Hearing the spacetime curvature in quantum noise

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Background

- Why is quantum gravity hard?

  *GR is structurally very different from quantum theories*

- Any bridge between GR and QT desirable.

- Here: build one of those bridges.
Idea

- Quantum fields fluctuate, have quantum noise

- Spacetime curvature affects that quantum noise

Key question:

- Can we get the curvature back from the quantum noise?

- Is the metric expressible in terms of quantum noise?
How could this work?

- First in flat spacetime:
  \[ \nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{m^2 c^2}{\hbar^2} \psi \nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{m^2 c^2}{\hbar^2} \psi \]

- Coupled harmonic oscillators

- Their quantum fluctuations are correlated

- Quantified by: 2-point functions such as the propagator
In curved spacetime

- Klein Gordon equation now:

\[
\frac{1}{\sqrt{g}} \partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu + \frac{m^2 c^2}{\hbar^2} \psi = 0.
\]

- Curvature affects the coupling of the oscillators

  ➜ Curvature affects the correlations of quantum noise

  ➜ Curvature affects the propagator

Does the propagator know all about the curvature?
Result

- For dimensions $D > 2$, the metric can be expressed in terms of the Feynman propagator:

\[
g_{ij}(y) = -\frac{1}{2} \left[ \frac{\Gamma(D/2 - 1)}{4\pi^{D/2}} \right]^{\frac{2}{D-2}} \lim_{x \to y} \frac{\partial}{\partial x^i} \frac{\partial}{\partial y^j} \left( G(x, y)^{\frac{2}{2-D}} \right)
\]

- Proof: covariance, geodesic coordinates, asymptotic behavior
- Also in paper: worked-out examples

⇒ In principle: Can replace the metric with the propagator!
Intuitively, why does this work?

• Recall that in 3+1 dimensions:

\[ \text{curvature} = \text{causal structure} + \text{scalar function} \]

• Propagator knows the causal structure

• But: propagator also indicates effective spacetime distances!

And knowing infinitesimal distances is to know the metric.
Interpretation

- Einstein built general relativity on rods and clocks

- But no rods and clocks at sub-atomic scales!
- Instead: as distance proxy, use strength of noise correlators:
Accelerators measure curvature

- Accelerators test Feynman rules, including the propagators
- Measuring the propagator locally is to measure the metric

*With a mobile accelerator one could measure the metric anywhere*
What can we use this tool for?

- We have a bridge between GR and QFT: From the propagator we can calculate the metric.

Assume some arbitrary model for the Planck scale.

How does bridge hold up when approaching the Planck scale?

- We can calculate the corresponding impact on the metric!
Example: sharp natural UV cutoff

\[ g_{ij}^{\Lambda}(y) \equiv -\frac{1}{2} \left[ \frac{\Gamma(D/2 - 1)}{4\pi^{D/2}} \right] \frac{2}{D-2} \lim_{x \to y} \frac{\partial}{\partial x^i} \frac{\partial}{\partial y^j} G_{\Lambda}(x, y) \frac{2}{2-D} \]

- Notice: the UV limits \( x \to y \) and \( \Lambda \to \infty \) compete!

- Do they commute?

- It’s more subtle: only “on average” they do
Example D=3 flat space:

\[ g_{\alpha\beta}(x, y) = \delta_{\alpha\beta}f_1(\Lambda r_{xy}) + \frac{(x_\alpha - y_\alpha)(x_\beta - y_\beta)}{r_{xy}^2} f_2(\Lambda r_{xy}) \]

- Oscillations. We recover usual metric “on average”
Oscillations visible in the CMB?

CMB’s structure originated close to Planck scale.

Hubble scale in inflation only about 5 orders from Planck scale.
Natural UV cutoffs in inflation

Multiple groups have non-covariant predictions for CMB.

- Effect could be first or second order in (Planck length/Hubble length)
  i.e. could be say $O(10^{-5})$ or $O(10^{-10})$

**Big challenge was:**

Predictions with local Lorentz covariant bandlimit cutoff!

**Recent paper with former students:**

Aidan Chatwin-Davies, (CalTech) and Robert Martin (U. Cape Town)

Results for sharp covariant cutoff

- Power law inflation: relative change in (tensor) spectrum
- Effect is linear in (Planck length/Hubble length)
- Plus characteristic oscillations.
Results for sharp covariant cutoff

- However, this was assuming decoherence at horizon crossing!

- If decoherence at re-heating, then spectrum is very robust.
Summary

- Quantum noise knows all about curvature
- Metric expressible through Feynman propagator
- Covariant natural UV cutoff ➔

Oscillations in inflationary predictions for CMB