Landauer's Principle in Repeated Interaction Systems

Eric Hanson[1]

Alain Joye[2], Yan Pautrat[3], Renaud Raquépas[1]

[1]McGill University, Montréal, QC,

[2]Institut Fourier, Saint-Martin d'Hères, France

[3]Université Paris-Saclay, Orsay, France

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Outline

Three ingredients

1. Landauer's Principle
2. Adiabatic theorems
3. Repeated Interaction Systems

Combining the ingredients

Two tools

1. An adiabatic theorem for RIS
2. Perturbation of relative entropy

Entropy production of RIS in adiabatic limit

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} our work arxiv/1510.00533

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Landauer's setup (1/4)

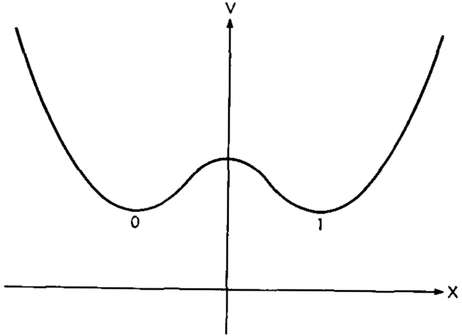


Figure 1 **Bistable potential well.**
x is a generalized coordinate representing quantity which is switched.

Landauer's setup (1/4)

Goal: Erasure process: $E : \{0, 1\} \to \{0\}$.

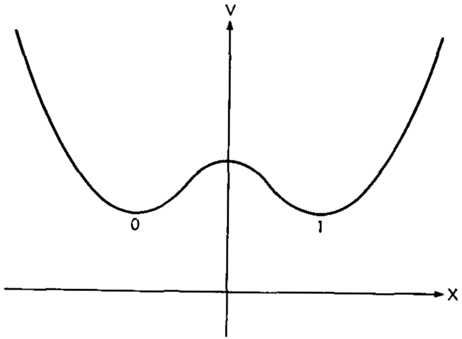
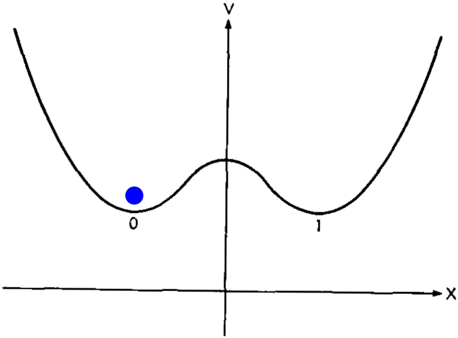


Figure 1 **Bistable potential well.**
x is a generalized coordinate representing quantity which is switched.

Landauer's setup (2/4)

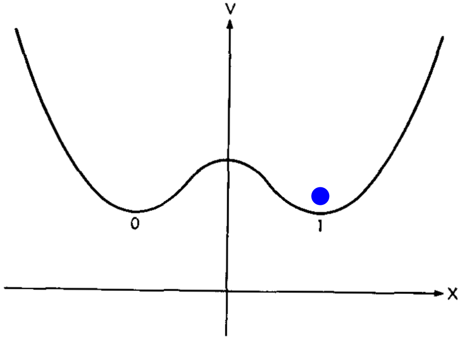
$0 \to 0$



If the system is in 0, we don't have to do anything to erase.

Landauer's setup (3/4)

$1 \to 0$



Now consider if the system starts in state 1

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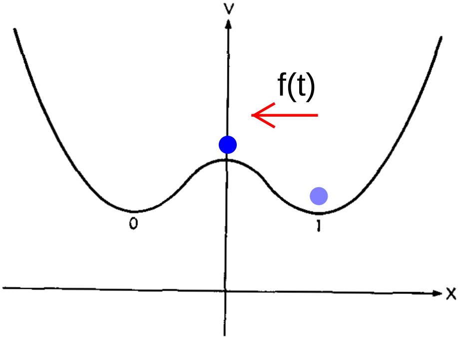
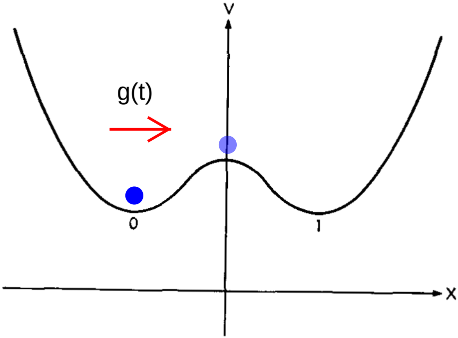


