**Historical Background:**

- **c. 1900:**
  - Classical mechanics became experimentally untenable:
    - Black body radiation ("Ultra-violet catastrophe")
    - Photoelectric effect (Ionization depends on color, not intensity)
    - Stability of matter \( \Delta x \Delta p \geq \frac{\hbar}{2} \) implies that e\(^{-} \)
      (do not spiral into the nuclei)

- **c. 1925:**
  - Heisenberg discoures nonrelativistic quantum mechanics (QM):

- **Equations of motion stay the same, e.g.:**
  \[
  m \ddot{x} = -k \dot{x} \quad \text{(harm. oscillator)}
  \]

- but we have noncommutativity:
  \[
  \left[ x, \hat{p} \right] = i\hbar \quad \text{"canonical commutation relation"}
  \]

**Remark:**

- Schrödinger discovered his equation
  \[
  i\hbar \frac{\partial}{\partial t} \psi(x,t) = -\frac{\hbar^2}{2m} \Delta \psi(x,t) + V(x,t) \psi(x,t)
  \]

  ½ year later.

- Dirac (same year) showed equivalence to Heisenberg's.
Quantization implied fundamental changes:

Math: \([\hat{x}(t), \hat{p}(t)] = i\hbar \Delta t \equiv 0 \Rightarrow \hat{x}(t), \hat{p}(t)\) not number-valued.

Q: Could \(\hat{x}(t), \hat{p}(t)\) take values in finite dimensional matrices?

A: \textcolor{red}{\textbf{No:}} \textcolor{red}{\textbf{If}} \(\hat{x}(t), \hat{p}(t)\) were \(N \times N\) matrices, \(\Rightarrow T_n(\{\hat{x}, \hat{p}\}) = T_n(i\hbar \Delta t) \Rightarrow 0 = i\hbar N \Rightarrow \hat{x}(t), \hat{p}(t)\) must not have well-defined trace, i.e., must act on infinite dimensional Hilbert space, i.e., must be operator-valued.

Physics: \(\Delta x; \Delta p \geq \frac{\hbar}{2}\Delta s_{ij}\)

\(\Rightarrow\) Uncertainty, i.e. "quantum fluctuations", are seen as being part of nature.

But: Non-relativistic quantum mechanics, i.e.,

\([\hat{x}_{ij}, \hat{p}_{ij}] = i\hbar \delta_{ij}\) and \(i\hbar \frac{\partial}{\partial x} \hat{f}(x, \hat{p}) = \{\hat{f}(x, \hat{p}), \hat{H}\}\)

soon became unsatisfactory.

Why? QM is not consistent with special relativity:

E.g. typical momentum of e\(^{-}\) in ground state of H-atom corresponds to \(\approx 1\%\) of speed of light.

\(\Rightarrow\) The effects of special relativity were soon spectroscopically measurable.

\(\Rightarrow\) measurable contradiction to QM!
Attempts to find a covariant generalization of the Schrödinger equation led to:

- "Dirac Equation"
- "Klein Gordon Equation" (see later)

They had some success, but suffer serious problems too:

- Energy not bounded from below \( \Rightarrow \text{instability} \)
- Unitarity of time evolution unclear
- Also: It remained unclear how particle creation and annihilation processes could be calculated.

Thus, a new idea was needed!

The idea of 2nd quantization: (Heisenberg and others, 1930s)

Observation:

In QM, all is subject to quantum fluctuations and therefore to uncertainty – except for the wave function \( \Psi(x,t) \):

Namely:

As in classical theories, if the wave function’s initial conditions are known, then the equation of motion (say the Schrödinger, Klein Gordon or Dirac equation) determines the evolution of \( \Psi(x,t) \) without any uncertainty.
**Idea:**

In 2nd quantization, quantize \( \psi \)!

**Program:**

Similar to \( \hat{p}_i = \hat{x}_i \) (in suitable units)

introduce a "momentum wave function":

\[
\hat{\pi}(x,t) = \hat{\psi}(x,t)
\]

Then, similar to \([\hat{\gamma}_i, \hat{p}_j] = i\hbar \delta_{ij}\), require:

\[
[\hat{\psi}(x,t), \hat{\pi}(x',t)] = i\hbar \delta(x-x')
\]

**Success!**

Problems with energy positivity, unitarity, etc. can be solved.

**Consequences:**

**Math:**

\[
\Rightarrow \hat{\psi}(x,t) \text{ and } \hat{\pi}(x,t) \text{ can no longer be number-valued.}
\]

\[
\Rightarrow \text{For each } x \text{ and } t \text{ the "value" } \hat{\psi}(x,t)
\]

is an operator on a Hilbert space!

<table>
<thead>
<tr>
<th>Notice:</th>
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<tr>
<td>(Result: The laws of motion stay the same)</td>
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<td>also in 1st quantization</td>
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The equations of motion (Schrödinger, Klein Gordon or Dirac equation) stay the same only now with \( \hat{\psi}, \hat{\pi} \) noncommutative.
Physics: 

\[ \Delta \Psi(x,\mathbf{v}) \Delta T(v,\mathbf{v}) \geq \frac{\hbar}{2} \delta(x-x') \]

\[ x = (x_1, x_2, x_3) \]

⇒ The "wave function" is now subject to quantum fluctuations and uncertainty!

⇒ New phenomena now predicted and described:

1) Regarding particles:

- Particle creation/annihilation (e.g. from a wave field)
- i.e. particle number not fixed

Existence of antiparticles ← the negative energy (or mass) states can be interpreted as particles propagating backwards in time, thus to us appearing to have positive energy (or mass).

2) Regarding fields:

Even in the lowest energy state (i.e. no particles, i.e. in the Vacuum, the statement

\[ \Psi(x,\mathbf{v}) = \langle \text{vacuum} | \hat{\Psi}(v,\mathbf{v}) | \text{vacuum} \rangle = 0 \]

allows for the values of \( \hat{\Psi}(x,\mathbf{v}) \) when measured, to fluctuate:
2 main uses of quantum field theory:

1) The Standard Model of Particle Physics

* EM, weak and strong forces

* Screening, anti-screening, and renormalization

* How fundamentally massless particles can effectively acquire a mass: "Spontaneous symmetry breaking"

  Namely: Ground state has less symmetry than the action: "Higgs" particle.

* Anomalies: Quantum fluctuations reduce symmetry of the action itself. The constrain the Standard Model of Particle Physics's structure.

2) The Standard Model of Cosmology

(The aim of this course)

Classical General Relativity + QFT

Mostly

i.e.: Accelerations, curvature, horizons + QFT

* Unruh Effect: What is a "particle"?

* Hawking Effect: Can nature destroy information?

* Cosmic Inflation: Where did it all come from?
Cosmic Inflation:

- A local quantum fluctuation of high potential $V(\phi)$ may occur.
- Acting as temporary cosm. constant, may spawn a rapidly-expanding daughter universe.
- Finally, $V(\phi) \rightarrow 0$, energy goes into particle production: plasma.
- Rapid expansion amplified quantum field fluctuations.
- These fluctuations imprinted on primordial plasma, seeding galaxy formation.

Our target.