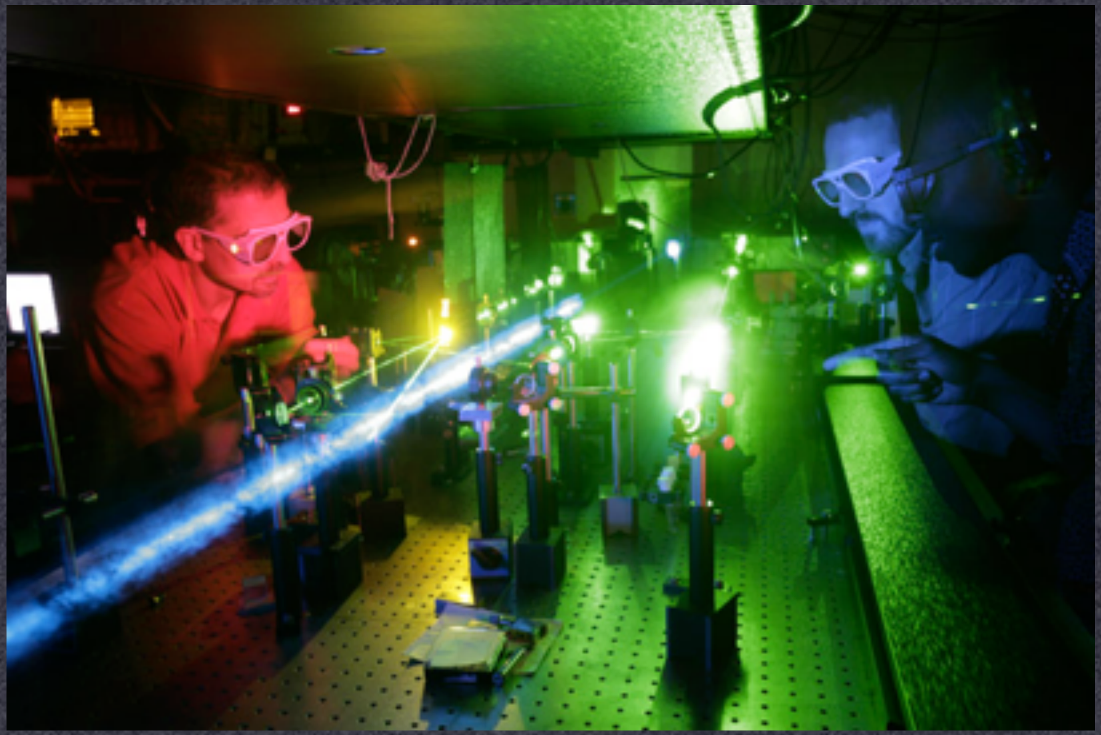
# (Some) Relativistic aspects of the Light-Matter interaction



Eduardo Martín-Martínez
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Perimeter Institute

# The full light-matter interaction

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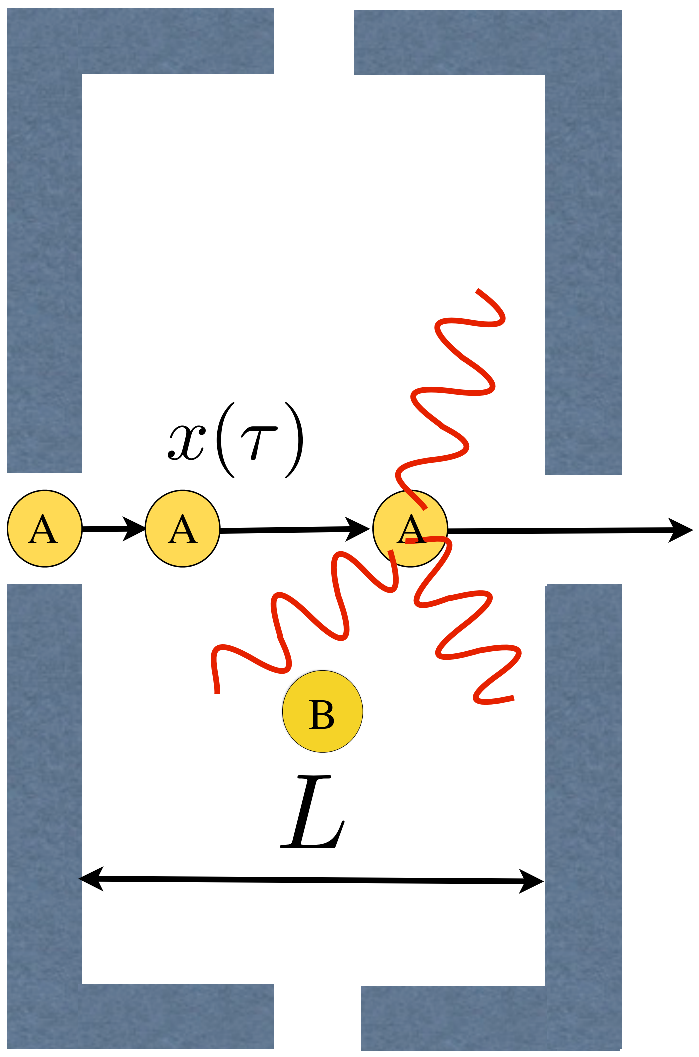

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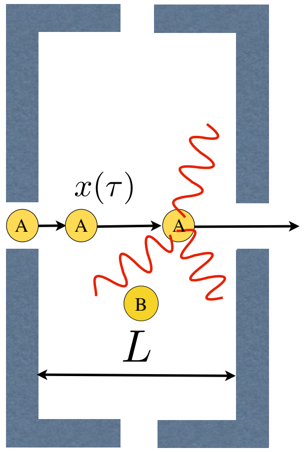