To get to zero, we could apply a force $f(t)$ to the left

Landauer's setup (3/4)

$1 \rightarrow 0$



Then remove the energy we added with a force $g(t)$ to the right

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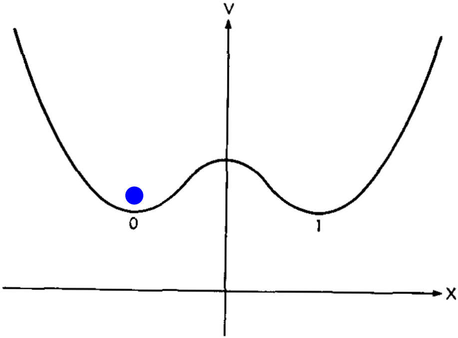

Resulting in the system "erased" in state zero without any net energy cost.

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- Landauer 1961: If we had friction, this would work (with $g(t) \equiv 0$)
  - On the other hand, our erasure map E cuts the size of phase space; since entropy shouldn't decrease, we must have heat output.

Modern (finite-dimensional) quantum formulation (1/3)

The process

Initial joint state: $\rho^\mathrm{i} \otimes \xi^\mathrm{i}$.

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The process

$\mathcal{S}$

$(\mathcal{H}_{\mathcal{S}}, h_{\mathcal{S}}, \rho^{\mathrm{i}})$

Arbitrary ρ^{i}

$\mathcal{E}$

$(\mathcal{H}_{\mathcal{E}}, h_{\mathcal{E}}, \xi^{\mathrm{i}})$

Thermal: $\xi^{\mathrm{i}} = \exp(-\beta h_{\mathcal{E}}) / \operatorname{Tr}(\ldots)$

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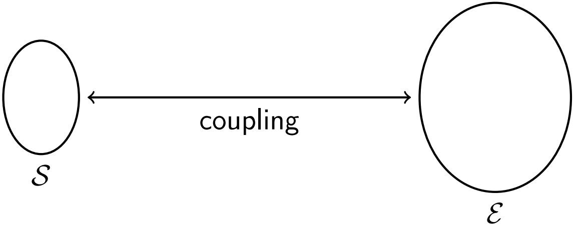

Time evolution by unitary U...

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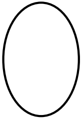
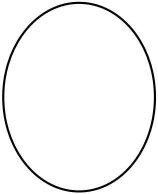


$\mathcal{S}$

$$\rho^{\mathsf{f}} = \mathrm{Tr}_{\mathcal{H}_{\mathcal{E}}}(U\rho^{\mathsf{i}} \otimes \xi^{\mathsf{i}} U^*)$$



$\mathcal{E}$

$$\xi^{\mathsf{f}} = \mathrm{Tr}_{\mathcal{H}_{\mathcal{S}}}(U\rho^{\mathsf{i}} \otimes \xi^{\mathsf{i}} U^*)$$

Final joint state: $U\rho^{\mathsf{i}} \otimes \xi^{\mathsf{i}} U^*$.

Modern (finite-dimensional) quantum formulation (2/3)

Quantities of interest

Def: $S(\rho) := -\operatorname{Tr} \rho \log \rho, \qquad S(\eta|\nu) := \operatorname{Tr} \big(\eta(\log \eta - \log \nu)\big)$

