Quantizing the electromagnetic field II

Hyeran Kong

Table of Contents

Quantization with different quantum variables

1. Discrete quantum variables $\hat{n} = \hat{a}^{\dagger}\hat{a} \quad [\hat{a}, \hat{a}^{\dagger}] = 1$

$$\hat{H} = \sum \hbar\omega(\hat{a}^{\dagger}\hat{a} + 1/2)$$

Energy ; photon counting
e.g. Single photon, Fock states ..

2. Continuous quantum variables $\hat{P}, \hat{Q}$

Phase and amplitude quadratures
e.g. Squeezed state, Schrodinger cat state

$$\hat{P} = \hat{a}^{\dagger} + \hat{a}$$
$$\hat{Q} = i(\hat{a}^{\dagger} - \hat{a})$$
$$[\hat{Q}, \hat{P}] = 2i$$

Qua

1. Discr

Energy ; p
e.g. Single

2. Cont

Phase anc
e.g. Squee

Plane wave expansions in cube of volume L^3

$$A(\vec{r},t) = \frac{1}{\epsilon_0^{1/2} L^{3/2}} \sum_{k_x, k_y, k_z} A_k(t) e^{i\vec{k}\bullet\vec{r}}$$

Homogeneous wave equation from Maxwell eqn. $\nabla^2 A(\vec{r},t) - \frac{1}{c^2}\frac{\partial}{\partial t^2} A(\vec{r},t) = 0$

$$A(\vec{r},t) = \frac{1}{2\epsilon_0^{1/2} L^{3/2}} \sum_k \sum_s [[q_{ks}(t) + \frac{i}{w} p_{ks}(t)] e^{i\vec{k}\bullet\vec{r}} + c.c]$$

complex analytical signal

$$u_{ks}(t) = c_{ks} e^{-iwt}$$

$$q_{ks}(t) = [u_{ks}(t) + u^*_{ks}(t)]$$

$$p_{ks}(t) = -iw[u_{ks}(t) - u^*_{ks}(t)]$$

$$\hat{H} = \tfrac{1}{2} \sum_k \sum_s [\hat{p}^2_{ks}(t) + w^2 \hat{q}^2_{ks}(t)]$$

$$[\hat{q}_{ks}(t), \hat{p}_{k's'}(t)] = i\hbar \delta^3_{kk'} \delta_{ss'}$$

$$[\hat{q}_{ks}(t), \hat{q}_{k's'}(t)] = 0$$

$$[\hat{p}_{ks}(t), \hat{p}_{k's'}(t)] = 0$$

$$\hat{a}_{ks}(t) = \frac{1}{(2\hbar w)^{1/2}} [w\hat{q}_{ks}(t) + i\hat{p}_{ks}(t)]$$

$$\hat{a}^\dagger_{ks}(t) = \frac{1}{(2\hbar w)^{1/2}} [w\hat{q}_{ks}(t) - i\hat{p}_{ks}(t)]$$

$$[\hat{a}_{ks}(t), \hat{a}^\dagger_{k's'}(t)] = \delta^3_{kk'} \delta_{ss'}$$

$$[\hat{a}_{ks}(t), \hat{a}_{k's'}(t)] = 0$$

$$[\hat{a}^\dagger_{ks}(t), \hat{a}^\dagger_{k's'}(t)] = 0$$

$$\hat{H} = \sum_k \sum_s \hbar w_{k,s} (\hat{a}^\dagger_{k,s} \hat{a}_{k,s} + 1/2)$$

$$\hat{n}_{ks}(t) = \hat{a}^\dagger_{ks} \hat{a}_{ks}$$

$$[\hat{a}_{ks}(t), \hat{n}_{k's'}(t)] = \hat{a}_{ks} \delta^3_{kk'} \delta_{ss'}$$

Qua

1. Discr

Energy ; p
e.g. Single

2. Cont

Phase anc
e.g. Sque

eigenstate of operator $\hat{n}_{ks}$ $\hat{n}_{ks}|n_{ks}\rangle = n_{ks}|n_{ks}\rangle$

Using the commutation relation $[\hat{a}_{ks}(t), \hat{n}_{k's'}(t)] = \hat{a}_{ks}\delta^3_{kk'}\delta_{ss'}$

$$\hat{n}_{ks}\hat{a}^\dagger_{ks}|n_{ks}\rangle = \hat{a}^\dagger_{ks}(\hat{n}_{ks}+1)|n_{ks}\rangle$$
$$\hat{n}_{ks}\underline{\hat{a}^\dagger_{ks}|n_{ks}\rangle} = (n_{ks}+1)\underline{\hat{a}^\dagger_{ks}|n_{ks}\rangle}$$

eigenstate of $\hat{n}_{ks}$ $\therefore\ \hat{a}^\dagger_{ks}|n_{ks}\rangle = g_{ks}|n_{ks}+1\rangle$

$$\langle n_{ks}|\hat{a}_{ks}\hat{a}^\dagger_{ks}|n_{ks}\rangle = |g_{ks}|^2\langle n_{ks}+1|n_{ks}+1\rangle = |g_{ks}|^2$$
$$= \langle n_{ks}|\hat{a}^\dagger_{ks}\hat{a}_{ks}+1|n_{ks}\rangle = \langle n_{ks}|\hat{n}_{ks}+1|n_{ks}\rangle = n_{ks}+1$$

