(t) = \frac{u-a}{\tau(u,a)} \tag{5}$$

where

$$\tau(u,a) = \begin{cases} t_1\hat{a} & u \geq a \\ t_2/\hat{a} & u < a \end{cases} \tag{6}$$

and

$$\hat{a} = 0.5 + 1.5a \tag{7}$$

The parameter values of $t_1 = 15\ ms$ and $t_2 = 50\ ms$ are taken from [10].

Tendon Force

The tendon force is normalized to muscle maximum isometric force F^m_{max}, and is represented as an exponential function of the tendon strain:

$$\tilde{f}^t = \begin{cases} \dfrac{\tilde{f}^t_{toe}}{e^{k_{toe}}-1}\left(e^{\dfrac{k_{toe}\varepsilon^t}{\varepsilon^t_{toe}}}-1\right) & \varepsilon^t \leq \varepsilon^t_{toe} \\ k_{lin}(\varepsilon^t - \varepsilon^t_{toe}) + \tilde{f}^t_{toe} & \varepsilon^t > \varepsilon^t_{toe} \end{cases} \tag{8}$$

where ε^t is engineering strain of tendon (calculated based on the slack length l_{slack}), ε^t_{toe} is a limit after which the tendon relation switches to the linear expression, k_{toe} is a shape factor, k_{lin} is the linear slope of the second condition, and $\tilde{f}^t_{toe}$ is the function value at ε^t=ε^t_{toe}. Values of the parameters are adopted from [10]: k_{toe}=3, $\tilde{f}^t_{toe}$=0.33, ε^t_0=0.04, ε^t_{toe}=0.609ε^t_0, and k_{lin}=1.712/ε^t_0.

Parallel Elastic Element

The relation for muscle passive force normalized to muscle maximum isometric force F^m_{max} is expressed as:

$$\tilde{f}^{pe} = \frac{e^{\dfrac{k^{pe}(\tilde{l}^{ce}-1)}{\varepsilon^m_0}}-1}{e^{k^{pe}}-1} \tag{9}$$

where $\tilde{l}^{ce}$ is the muscle fiber length normalized to l^{ce}_{opt}, k^{pe} is a shape parameter set to 5, ε^m_0 is called the passive muscle strain and adopted to be 0.6 (for young adults).

Force-Length-Velocity Relation

The force-length relation is written as:

$$f^{ce}_{isom} = e^{-\dfrac{(\tilde{l}^{ce}-1)^2}{\gamma}} \tag{10}$$

where γ is a shape factor and is set to be 0.45.

Afterwards, the total force-length-velocity in this muscle model can be formulated as the following:

$$v^{ce} = (0.25 + 0.75a)\, v^{ce}_{max} \frac{\tilde{f}^{ce} - a f^{ce}_{isom}}{b} \tag{11}$$

where

$$b = \begin{cases} a f^{ce}_{isom} + \tilde{f}^{ce}/A_f & \tilde{f}^{ce} \leq a f^{ce}_{isom} \\ \dfrac{(2 + 2/A_f)(a f^{ce}_{isom} \hat{f} - \tilde{f}^{ce})}{\hat{f} - 1} & \tilde{f}^{ce} > a f^{ce}_{isom} \end{cases} \tag{12}$$

Here, v^{ce} is the fiber velocity (velocity of CE element), $\tilde{f}^{ce}$ is the force of CE element normalized to maximum isometric force, $\hat{f}$ is the normalized asymptotic eccentric force (equal to 1.4 for young adults), A_f is a shape parameter (adopted to be 0.25), v^{ce}_{max}=10 l^{ce}_{opt} m/s is the maximum contraction velocity of the muscle fiber.