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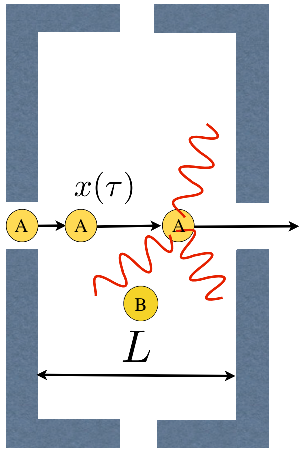

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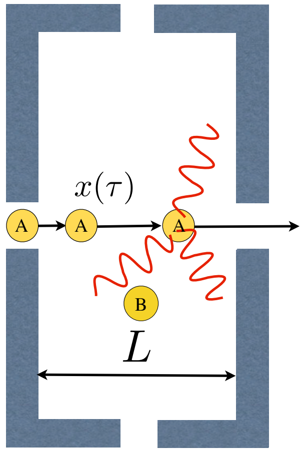

# Relativity: Who measures what?

# Same Physics, Different Descriptions

Bell Rocket "Paradox"


Does the rope break or not??

Why??

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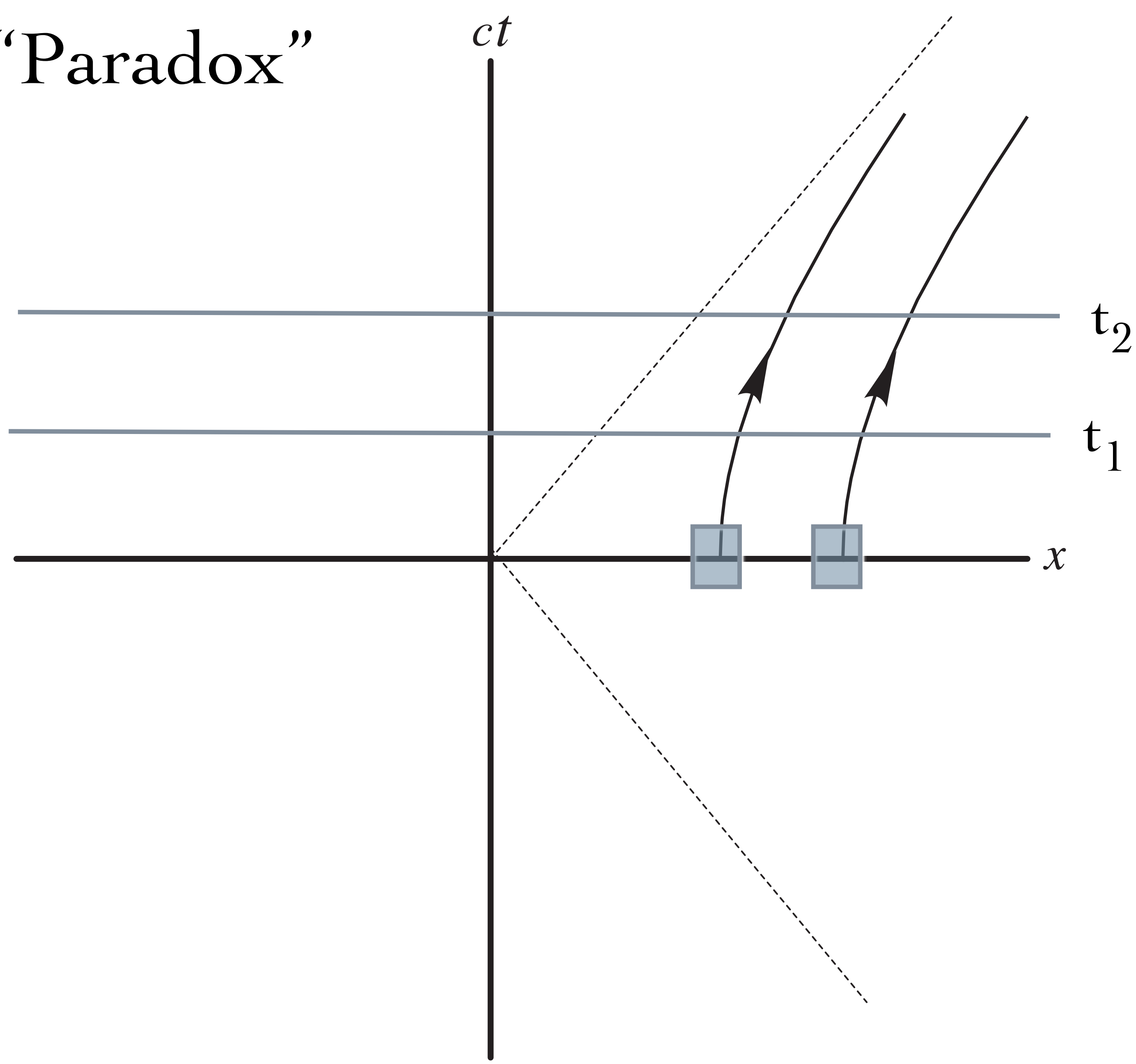