$$\therefore g_{ks} = \sqrt{n_{ks}+1}$$

$$\therefore\ \hat{a}^\dagger_{ks}|n_{ks}\rangle = \sqrt{n_{ks}+1}|n_{ks}+1\rangle$$

creation operator

$$\hat{a}_{ks}|n_{ks}\rangle = \sqrt{n_{ks}}|n_{ks}-1\rangle$$

annihilation operator

$$\hat{H} = \sum_k \sum_s \hbar w_{k,s}(\hat{a}^\dagger_{k,s}\hat{a}_{k,s}+1/2)$$

eigenvalue is quantized

Quantization with different quantum variables

1. Discrete quantum variables

Fock state representation

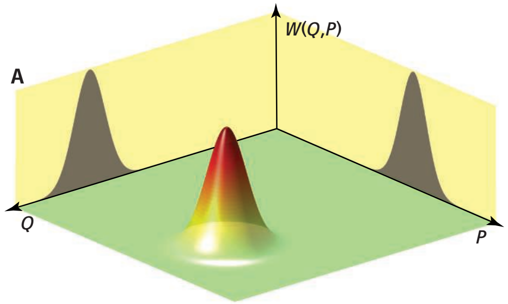
$$\rho_{nm} = \sum P_{nm}|n\rangle\langle m|$$

Energy ; photon counting
e.g. Single photon, Fock states ..

2. Continuous quantum variables

Phase and amplitude
e.g. Squeezed state, Schrodinger cat state

Coherent state $|\alpha\rangle$



$$\langle(\triangle\hat{Q})^2\rangle^{1/2} = 1,\ \langle(\triangle\hat{P})^2\rangle^{1/2} = 1$$

Quantization with different quantum variables

- Phase ?

$\hat{a}^\dagger = e^{-i\hat{\phi}}\ \hat{g}$ does not satisfy uncertainty principle $\langle(\triangle\hat{n})^2\rangle^{1/2}\langle(\triangle\hat{\phi})^2\rangle^{1/2} \overset{?}{\geq} \frac{1}{2}$

$\hat{\phi}$ is not Hermitian

$$\hat{\mathcal{C}} \equiv \frac{1}{2}[\widehat{\exp(i\phi)} + \widehat{\exp(i\phi)}^\dagger]$$

$$\hat{\mathcal{S}} \equiv \frac{1}{2i}[\widehat{\exp(i\phi)} - \widehat{\exp(i\phi)}^\dagger]$$

do not commute

$$\hat{\phi}_c = arc\ cos\ \hat{\mathcal{C}}$$

$$\hat{\phi}_s = arc\ sin\ \hat{\mathcal{S}}$$

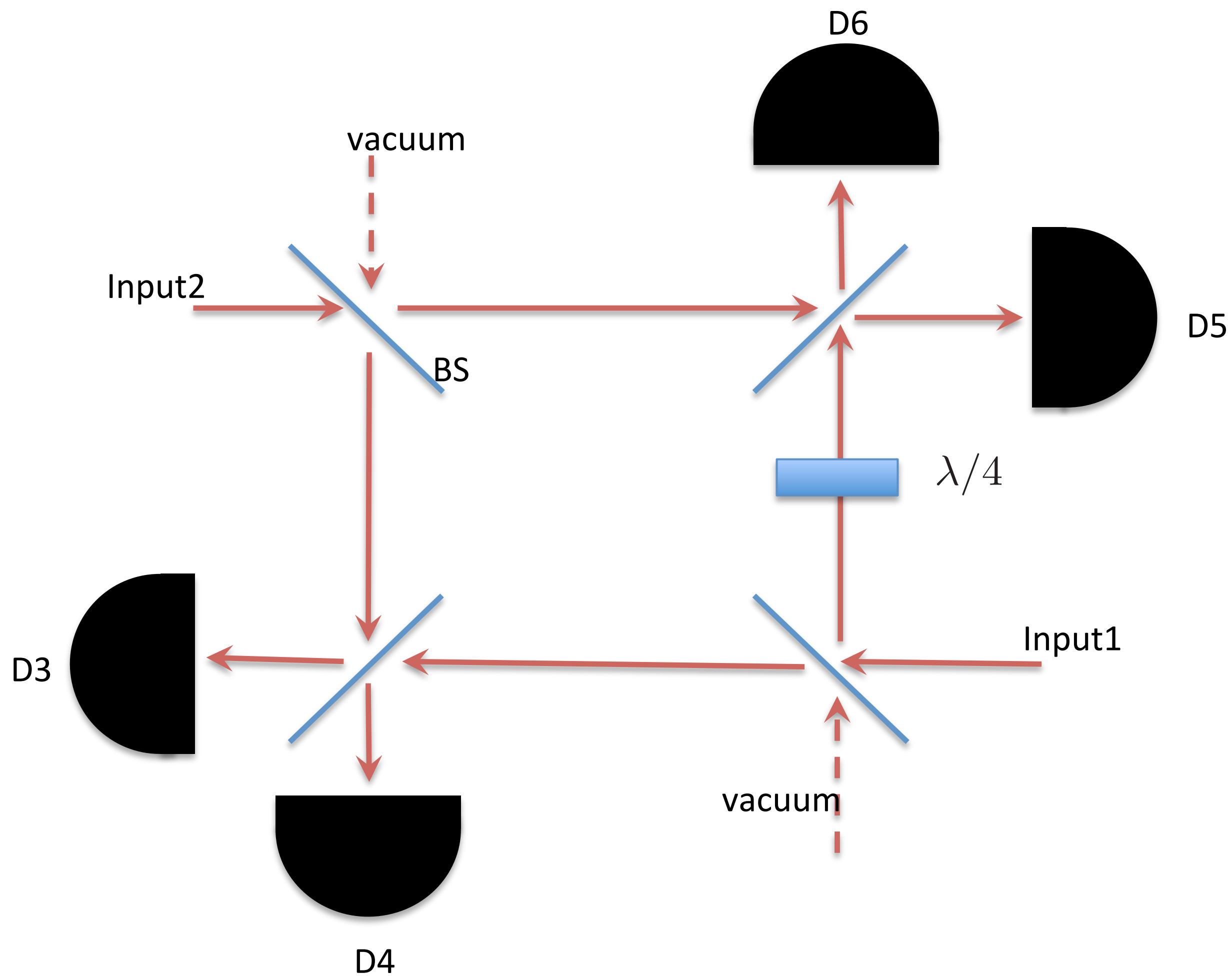
commute !