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  ↑ key object #1

- Computation of σ using ξ^{i} is Gibbs:

$$\begin{aligned}
\sigma &= -S(U\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}} U^*) - \operatorname{Tr}\left(U\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}} U^* \left(\log \rho^{\mathrm{f}} \otimes \mathrm{Id}\right)\right) \\
&\qquad - \operatorname{Tr}\left(U\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}} U^* \left(\mathrm{Id} \otimes \log \xi^{\mathrm{i}}\right)\right) \\
&= -S(\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}}) + S(\rho^{\mathrm{f}}) - \operatorname{Tr}(\xi^{\mathrm{f}} \log \xi^{\mathrm{i}}) \\
&= -S(\rho^{\mathrm{i}}) - S(\xi^{\mathrm{i}}) + S(\rho^{\mathrm{f}}) - \operatorname{Tr}(\xi^{\mathrm{f}} \log \xi^{\mathrm{i}}) \\
&= -\Delta S_{\mathcal{S}} + \beta \Delta Q_{\mathcal{E}}.
\end{aligned}$$

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$$\boxed{\sigma = -\Delta S_{\mathcal{S}} + \beta\Delta Q_{\mathcal{E}}.} \qquad \textit{the balance equation}$$

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Special case: Erasure process of qubit: $\mathcal{H}_{\mathcal{S}} = \mathbb{C}^2$, $\rho^{\mathrm{i}} = \frac{1}{2}\mathrm{Id}$, $\rho^{\mathrm{f}} = |0\rangle\langle 0|$. Then $\Delta S_{\mathcal{S}} = \log 2$, and LP becomes

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- When $\sigma = 0$, which in this process occurs only when $\Delta S_{\mathcal{S}} = \Delta Q_{\mathcal{E}} = 0$ [JP14].
- In fact, tighter bounds exist in finite dimensions [RW14].

Outline

The adiabatic limit

- In this unitary time evolution set up, we write Schrödinger's equation

$$i\frac{\mathrm{d}}{\mathrm{d}s}U(s) = h(s)U(s), \ s \in [0,1], \text{ with } U(0) = \mathrm{Id}.$$

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- Then *adiabatic limit* concerns the solution $U_T(s)$ of the rescaled Schrödinger equation

$$i\frac{\mathrm{d}}{\mathrm{d}t}U_T(t) = h(t/T)\,U_T(t),\ t \in [0,T],\ \text{with}\ U_T(0) = \mathrm{Id},$$

in the limit $T \to \infty$.

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1. Intertwine with P: $W(t)P(0) = P(t/T)W(t)$.
2. Approximate time evolution on Ran $P(0)$:

$$\Big(U_T(t) - \exp\big(-\mathrm{i}T\int_0^{t/T} e(s)\,\mathrm{d}s\big)\,W(t)\Big)\,P(0) = O(T^{-1})$$

uniformly in $t \in [0, T]$.

Adiabatic limit of Landauer's Principle

At fixed $T > 0$, we can consider the quantities ΔS_T, ΔQ_T defined through the time evolution $U_T = U_T(1)$. Then we obtain our balance equation

$$\Delta S_T + \sigma_T = \beta \Delta Q_T,$$

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- In [JP14], the authors use a different adiabatic theorem and show $\sigma_T \to 0$, when the reservoir is infinite dimensional, with the help of an ergodicity assumption.

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$$U_k := \exp\big(-i\tau_k(h_{\mathcal{S}} \otimes \mathrm{Id} + \mathrm{Id} \otimes h_{\mathcal{E}_k} + \lambda_k v_k)\big)$$

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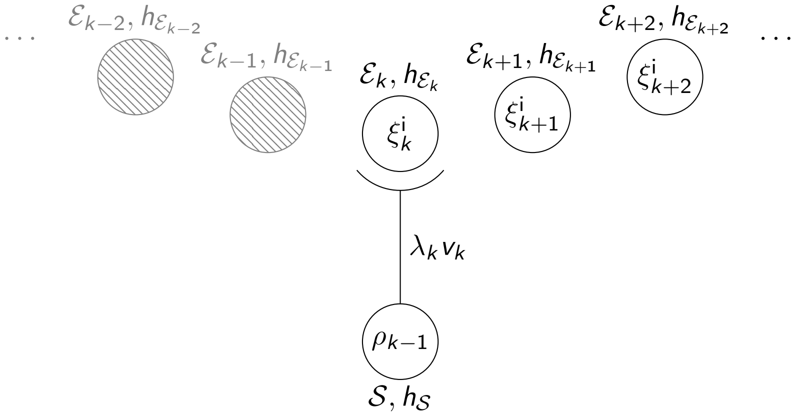
4. We trace out $\mathcal{E}_k$ to obtain the system state

$$\rho_k = \mathrm{Tr}_{\mathcal{E}}\big(U_k(\rho_{k-1} \otimes \xi_k^{\mathrm{i}})U_k^*\big).$$