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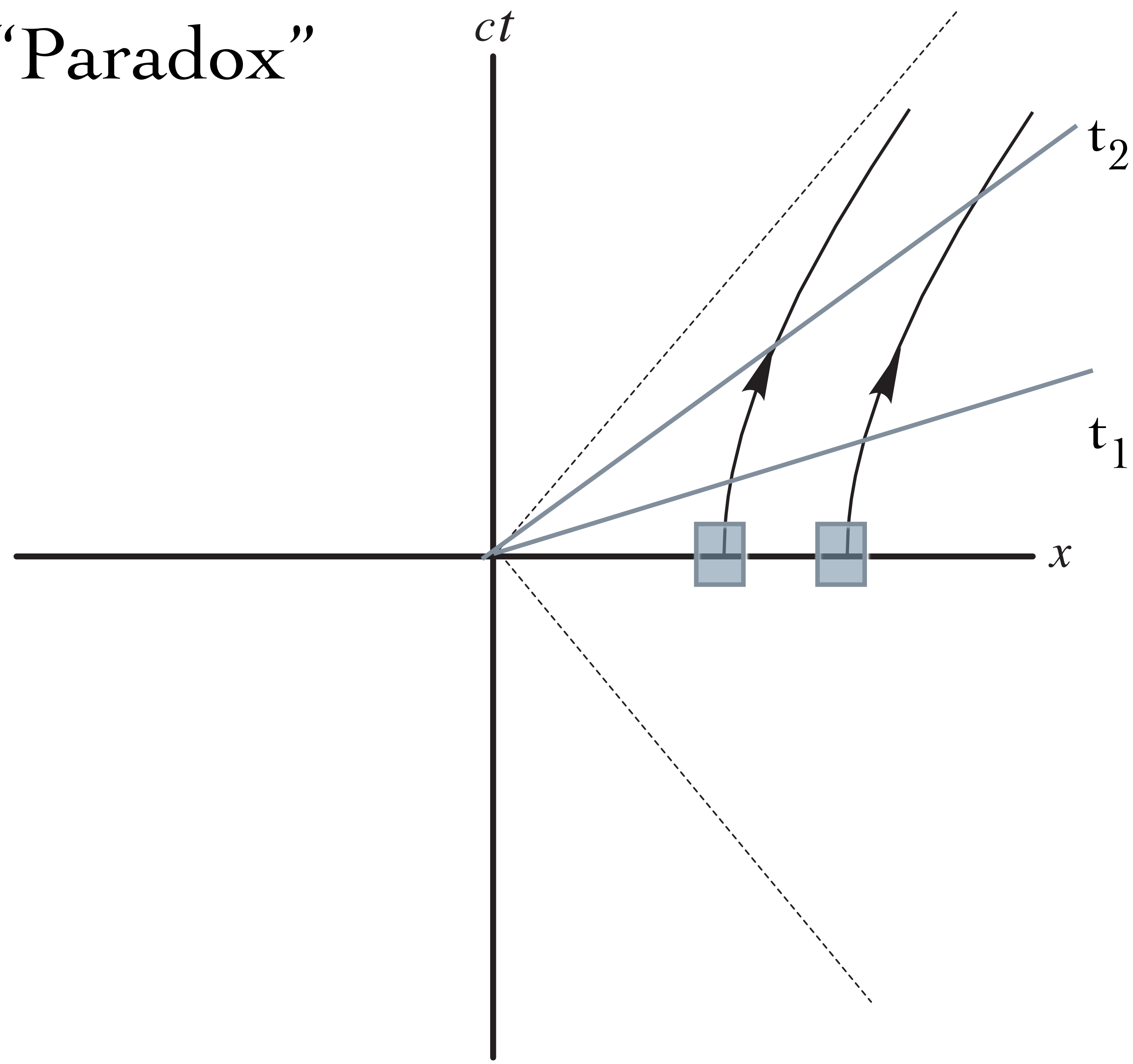


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PHYSICS:

The rope breaks, all right!

# Who measures what?

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PHYSICS:

The rope breaks, all right!

PHENOMENOLOGY:

For the accelerated observer A: Because rocket B is faster than us!

For the observer on the ground: Because both rockets go equally faster and faster, the length of the rope Lorentz-contracts!

# Quantum Mechanics and GR

# Same Physics, Different Descriptions

In non-relativistic quantum theory, time is a parameter!

$$\frac{d}{dt}\hat{A}(t) = \frac{i}{\hbar}\left[\hat{H}(t), \hat{A}(t)\right] + \frac{\partial \hat{A}(t)}{\partial t}.$$

# Quantum Mechanics and GR

# Same Physics, Different Descriptions

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Excuse me... Time?

Whose time?

Mine? Yours?

$$\frac{d}{dt}\hat{A}(t) = \frac{i}{\hbar}\left[\hat{H}(t), \hat{A}(t)\right] + \frac{\partial \hat{A}(t)}{\partial t}.$$

$$\frac{d}{d\tau}\hat{A}\left[t(\tau)\right] = \frac{dt}{d\tau}\frac{d}{dt}\hat{A}(t)\bigg|_{t=t(\tau)} = \frac{dt}{d\tau}\frac{i}{\hbar}\left[\hat{H}(t), \hat{A}(t)\right] + \frac{dt}{d\tau}\frac{\partial \hat{A}(t)}{\partial t}\bigg|_{t=t(\tau)}$$

$$\frac{d}{d\tau}\hat{A}\left[t(\tau)\right] = \frac{i}{\hbar}\left[\left(\frac{dt}{d\tau}\hat{H}\left[t(\tau)\right]\right), \hat{A}\left[t(\tau)\right]\right] + \frac{\partial \hat{A}\left[t(\tau)\right]}{\partial \tau}$$

# Quantum Mechanics and GR
# Same Physics, Different Descriptions

## Rule of thumb

Thus, we can start with the Hamiltonian $\hat{H}(t)$, which generates translations in the global time coordinate $t$, and then define

$$\hat{H}(\tau) := \frac{dt}{d\tau}\hat{H}\left[t(\tau)\right]$$

as the Hamiltonian which generates evolution for the entire system with respect to the time coordinate $\tau$