$$[\exp(\widehat{i\phi_1})[\exp(\widehat{i\phi_2})]^\dagger, \hat{n}_1 + \hat{n}_2] = [(\hat{\mathcal{C}}_1 + i\hat{\mathcal{S}}_1)(\hat{\mathcal{C}}_2 - i\hat{\mathcal{S}}_2), \hat{n}_1 + \hat{n}_2] = 0$$

$$|\theta\rangle = \frac{1}{(s+1)^{1/2}}\sum_{n=0}^{s} e^{in\theta}|n\rangle \qquad \theta_m = \theta_0 + \frac{2\pi m}{s+1} \quad , m = 0, 1, 2..s$$

commute, orthogonal

Quantization with different quantum variables



J.W. Noh, A. Fougeres, L. Mandel, *Phys. Rev. Lett.* **67**, 1467 (1991).

Quantization with different quantum variables

$$\hat{\mathcal{C}}_M = (\hat{n}_4 - \hat{n}_3)[(\hat{n}_4 - \hat{n}_3)^2 + (\hat{n}_6 - \hat{n}_5)^2]^{-1/2}$$
$$\hat{\mathcal{S}}_M = (\hat{n}_6 - \hat{n}_5)[(\hat{n}_4 - \hat{n}_3)^2 + (\hat{n}_6 - \hat{n}_5)^2]^{-1/2}$$

$$\frac{\langle(\triangle\hat{\mathcal{C}}_M)^2\rangle^{1/2}}{\langle\hat{\mathcal{C}}_M\rangle} \geq 1$$
$$\frac{\langle(\triangle\hat{\mathcal{S}}_M)^2\rangle^{1/2}}{\langle\hat{\mathcal{S}}_M\rangle} \geq 1$$

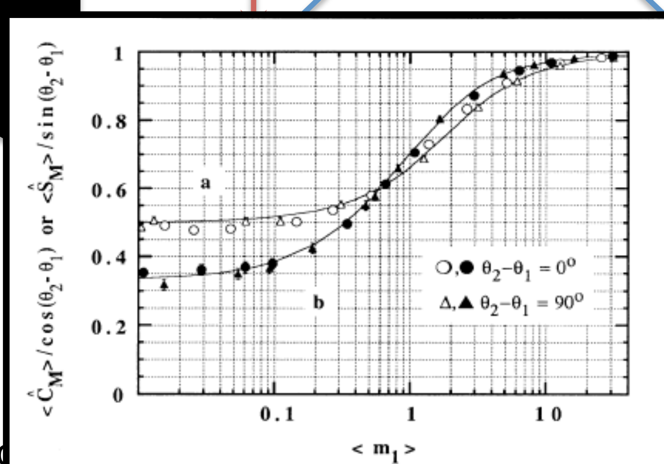
In the regime, $\langle\hat{n}_1\rangle, \langle\hat{n}_2\rangle \ll 1$