RIS Setup (2/2)


$\dots$
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ξ_k^{i}
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$\dots$
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We will consider both potentials.

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- Then

$$\rho_k = \mathcal{L}_k \mathcal{L}_{k-1} \cdots \mathcal{L}_1 \rho^{\mathrm{i}}.$$

  This is a *Markovian* form for the sequence of states $(\rho_k)_k$.

Reduced dynamics in our favorite examples

- With both v_{RW} and v_{FD}, we've computed a 4x4 matrix representation of the reduced dynamics $\mathcal{L}(\beta)$ in terms of E, E_0, β, τ and λ. Recall only β changes with the step, so we may obtain $\mathcal{L}_k = \mathcal{L}(\beta_k)$.

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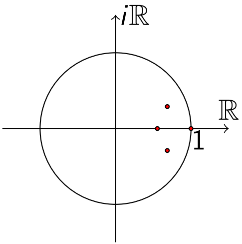

Numerically obtained eigenvalues of $\mathcal{L}_{\mathrm{FD}}$ with $\lambda = 2$, $\tau = 0.5$, $E_0 = 0.8$, and $E = 0.9$. The evals of $\mathcal{L}_{\mathrm{FD}}$ are independent of β.

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- However, in general, we could have $\|\mathcal{L}_k\| > 1$ when considered as an operator on $(\mathcal{I}_1(\mathcal{H}_\mathcal{S}), \|\cdot\|_2)$, where $\|A\|_2 = \sqrt{\mathrm{Tr}(A^*A)}$.

Outline

Landauer's Principle at step k of an RIS

- The entropy change of $\mathcal{S}$ and energy change of $\mathcal{E}_k$ at step k is given by

$$\Delta S_k := S(\rho_{k-1}) - S(\rho_k) = S(\rho_{k-1}) - S(\mathcal{L}_k(\rho_{k-1})),$$
$$\Delta Q_k := \mathrm{Tr}\left(h_{\mathcal{E}_k} \underbrace{\mathrm{Tr}_{\mathcal{S}}\big(U_k(\rho_{k-1} \otimes \xi_k^{\mathrm{i}})U_k^*\big)}_{\xi_k^{\mathrm{f}}}\right) - \mathrm{Tr}(h_{\mathcal{E}_k} \xi_k^{\mathrm{i}}),$$

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- So the balance equation holds at step k

$$\Delta S_k + \sigma_k = \beta_k \Delta Q_k,$$

with

$$\sigma_k := S\big(U_k(\rho_{k-1} \otimes \xi_k^{\mathrm{i}})U_k^* | \mathcal{L}_k(\rho_{k-1}) \otimes \xi_k^{\mathrm{i}}\big).$$

Adiabatic RIS

- To consider an "adiabatic limit" of an RIS process, we will consider $T \gg 0$ interactions (i.e., $\mathcal{S}$ interacts with $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_T$), and

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- We will sample the probe and interaction parameters from C^2 functions. That is, define C^2 functions

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for $s \in [0, 1]$, and choose

$$h_{\mathcal{E}_{k,T}} = h_{\mathcal{E}}(k/T), \qquad \beta_{k,T} = \beta(k/T), \qquad v_{k,T} = v(k/T).$$

LP in RIS in the adiabatic limit (1/2)

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- We are interested in $\lim_{T\to\infty} \sigma_T$. Does this vanish?

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- Each $\sigma_{k,T} \geq 0$, so we need vanishing entropy production at each step.
  - In fact, want $\sigma_{k,T} = O(1/T^2)$, so $\sigma_T = O(1/T)$.