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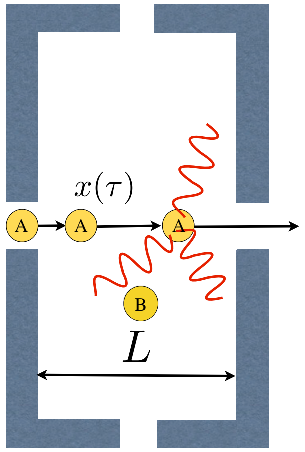

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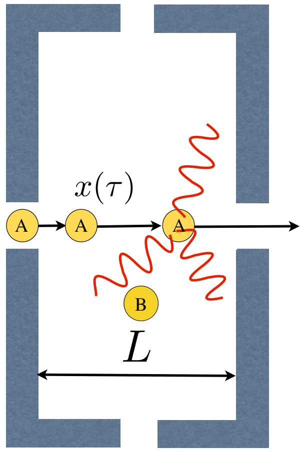

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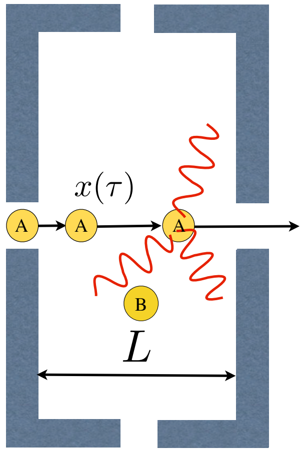


$$H_0 = \Omega\, \sigma^+ \sigma^- + \frac{\mathrm{d}\tau(t)}{\mathrm{d}t} \sum_{j=1}^{\infty} \omega_j\, a_j^\dagger a_j$$

# The full light-matter interaction

What about the interaction Hamiltonian?

It is easiest (but not always possible) to write the Hamiltonian that generates translations with respect to the atom's proper time. In the interaction picture

$$H_I = \lambda \left(\sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}\right) \sum_{j=1}^{\infty} \left[a_j^\dagger e^{i\omega_j t(\tau)} + a_j e^{-i\omega_j t(\tau)}\right] \sin k_j x(\tau),$$

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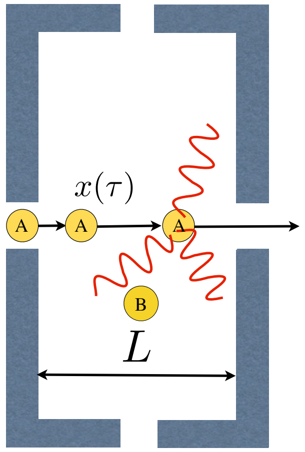

# The full light-matter interaction

But in my experiments atoms
do not travel anywhere near c. Should I care?

Atom at rest: $x(\tau) = x_0, \quad t(\tau) = \tau$

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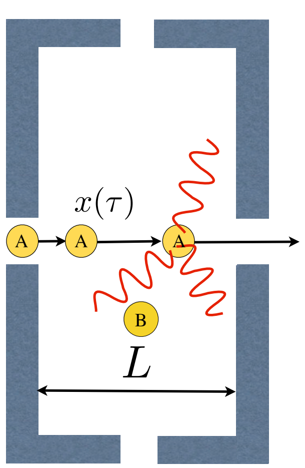

# Conservation of Energy?

$$H_0 = \frac{\Omega}{2}\sigma_z + \sum_j \omega_j a_j^\dagger a_j$$

$\hbar\omega_j$ $\quad -\hbar\Omega/2$

$$H_I = \lambda\left(\sigma^+ + \sigma^-\right)\sum_{j=1}^{\infty}\left[a_j^\dagger + a_j\right]\sin k_j x,$$

$$\sigma^+ a_\omega \left|1_\omega\right\rangle \left|g\right\rangle \longrightarrow \left|0\right\rangle \left|e\right\rangle$$

$$\sigma^+ a_\omega^\dagger \left|0\right\rangle \left|g\right\rangle \longrightarrow \left|1_\omega\right\rangle \left|e\right\rangle$$

$0 \qquad\qquad \hbar(\Omega + \omega_j)$

$\sigma^+ a_j, \;\; \sigma^- a_j^\dagger$ Rotating-wave terms

$\sigma^- a_j, \;\; \sigma^+ a_j^\dagger$ Counter-rotating wave terms

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Neglect the CRW terms because "They do not conserve energy… for some time that's fine (uncertainty principle) but not for long times…"