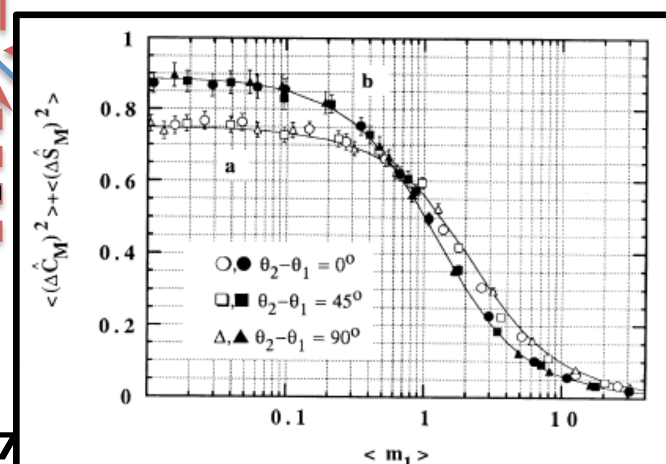
Phase difference ill defined

Input2

D3

um





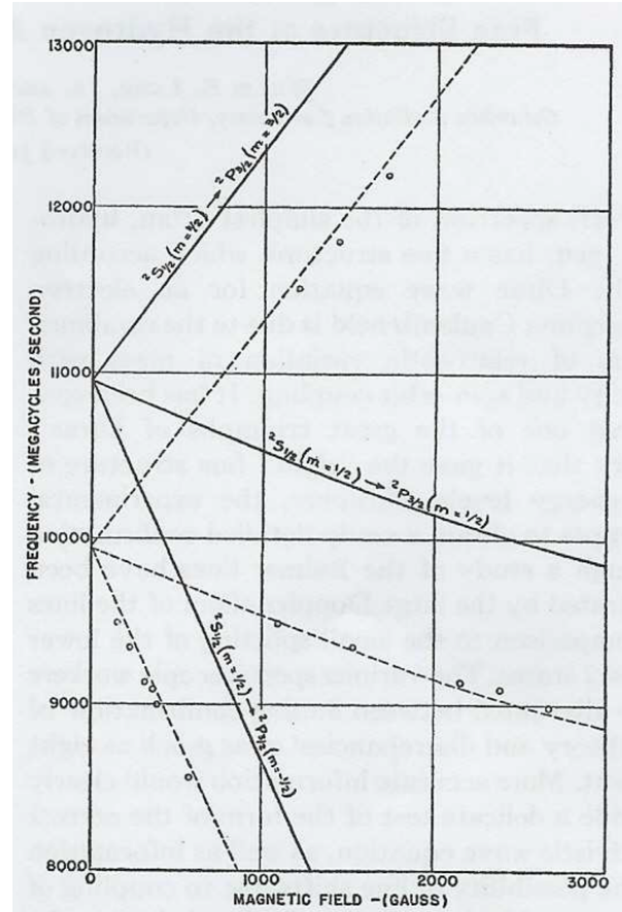
J.W. Noh, A. Fo

67

Lamb Shift

- "Experimental Result"

Energy difference between $2S_{1/2}, 2P_{1/2}$
by level shift



- Vacuum Fluctuation Theory
  (by Welton, 1957)

W E. Lamb, JR., and R C. Rutherford, *Phys. Rev. Lett.* **72**,241 (1947)
W E. Lamb, JR. Nobel Lecture, (1955)

Lamb Shift

- "Experimen

Energy difference be

by level shift

- Vacuum Flu
  (by Welto

Electron orbit perturbed by vacuum fluctuation

perturbation δr

$$\delta V = V(r+\delta r) - V(r)$$

$$< \delta V >= 1/6 < \bigtriangledown^2 V >< (\delta r)^2 >$$

Non-relativistic eqn. of motion for electron

$$m_e \ddot{r} = -eE_\omega cos\omega t$$

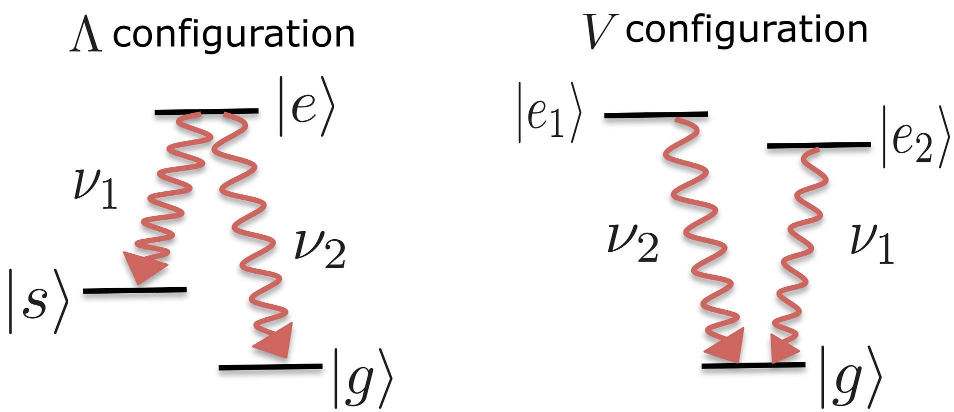
$$\triangle E = \frac{\hbar e^4}{12\pi^2 \epsilon_0^2 m^2 c^3} |\psi(0)|^2 ln(mc^2/\hbar\omega_0)$$

~1GHz in good agreement with Lamb shift measurement

W E. Lamb, JR., and R C. Rutherford, *Phys. Rev. Lett.* **72**,241 (1947)
W E. Lamb, JR. Nobel Lecture, (1955)

Quantum beat

- Semiclassical Theory (SCT)



$$E^{(+)} = \mathcal{E}_1 \exp\left(-i\nu_1 t\right) + \mathcal{E}_2 \exp\left(-i\nu_2 t\right)$$

$$|E^{(+)}|^2 = |\mathcal{E}_1|^2 + |\mathcal{E}_2|^2 + \underline{(\mathcal{E}_1^* \mathcal{E}_2 \exp\left(-i(\nu_1 - \nu_2)t\right) + \text{c.c})}$$

