Projects for QO course

Adrian Lupascu

May 11, 2015

Project 1: Decoherence due to low-frequency noise

Consider a two-level system (qubit) coupled to a bath. The qubit-bath coupling is such that it leads to standard Markovian decoherence. The qubit Hamiltonian, when expressed in the energy eigenbasis, is given by

$$H_{\mathrm{qb}} = -\frac{\omega_{01} + \xi(t)}{2}\sigma_z, \tag{1}$$

with ω_{01} a fixed component to the transition frequency and $\xi(t)$ a noisy component in the transition frequency.

(a) Show that the decay of the coherences of the density matrix of the qubit can be written as the product of an exponential decay factor with a dephasing rate determined by the bath and of a *coherence function*. The coherence functions is defined as $C(\tau) = \langle \exp\left(-i \int_0^\tau \xi(t)dt\right)\rangle$ where the average $\langle ..\rangle$ is an average over the different realizations of the stochastic process.

(b) Discuss the properties of the coherence function when the noise is Gaussian. A good discussion of the properties of Gaussian noise is given in ref Kubo *et al.* (1991) (sec 1.4). Show that the coherence function decay depends only on the second order correlation function, and consequently on the power spectral density (PSD) of the noise.

(c) Discuss the decoherence induced by a source of white noise with a PSD given by a constant A. What is the effect of a finite frequency sharp cutoff at frequency B?

(d) Discuss decoherence due to a random telegraph noise (RTN) process. The noise process $\xi(t)$ takes either of two values, ξ_1 or ξ_2; transitions between these two values take place with rates γ_{12} and γ_{21}. Is this noise Gaussian? Calculate the coherence function decay. In what limit does the coherence decay becomes the same as that due to a Gaussian process with equivalent spectral density.

Project 2: Implementations of circuit quantum electrodynamics

Review the implementation of cQED in three types of physical systems:

(a) optical cavities and alkali atoms

(b) microwave cavities and Rydberg levels

(c) superconducting resonators and qubits

Discuss the physics of the implementations. What are the challenges in each of this application? What are the most promising practical applications.

Project 3: Quantum limited amplifiers

Review the Caves treatment of quantum limited amplifiers. Apply this theory to a practical amplifier. You could consider for example the amplifier of Castellanos-Beltran et al. (Nature Physics 4, 929, 2008) or Bergeal et al. (Nature 465, 64, 2010).

References

R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics II*, 2nd ed., edited by H. K. V. Lotsch, Springer Series in Solid-State Sciences, Vol. 2 (Springer Verlag, 1991).