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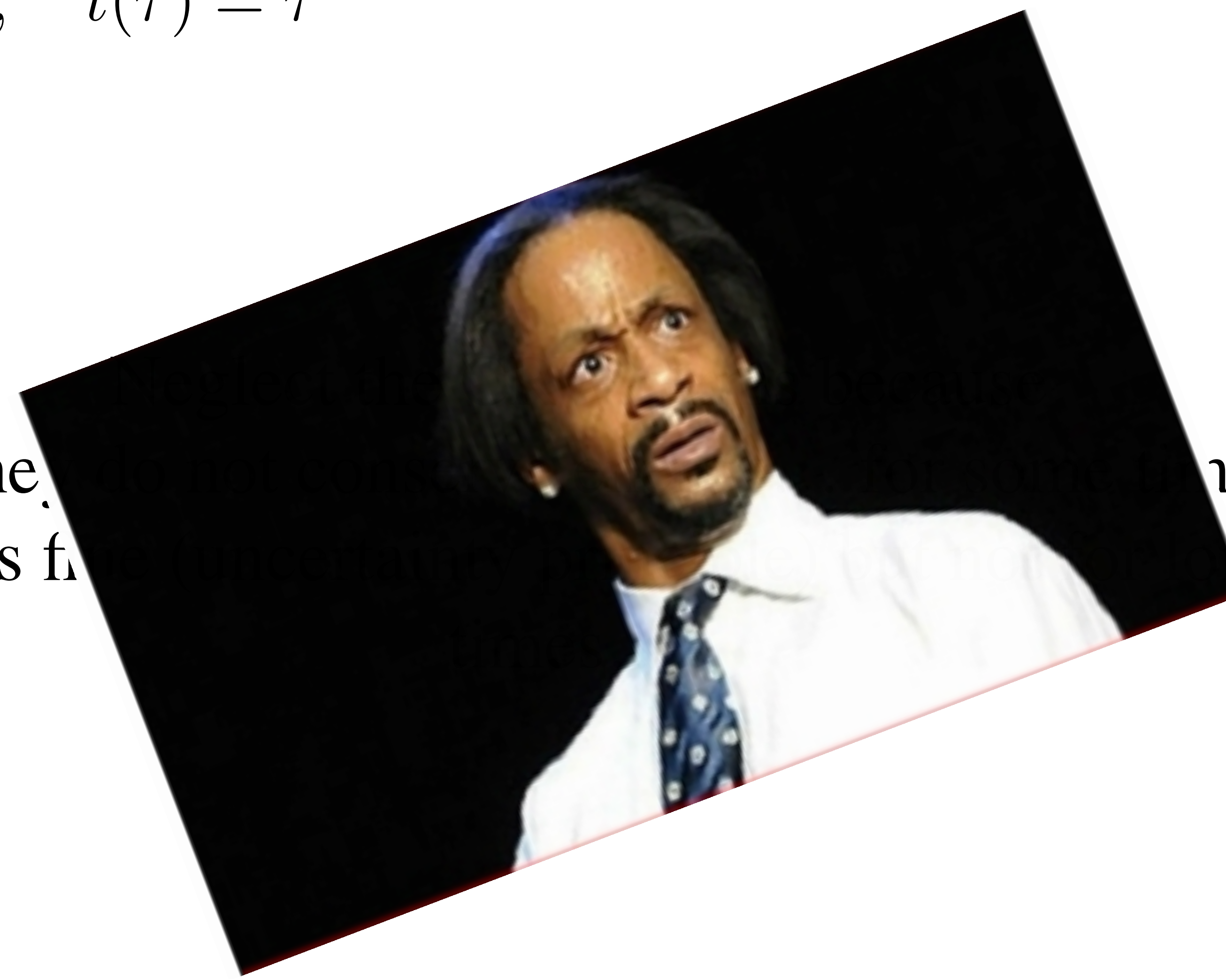
“The ... ne
that’s f ... g



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People like Sakurai say that…

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Is there a problem with energy conservation?

$\hbar\omega_j$ $\hbar\Omega$

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Time independent Hamiltonian… Conserves energy at all times

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$$\Delta E \Delta t \geq \frac{1}{2}\hbar$$

This is the way to waive your hands about Heisenberg principle… but no 'non-conservation' of energy

# usual approximations in QO

Let us see it more rigorously!

First order Dyson Series

$$U = U^{(0)} + U^{(1)} + U^{(2)} + \mathcal{O}(\lambda^3)$$

$$U^{(1)} = -i \int_{t_0}^{t} \mathrm{d}t \, H_I(t)$$

Excitation probability

$$\sum_{\phi} \left| \langle \phi, e| \, U^{(1)} \, |0, d\rangle \right|^2$$

# usual approximations in QO

Typical approximations made in quantum optics when $\Omega = \omega_j$

$\Delta T \gg \Omega^{-1}$ Single mode approximation

$$\lim_{\substack{t\to\infty \\ t_0\to-\infty}} \int_{t_0}^{t} \mathrm{d}t_1\, e^{\mathrm{i}(\Omega-\omega_j)t_1} \sim \Delta T$$

$$\lim_{\substack{t\to\infty \\ t_0\to-\infty}} \int_{t_0}^{t} \mathrm{d}t_1\, e^{\mathrm{i}(\Omega-\omega_n)t_1} \sim \Delta T \left\langle e^{-\mathrm{i}(\Omega-\omega_n)t_1} \right\rangle \sim \frac{1}{(\Omega-\omega_n)}$$