Interference term

Quantum bea

•

$$|\psi_\Lambda\rangle = \sum_{i=g,s,e} c'_i|i,0\rangle + c'_1|g,1_{\nu_1}\rangle + c'_2|s,1_{\nu_2}\rangle$$

$$E_1^{(-)}(t)E_2^{(+)}(t)|\psi_\Lambda\rangle$$

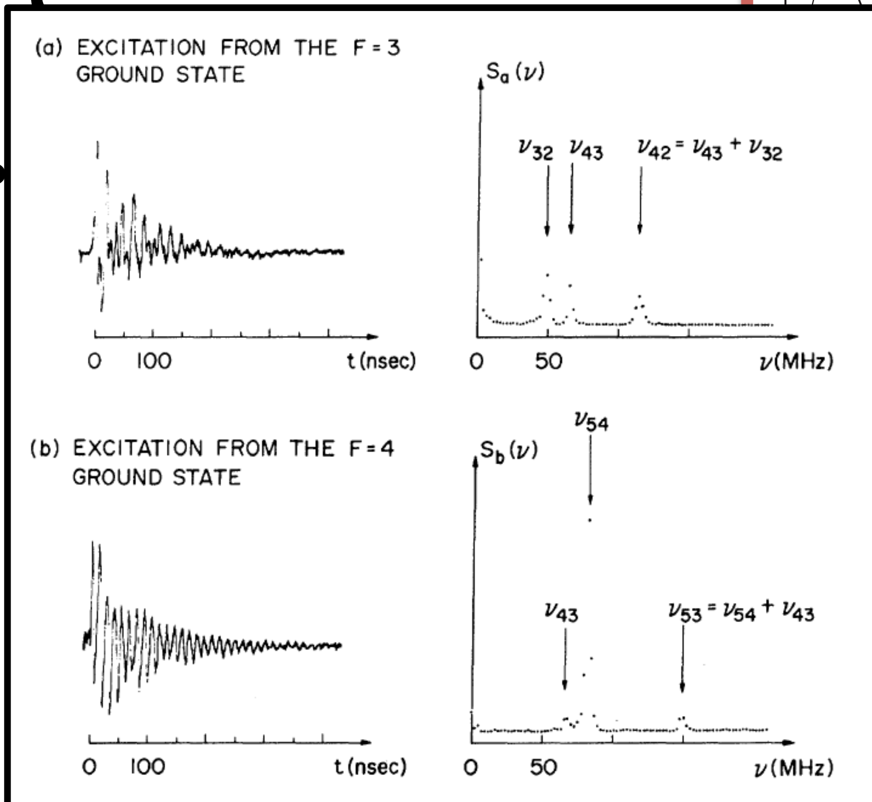
$$_{\nu_1}0_{\nu_2}|a_1^\dagger a_2|0_{\nu_1}1_{\nu_2}\rangle \exp(i(\nu_1-\nu_2)t)\langle g|s\rangle$$

$$|E_1^{(-)}(t)E_2^{(+)}(t)|\psi_\Lambda\rangle = 0$$

ν_1

$$= \sum_{i=g,e_1,e_2} c_i|i,0\rangle + c_1|g,1_{\nu_1}\rangle + c_2|g,1_{\nu_2}\rangle$$

$$E_1^{(-)}(t)E_2^{(+)}(t)|\psi_V\rangle = \kappa \exp(i(\nu_1-\nu_2)t)$$



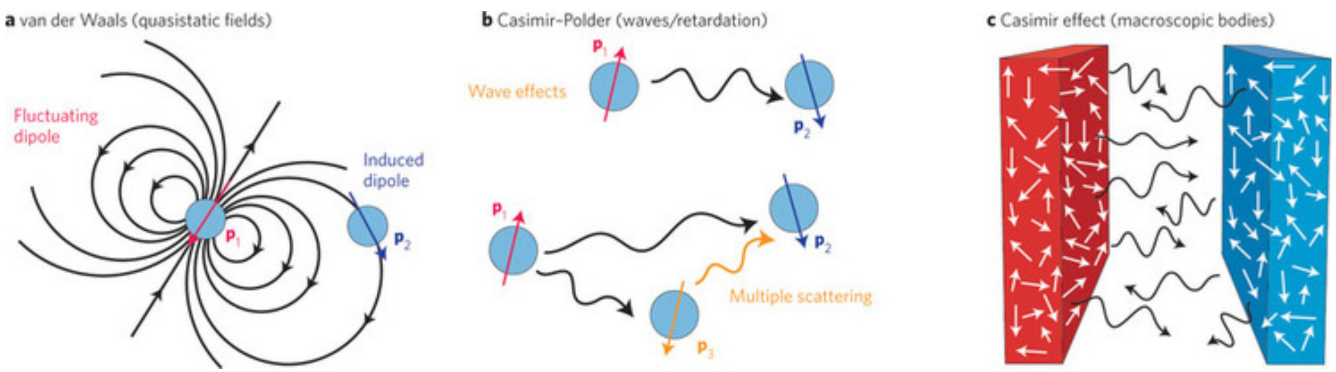
$$|\psi_\Lambda\rangle = \sum_{i=g,s,e} c'_i$$

$$|\psi_V\rangle = \sum_{i=g,e_1,e_2} c_i|i,0\rangle + c_1|g,1_{\nu_1}\rangle + c_2|g,1_{\nu_2}\rangle$$

S. Haroche, J.A. Paisner, A.L. Schawlow *Phys. Rev. Lett.* **30**, 948 (1973).

Casimir effect

Casimir effect manifested over different scales.