Strategy

- Our key object:

$$\sigma_{k,T} = S\big(U_{k,T}(\rho_{k-1,T} \otimes \xi^{\mathrm{i}}_{k,T})U^*_{k,T} | \mathcal{L}_{k,T}(\rho_{k-1,T}) \otimes \xi^{\mathrm{i}}_{k,T}\big),$$

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- Then $\sigma_{k,T}$ approximately only depends on parameters at step k, and *not* on steps $0,\ldots,k-1$.

Outline

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Three ingredients
1. Landauer's Principle
2. Adiabatic theorems
3. Repeated Interaction Systems

Combining the ingredients

Two tools
1. An adiabatic theorem for RIS
2. Perturbation of relative entropy

Entropy production of RIS in adiabatic limit

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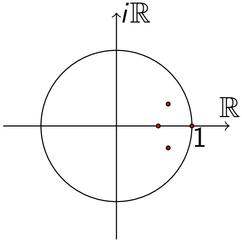

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Notation: $P^j_{k,T}$ is the projection on to the jth peripheral eigenvalue at step k, with adiabatic parameter T.

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Notation: $P_{k,T}^{j}$ is the projection on to the jth peripheral eigenvalue at step k, with adiabatic parameter T.

2. The theorem: there exist constants $T_0 > 0$ and $C > 0$ such that for all $T > T_0$, there exists maps $(A_{k,T})_{k=1}^{T}$ such that

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Perturbation theory applied to relative entropy

For any state η and small enough perturbations D_1, D_2,

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where $F_\eta(A) := F_\eta(A, A)$ for

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2nd order term: $\|F_\eta(A)\| = O(\|A\|^2)$.

Outline

Three ingredients

1. Landauer's Principle
2. Adiabatic theorems
3. Repeated Interaction Systems

Combining the ingredients

Two tools

1. An adiabatic theorem for RIS
2. Perturbation of relative entropy

Entropy production of RIS in adiabatic limit

Perturbing $\sigma_{k,T}$

Now we want to write $\sigma_{k,T} = S(\eta + D_1|\eta + D_2)$. But

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So we want to approximate $\rho_{k-1,T} \approx \rho_{k,T} \approx \rho^{\mathrm{inv}}_k$. The error is the difference

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Using our adiabatic theorem and perturbation of entropy, we can find

$$\sigma_{k,T} = F_{\rho^{\mathrm{inv}}_{k,T} \otimes \xi^{\mathrm{i}}_{k,T}}(D_{k,T} - X_{k,T}) + O(1/T^3).$$

Back to our examples

Consider our 2x2 examples.

- With v_{RW}, $X_{k,T} \equiv 0$. Then

$$\begin{aligned} \sigma_{k,T} &= F_{\rho_{k,T}^{\text{inv}}}(D_{k,T}) + O(1/T^3) \\ &= O(1/T^2) \end{aligned}$$

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  and the entropy production $\sigma_T \to 0$.
- With v_{FD}, $X_{k,T} = O(\lambda)$. We in fact find $\sigma_T \to \infty$, even in the small coupling limit $\lambda^2 T \to 0$ which I have not discussed.[1]

[1] There are subtleties about A2 that require a modification to the setup.

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- Since any qubit state $\rho^{\text{f}} > 0$ may be written as a Gibbs state at some temperature β^{f}, we may choose our temperatures so that $\beta_{T,T} = \beta(1) = \beta^{\text{f}}$. Then the system will be driven to $\rho_{T,T}^{\text{inv}} = \rho^{\text{f}}$, and thus drive initial states to our target state.

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- If the initial state is in the eigenspace of the invariant state, our adiabatic theorem tells us we create $\sigma_T \sim \frac{1}{T}$ entropy production, and approximate $\rho_{T,T} = \rho^{\text{f}} + O(T^{-1})$.
- Otherwise, we create $\sigma_T \sim \frac{1}{T} + \frac{1}{(1-\ell)}$ entropy production, and approximate $\rho_{T,T} = \rho^{\text{f}} + O(1/T) + O(\ell^T)$.

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