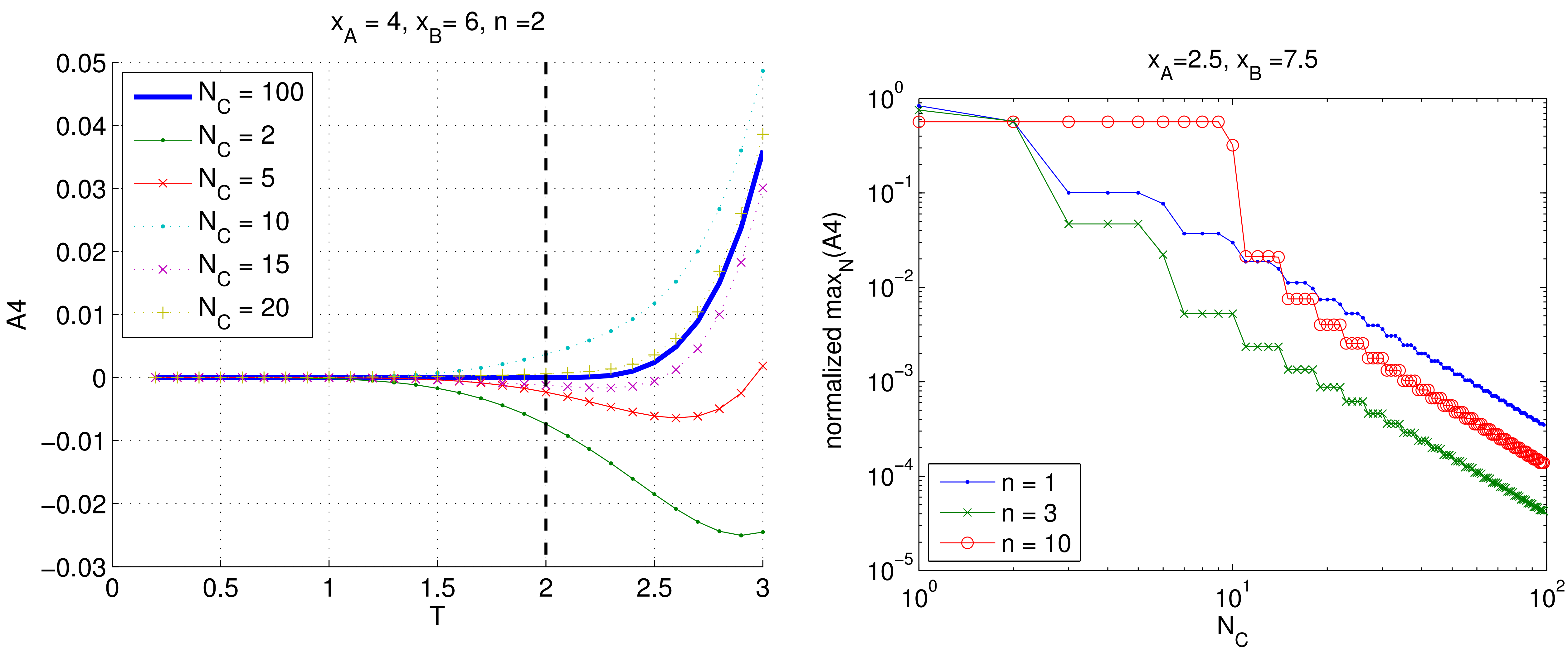
$\Delta T \gg (2\Omega)^{-1}$ Rotating-wave approximation

$$\lim_{\substack{t\to\infty \\ t_0\to-\infty}} \int_{t_0}^{t} \mathrm{d}t_1\, e^{\mathrm{i}(\Omega+\omega_n)t_1} \sim \Delta T \left\langle e^{\mathrm{i}(\Omega+\omega_n)t_1} \right\rangle \sim \frac{1}{(\Omega+\omega_n)}$$

Rotating-wave approximation $\Rightarrow$ No ‘vacuum’ fluctuations

# usual approximations in QO

QED is a covariant theory
These approximations destroy that




Robert, Achim and me, Phys. Rev. A 89, 022330 (2014)