H.B.G. Casimir, and D. Polder, *Phys. Rev. Lett.* **73**, 360 (1947).
V. Sandoghdar, C. I. Sukenik, E. A. Hinds, S. Haroche, *Phys. Rev. Lett.* **68**, 3432 (1992).

Two flavors of quantum variables

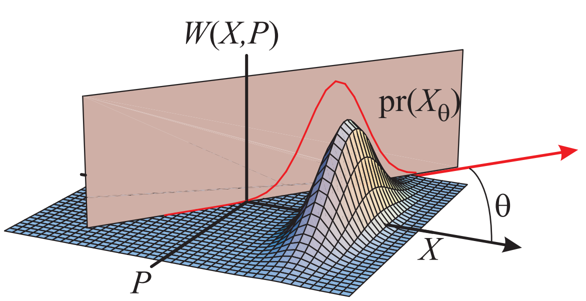
1. Energy → Discrete quantum variables $[\hat{a}, \hat{a}^\dagger] = 1$ *photon counting*

In Fock-state representation $\rho_{nm} = \sum_{n,m} P_{nm} |n\rangle \langle m|$

2. Fields (phase-amplitude) → Continuous quantum variables

quadratures $\hat{P} = \hat{a}^\dagger + \hat{a}$, $\hat{Q} = i(\hat{a}^\dagger - \hat{a})$ with $[\hat{Q}, \hat{P}] = 2i$

e.g., Wigner function for squeezed states



L. Mandel., and E. Wolf, "Optical Coherence and Quantum Optics"

Optical equivalence theorem

VOLUME 10, NUMBER 7 PHYSICAL REVIEW LETTERS 1 APRIL 1963

EQUIVALENCE OF SEMICLASSICAL AND QUANTUM MECHANICAL DESCRIPTIONS OF STATISTICAL LIGHT BEAMS

E. C. G. Sudarshan
Department of Physics and Astronomy, University of Rochester, Rochester, New York
(Received 1 March 1963)

We can make use of the overcompleteness[6] of the states to represent every density matrix,

$$\rho = \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} \rho(n,n') \psi(n) \psi^{\dagger}(n'),$$

in the "diagonal" form

$$\rho = \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} \rho(n,n') \frac{(n!n'!)^{1/2}}{(n+n')!} \left\{ \left(\frac{\partial}{\partial r}\right)^{n+n'} \int \frac{d\theta}{2\pi} \exp[r^2 + i(n'-n)\theta] \, |re^{i\theta}\rangle\langle re^{i\theta}| \right\}\Big|_{r=0}.$$

This form is particularly interesting since if $O = (a^{\dagger})^{\lambda} a^{\mu}$ be any normal ordered operator (i.e., all creation operators to the left of all annihilation operators), its expectation value in the statistical state represented by the density matrix in the "diagonal" form

$$\rho = \int d^2 z \, \phi(z) \, |z\rangle\langle z| \qquad (4)$$

is given by

$$\mathrm{tr}\{\rho O\} = \mathrm{tr}\{\rho (a^{\dagger})^{\lambda} a^{\mu}\} = \int d^2 z \, \phi(z) (z^*)^{\lambda} z^{\mu}. \qquad (5)$$

Glauber-Sudarshan function

- *Universal* character of all states of EM fields
- *Quasi-probability density in the "diagonal" basis*

Representation of quantum states in the coherent-state basis
Prediction of *genuine* quantum effects, which cannot be described classically.

E. C. G. Sudarshan, *Phys. Rev. Lett.* **10**, 277 (1963)
R. J. Glauber, *Phys. Rev.* **131**, 2766 (1963).

Optical equivalence theorem

VOLUME 10, NUMBER 7 PHYSICAL REVIEW LETTERS 1 APRIL 1963

EQUIVALENCE OF SEMICLASSICAL AND QUANTUM MECHANICAL DESCRIPTIONS OF STATISTICAL LIGHT BEAMS

E. C. G. Sudarshan
Department of Physics and Astronomy, University of Rochester, Rochester, New York
(Received 1 March 1963)

We can make use of the overcompleteness[6] of the states to represent every density matrix,

$$\rho = \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} \rho(n,n')\psi(n)\psi^{\dagger}(n'),$$

in the "diagonal" form

$$\rho = \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} \rho(n,n') \frac{(n!n'}{(n+n}$$

This form is
$O=(a^{\dagger})^{\lambda}a^{\mu}$ be
all creation o
operators), it
state represe
"diagonal" fo

is given by

$\mathrm{tr}\{\rho O\}=\mathrm{tr}$

Classical domain: A *well-behaved* *P*-function can be interpreted as a probability density to find ρ in $|z\rangle$

vs.

Quantum domain: *Pathological examples* of *P*-functions such as photon number-state

$$\phi(z) = \frac{e^{|z|^2}}{n!}\frac{\partial^{2n}}{\partial z^n \partial z^{*n}}\delta^{(2)}(z).$$

Representation of quantum states in the coherent-state basis
Prediction of *genuine* quantum effects, which cannot be described classically.

