# QFT for Cosmology, Achim Kempf, Lecture 1

Note Title

## Historical background:

□ ≈ 1900:

Classical mechanics became experimentally untenable:

- Black body radiation ("Ultraviolet catastrophe")
- Photoelectric effect (Ionization depends on color, not intensity)
- Stability of matter ($\Delta x \Delta p \geq \frac{\hbar}{2}$ implies that $e^-$ do not spiral into the nuclei)

Finally, in 1925:

Heisenberg discovers nonrelativistic quantum mechanics (QM)

In essence:

- Equations of motion stay the same, e.g.:

$$m\ddot{\hat{x}} = -k\hat{x} \qquad \text{(harm. oscillator)}$$

- but we have noncommutativity:

$$[\hat{x}, \underbrace{m\dot{\hat{x}}}_{=\hat{p}}] = i\hbar \qquad \text{"canonical commutation relation"}$$

o A few months later:

Schrödinger discovered his equation

$$i\hbar \frac{d}{dt}\psi(x,t) = -\frac{\hbar^2}{2m}\Delta\psi(x,t) + V(x,t)\psi(x,t)$$

o A few more months later:

Dirac showed equivalence to Heisenberg's.

## Quantization implied fundamental changes:

Math: $[\hat{x}(t), \hat{p}(t)] = i\hbar 1 \neq 0 \Rightarrow \hat{x}(t), \hat{p}(t)$ not number-valued.

Q: Could $\hat{x}(t), \hat{p}(t)$ take values in finite dimensional matrices?

A: No: If $\hat{x}(t), \hat{p}(t)$ were $N\times N$ matrices, then:

$$\boxed{Tr([\hat{x},\hat{p}]) = Tr(i\hbar 1) \Rightarrow 0 = i\hbar N} \quad \lightning$$

$\Rightarrow \hat{x}(t), \hat{p}(t)$ must not have well-defined trace, i.e., must act on $\infty$ dim. Hilbert space, i.e., must be operator-valued.

Physics: $\Delta x_i \Delta p_j \geq \frac{\hbar}{2}\delta_{ij}$

$\rightsquigarrow$ Uncertainty, i.e. "quantum fluctuations", are seen as being part of nature.

- **But:** Nonrelativistic quantum mechanics, i.e.,

  $$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij} \quad \text{and} \quad i\hbar\frac{d}{dt}\hat{f}(\hat{x},\hat{p}) = [\hat{f}(\hat{x},\hat{p}), \hat{H}]$$

  soon became unsatisfactory.

- **Why?** QM is not consistent with special relativity:

  E.g. typical momentum of $e^-$ in ground state of H-atom corresponds to $\approx 1\%$ of speed of light.

  $\Rightarrow$ The effects of special relativity were soon spectroscopically measurable.

  $\Rightarrow$ measurable contradiction to QM!

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- Attempts to find a covariant generalization of the Schrödinger equation led to:
  - "Dirac Equation"
  - "Klein Gordon Equation" (see later)

- They had some success, but suffer serious problems too:
  - Energy not bounded from below $\Rightarrow$ "instability"
  - Unitarity of time evolution unclear
  - Also: It remained unclear how particle creation and annihilation processes could be calculated.

- Thus, a new idea was needed!

# The idea of 2nd quantization: (Heisenberg and others, 1930s)

- **Observation:**

  In QM, all is subject to quantum fluctuations and therefore to uncertainty – except for the wave function $\psi(x,t)$:

  Namely:

  As in classical theories, if the wave function's initial conditions are known, then the equation of motion (say the Schrödinger, Klein Gordon or Dirac equation) determines the evolution of $\psi(x,t)$ without any uncertainty.

- **Idea:**

  In 2nd quantization, quantize $\psi$ !

- **Program:**

  Similar to $\hat{p}_i = \dot{\hat{x}}_i$ (in suitable units)

  introduce a "momentum wave function":

  $$\hat{\pi}(x,t) = \dot{\hat{\psi}}(x,t)$$

  Then, similar to $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, require:

  $$\left[\hat{\psi}(x,t), \hat{\pi}(x',t)\right] = i\hbar\,\delta(x-x')$$

- **Success!**

  Problems with energy positivity, unitarity etc can be solved.