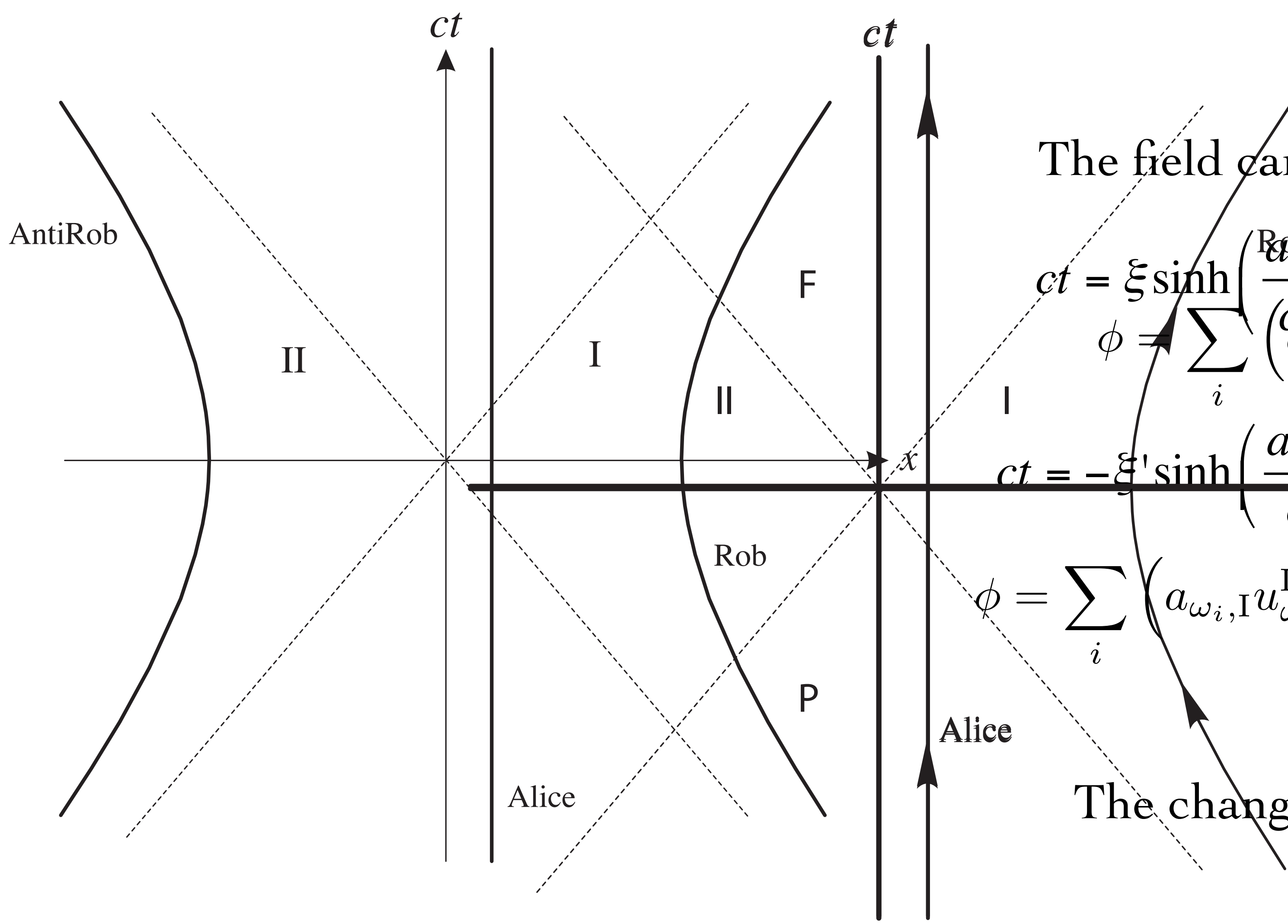
# Causality, Covariance and the SMA/RWA

Every time you make one of those approximations

Einstein kills a kitten…

# Field Quantization: Observer dependence

- Example: different observers of flat spacetime




The field can be expanded in either basis

$ct = \xi \sinh\left(\frac{a\tau}{c}\right), \qquad x = \xi \cosh\left(\frac{a\tau}{c}\right)$

$$\phi = \sum_i \left(a_{\hat\omega_i,\mathrm{M}} u^{\mathrm{M}}_{\hat\omega_i} + a^\dagger_{\hat\omega_i,\mathrm{M}} u^{\mathrm{M}*}_{\hat\omega_i}\right)$$

$ct = -\xi' \sinh\left(\frac{a\tau'}{c}\right), \qquad x = -\xi' \cosh\left(\frac{a\tau'}{c}\right)$

or equivalently

$$\phi = \sum_i \left(a_{\omega_i,\mathrm{I}} u^{\mathrm{I}}_{\omega_i} + a^\dagger_{\omega_i,\mathrm{I}} u^{\mathrm{I}*}_{\omega_i} + a_{\omega_i,\mathrm{II}} u^{\mathrm{II}}_{\omega_i} + a^\dagger_{\omega_i,\mathrm{II}} u^{\mathrm{II}*}_{\omega_i}\right)$$

The change of basis is not trivial

Positive freq. solutions $\nabla^\mu \nabla_\mu u = 0$

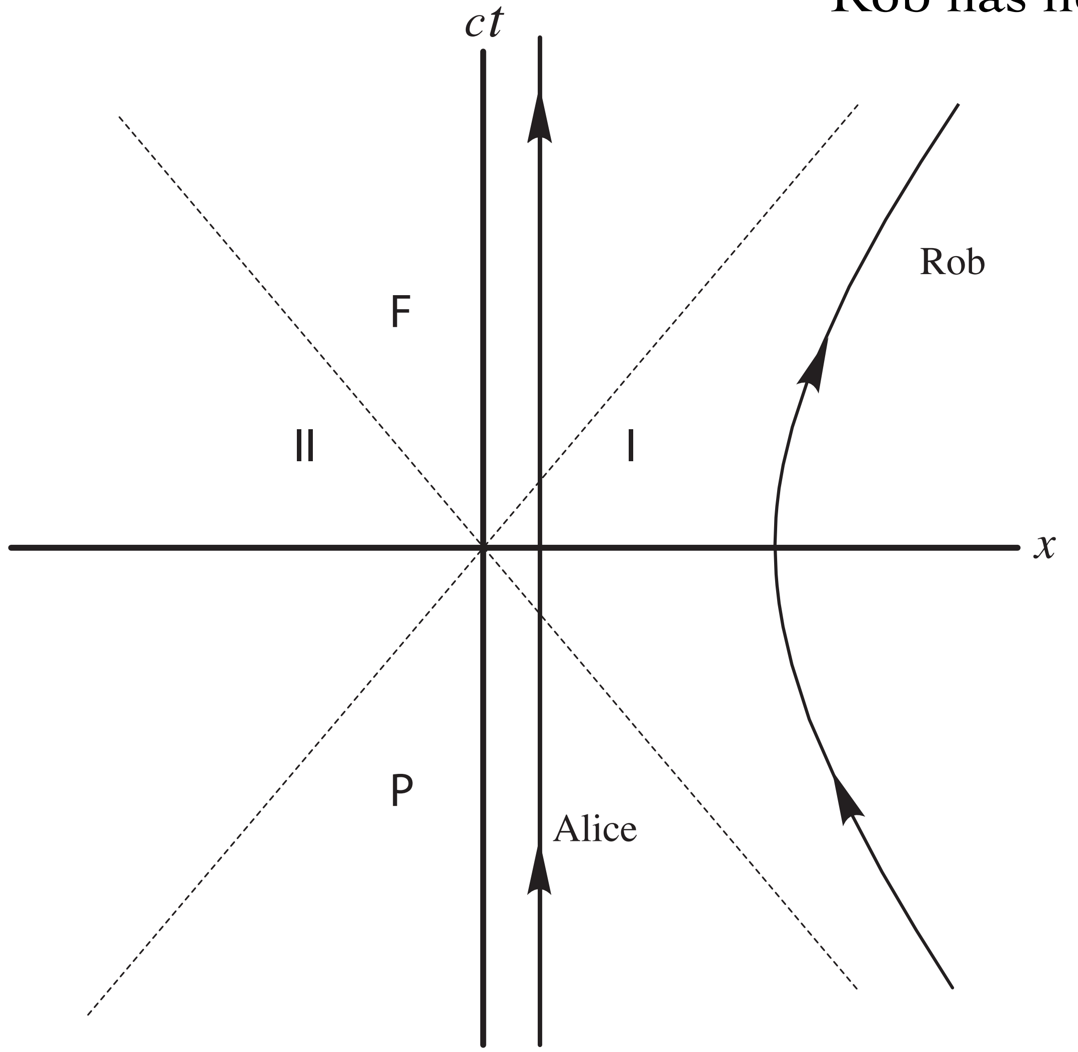
Basis{

Basis{

$$(\xi, \tau) \rightarrow u^{\mathrm{II}}_{\hat\omega} \propto e^{i\hat\omega\tau}$$