E. C. G. Sudarshan, *Phys. Rev. Lett.* **10**, 277 (1963)
R. J. Glauber, *Phys. Rev.* **131**, 2766 (1963).

A model-independent line between classical and manifestly quantum domains

Manifestly Quantum Domain *Nonclassical Light*	Classical Domain
Number states $\|n\rangle$ Squeezed states EPR states "Twin" beams …	Blackbody radiation Radio waves Cell phone emissions Laser light …
$P(\alpha) < 0$	$P(\alpha) \geq 0$
No classical description of $P(\alpha)$ with positive probability distribution for the field amplitudes	$P(\alpha)$ provides the basis for a classical statistical theory of the EM field $P(\alpha)$=probability distribution for field amplitudes

Glauber-Sudarshan

Phase-Space Function $P(\alpha)$

$$\hat{\rho} = \int d^2 z P(\alpha) |\alpha\rangle\langle\alpha|$$

Coherent State Representation

Fock state $|n\rangle$ representation $\quad \hat{a}|v\rangle = v|v\rangle, \quad \langle v|\hat{a}^\dagger = v^*\langle v|$

$$|v\rangle = \sum_{n=0}^{\infty} c_n |n\rangle, \quad \therefore \sum_{n=0}^{\infty} c_n \sqrt{n} |n-1\rangle = v \sum_{n=0}^{\infty} c_n |n\rangle$$

<coherent state>

$$c_n = \frac{v}{\sqrt{n}} c_{n-1} = \frac{v^2}{\sqrt{n(n-1)}} c_{n-2} = \dots = \frac{v^n}{\sqrt{n!}} c_0 \rightarrow |v\rangle = e^{-|v|^2/2} \sum_{n=0}^{\infty} \frac{v^n}{\sqrt{n!}} |n\rangle$$

normalize factor

$$|c_0| = e^{-|v|^2/2}$$

Probability follows Poisson distribution $\quad p(n) = |\langle n|v\rangle|^2 = \frac{|v|^{2n}}{n!} e^{-|v|^2}$

$$\sum_{n=0}^{\infty} np(n) = |v|^2 = \langle v|\hat{a}^\dagger \hat{a}|v\rangle$$ annihilation without changing state !

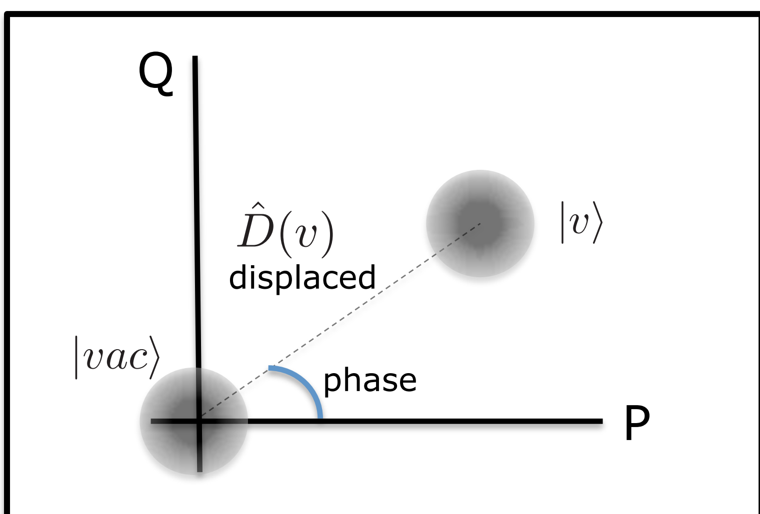
Coherent State Representation

- Displacement operator

$$|v\rangle = e^{-|v|^2/2}\sum_{n=0}^{\infty}\frac{v^n}{\sqrt{n!}}|n\rangle = e^{-|v|^2/2}\sum_{n=0}^{\infty}\frac{v^n\hat{a}^{\dagger n}}{\sqrt{n!}}|vac\rangle$$

$$= e^{-|v|^2/2}e^{v\hat{a}^\dagger}e^{-v^*\hat{a}}|vac\rangle = \hat{D}(v)|vac\rangle$$

