Homework 1 - Review material.
Not to be handed in

Linear algebra

Notation: $(\hat{\imath}, \hat{\jmath}, \hat{k})$ are the unit basis vectors of the Cartesian coordinate system.

1. The angular momentum vector $\vec{H}$ of a particle of mass m is defined by $\vec{H} = \vec{r} \times (m\vec{v})$, where $\vec{v} = \vec{\omega} \times \vec{r}$.
   (a) Using the result,
   $$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = (\vec{a} \cdot \vec{c})\,\vec{b} - \left(\vec{a} \cdot \vec{b}\right)\vec{c},$$
   show that if $\vec{r}$ is perpendicular to $\vec{\omega}$, then $\vec{H} = mr^2\vec{\omega}$.
   (b) Given that $m = 100$, $\vec{r} = 0.1(\hat{\imath} + \hat{\jmath} + \hat{k})$ and $\vec{\omega} = 2\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$, calculate $\vec{r} \cdot \vec{\omega}$ and $\vec{H}$.
2. Given $\vec{a} = -1\hat{\imath} - 3\hat{\jmath} - 1\hat{k}$, $\vec{b} = q\hat{\imath} + 1\hat{\jmath} + 1\hat{k}$ and $\vec{c} = 1\hat{\imath} + 1\hat{\jmath} + q\hat{k}$,
   (a) Calculate $\vec{a} \times \vec{b}$.
   (b) Determine the values of q for which $\vec{a}$ is perpendicular to $\vec{b}$.
   (c) Determine the values of q for which $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{0}$.

Integration

3. Evaluate the following,
$$\text{a)} \int_0^1 x\sqrt{2x+1}\,dx \quad \text{b)} \int \frac{\sin(\ln x)}{x}\,dx \quad \text{c)} \int_1^e (\ln x)^2\,dx$$
4. Evaluate the following, where R is bounded by $0 \le x \le 1, x^2 \le y \le x$,
$$\iint_R \frac{x}{y}\,\mathrm{e}^y\,dxdy.$$

Differentiation

5. Given a scalar function $F(x(t), y(t))$, that depends on two variables (x, y) that are themselves functions of t,
   (a) Determine the Taylor polynomial approximation of $F(x + \Delta x, y + \Delta y)$ up to and including first-order terms in Δx and Δy.
   (b) Calculate the derivative of $F(x(t), y(t))$ with respect to t: dF/dt.
6. Find the one-parameter family of solutions for the following differential equations,
$$\text{a)}\ \frac{dy}{dx} = \frac{x}{y} \quad \text{b)}\ \frac{dy}{d\theta} = \frac{e^y \sin^2\theta}{y \sec\theta}$$