# The Importance of the Horizon

Rob has no access to region II




$$\tanh r_\omega = \exp\left(-\frac{\pi c \omega}{a}\right)$$

Example: Minkowskian vacuum. Rob's perspective

$$|0\rangle_{\mathrm{M}}$$

First: change of Fock basis

$$|0\rangle_{\mathrm{M}} = \bigotimes_{\omega} \frac{1}{\cosh r_\omega} \sum_{n=0}^{\infty} \tanh^n r_\omega |n\rangle_{\mathrm{I}} |n\rangle_{\mathrm{II}}$$

Second: Trace out the disconnected region

$$\rho_{\mathrm{R},\omega} = \mathrm{Tr}_{\mathrm{II}}\left(|0_\omega\rangle\langle 0_\omega|\right) = \frac{1}{\cosh^2 r_\omega} \sum_n \tanh^{2n} r_\omega |n_\omega\rangle_{\mathrm{I}} \langle n_\omega|_{\mathrm{I}}$$

Result: thermal state

$$\langle N_{\omega,\mathrm{R}}\rangle = \frac{1}{e^{2\pi c/\omega a} - 1} \qquad T_{\mathrm{U}} = \frac{\hbar a}{2\pi K_{\mathrm{B}}}$$

# The Unruh-DeWitt detector model

$$H_I = \lambda \left(\sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}\right) \sum_{j=1}^{\infty} \left[a_j^\dagger e^{i\omega_j t(\tau)} + a_j e^{-i\omega_j t(\tau)}\right] \sin k_j x(\tau),$$

Models the interaction of a two-level system with a scalar field

| | | |
|---|---|---|
| $\sigma^+ a_j, \;\; \sigma^- a_j^\dagger$ | Rotating-wave terms | $e^{i[\Omega\tau - \omega_j t(\tau)]}$ |
| $\sigma^- a_j, \;\; \sigma^+ a_j^\dagger$ | Counter-rotating wave terms | $e^{i[\Omega\tau + \omega_j t(\tau)]}$ |

Detector at rest (or inertial): $x(\tau) = x_0, \quad t(\tau) = \tau$

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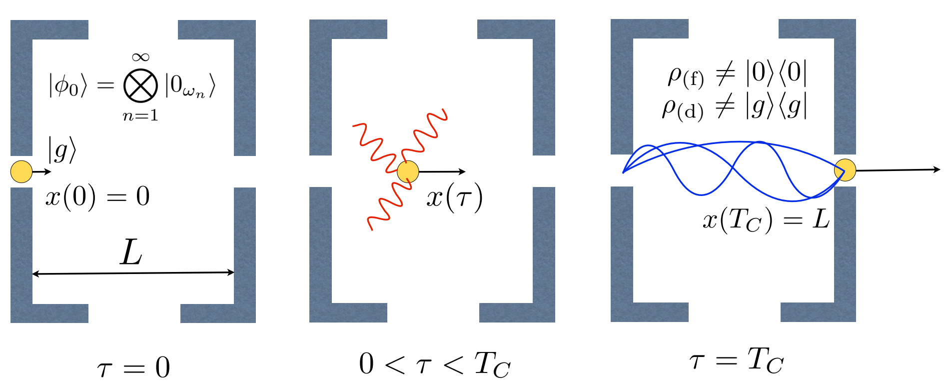
What if the trajectory is constantly accelerated?




$$x(\tau) = a^{-1}(\cosh a\tau - 1)$$

$$t(\tau) = a^{-1}\sinh a\tau$$

$\sigma^+ a_j, \ \ \sigma^- a_j^\dagger$ Rotating-wave terms $e^{i[\Omega\tau - \omega_j t(\tau)]}$

$\sigma^- a_j, \ \ \sigma^+ a_j^\dagger$ Counter-rotating wave terms $e^{i[\Omega\tau + \omega_j t(\tau)]}$

Both highly oscillatory and become not-negligible very soon.

Counter-rotating terms allow for vacuum excitations

# Accelerated atom in the vacuum

$$H_I = \lambda \left(\sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}\right) \sum_{j=1}^{\infty} \left[a_j^\dagger e^{i\omega_j t(\tau)} + a_j e^{-i\omega_j t(\tau)}\right] \sin k_j x(\tau),$$

For an accelerated detector

$$x(\tau) = a^{-1}(\cosh a\tau - 1)$$

$$t(\tau) = a^{-1} \sinh a\tau$$

- The counter-rotating and rotating terms quickly become comparable.

An accelerated detector probing the vacuum detects field quanta due to the contribution of the counter-rotating terms. This is the ‘Unruh effect’

# The Unruh Effect

Too small to be detected!!

Inertial frame

Accelerated frame

- Alice Observes the field vacuum.
- Rob observes a thermal bath of temperature $T_{\mathrm{U}} \propto a$