## Consequences:

**Math:**

→ $\hat{\psi}(x,t)$ and $\hat{\pi}(x,t)$ can no longer be number-valued.

→ For each $x$ and $t$ the "value"

$$\hat{\psi}(x,t)$$

is an operator on a Hilbert space!

**Notice:** (Recall: The eqns of motion stay the same also in 1st quantization)

The equations of motion (Schrödinger, Klein Gordon or Dirac equation) stay the same only now with $\hat{\psi}, \hat{\pi}$ noncommutative.

**Physics:**

we'll need to discuss that

$$\Delta\psi(x,t)\,\Delta\pi(x',t) \geq \frac{\hbar}{2}\,\delta^3(x-x')$$

$x = (x_1, x_2, x_3)$

⇒ The "wave function" is now subject to quantum fluctuations and uncertainty!

## ⇒ New phenoma now predicted and described:

1.) Regarding particles:

Particle creation/annihilation (E.g. norm of wave fctn i.e. particle number no longer fixed)

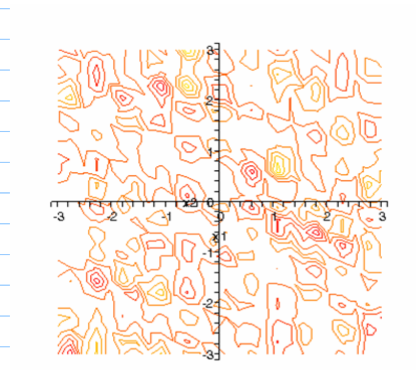
Existence of antiparticles ← (the negative energy (or mass) states can be interpreted as particles propagating backwards in time, thus to us appearing to have positive energy (or mass).)

## 2.) Regarding fields:

Even in the lowest energy state (i.e. no particles, i.e. in the Vacuum, the statement

$$\bar{\psi}(x,t) = \langle \text{vacuum} | \hat{\psi}(x,t) | \text{vacuum} \rangle = 0$$

allows for the values of $\hat{\psi}(x,t)$ when measured, to fluctuate:



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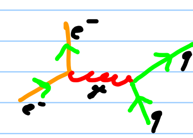
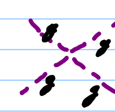
⇝ 2 main uses of quantum field theory:

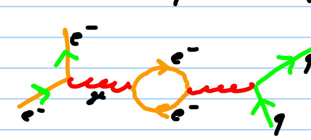
### 1) The Standard Model of Particle Physics

* EM, weak and strong forces

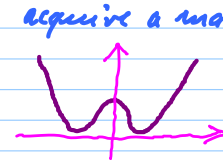




* Screening, anti-screening and renormalization



* How fundamentally massless particles can effectively acquire a mass:
"Spontaneous symmetry breaking"
Namely: Groundstate has less symmetry than the action:



"Higgs" particle.

* Anomalies: Quantum fluctuations reduce symmetry of the action itself.
The contrain the Standard Model of Particle Physics' structure.

# 2) The Standard Model of Cosmology

(the aim of this course)

Classical General Relativity + QFT

Mostly

i.e.: Accelerations, curvature, horizons + QFT

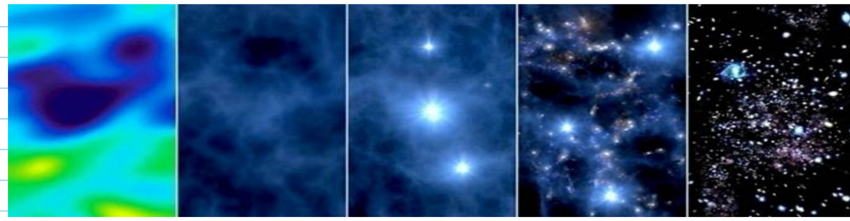
* Unruh Effect: What is a "particle"?

* Hawking Effect: Can nature destroy information?

* Cosmic Inflation: Where did it all come from?

Cosmic Inflation:

- A local quantum fluctuation of high potential $V(\phi)$ may occur.
- Acting as temporary cosm. constant, may spawn a rapidly-expanding daughter universe.
- Finally, $V(\phi) \to 0$, energy goes into particle production: plasma
- Rapid expansion amplified quantum field fluctuations.
- These fluctuations imprinted on primordial plasma, seeding galaxy formation:



Our target