displaced vacuum state



te

$$\hat{D}(v)\hat{D}^\dagger(v) = 1 = \hat{D}^\dagger(v)\hat{D}(v)$$

$$Tr(\hat{D}(v)\hat{D}^\dagger(v')) = \pi\delta^2(v-v')$$

$$\hat{D}(v)\hat{D}(v') = e^{(vv'^* - v^*v')/2}\hat{D}(v+v')$$

Coherent State Representation

- Unce

$\langle v|(\triangle\hat{a}(t))^2|v$

$\langle v|(\triangle\hat{p}(t))^2|v\rangle$ $\frac{1}{2}\hbar$

Minimum un

Unitary

$\hat{U}^\dagger(\theta)\hat{a}\hat{U}$ $\cdots)$

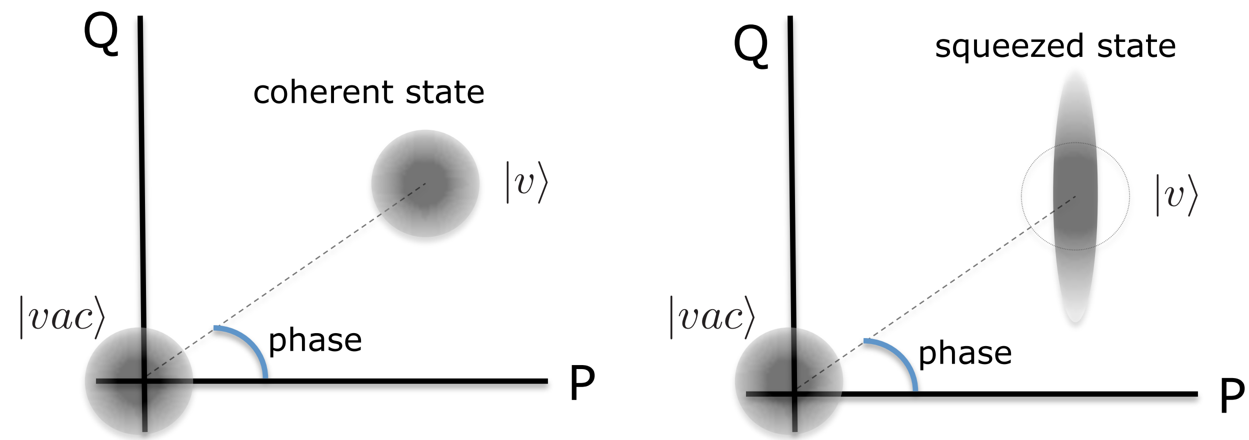
$\langle v,\theta|(\triangle\hat{q}(t))^2$

$\langle v,\theta|(\triangle\hat{p}(t))^2$

$$\hat{P} = \sqrt{\frac{2w}{\hbar}}\hat{p}\ ,\quad \hat{Q} = \sqrt{\frac{2}{\hbar w}}\hat{q}$$

$\langle(\triangle Q)^2\rangle^{1/2} = 1,\ \langle(\triangle P)^2\rangle^{1/2} = 1$ coherent state

$\langle(\triangle Q)^2\rangle^{1/2} = e^{-2\theta},\ \langle(\triangle P)^2\rangle^{1/2} = e^{2\theta}$ squeezed state



$$\therefore\ \langle v,\theta|(\triangle\hat{p}(t))^2|v,\theta\rangle^{1/2}\langle v,\theta|(\triangle\hat{q}(t))^2|v,\theta\rangle^{1/2} = \frac{1}{2}\hbar$$

Optical equivalence theorem

Glauber-Sudarshan P-representation for "all" EM field

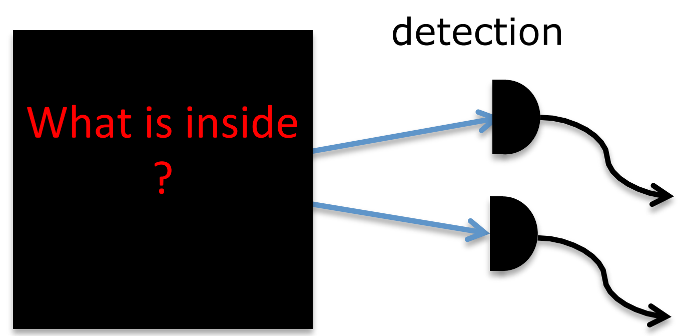
$$\hat{\rho} = \int P(v)|v\rangle\langle v|d^2v \ , \quad d^2v = d(Im(v))d(Re(v))$$

phase space

Non-orthogonality $\quad |\langle v_2|v_1\rangle|^2 = e^{-|v_1-v_2|^2}$

Completeness $\quad \int d^2v|v\rangle\langle v| = 1$

Observable $\quad \hat{\mathcal{O}} = \sum_{m,n} C_{mn}\ \hat{a}^{\dagger m}\hat{a}^n$



Optical equivalence theorem

$$\langle\hat{\mathcal{O}}\rangle = Tr(\hat{\mathcal{O}}\hat{\rho}) = \int P(v)\sum_{n,m} C_{mn}\langle v|\hat{a}^{\dagger m}\hat{a}^n|v\rangle d^2v \ = \int P(v)\sum_{n,m} C_{mn}v^{*m}v^n d^2v$$

$$= \int P(v)\mathcal{O}(v^*, v)d^2v$$

Optical equivalence theorem

Photon number fluctuation – Mandel Q parameter

$$\langle\hat{\mathcal{O}}\rangle = (\hat{n} - \langle\hat{n}\rangle)^2 = (\hat{a}^\dagger\hat{a} - \langle\hat{a}^\dagger\hat{a}\rangle)^2 \geq 0$$

$$= \int P(v)(|v|^2 - \langle|v|^2\rangle)^2 d^2v$$

at, $P(v) \geq 0$
(classical probability density)

2nd order coherence

$$g^{(2)}(0) = \frac{\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle}{|\langle\hat{a}^\dagger\hat{a}\rangle|^2} = 1 + \frac{\int P(v)[|v|^2 - \langle|v|^2\rangle]^2 d^2v}{[\int P(v)|v|^2 d^2v]^2}$$

$g^{(2)}(0) \geq 1$ for all classical EM fields

$g^{(2)}(0) < 1$ anti-bunching $\Rightarrow P(v) < 0$ Quasi-probability density

Can't be explained by classical statistics

Glauber-Sudarshan P-representation

Quasi-probability density function

from $\hat{\rho} = \int P(v)|v\rangle\langle v|d^2v$ with coherent state $|u\rangle$

$$\langle -u|\hat{\rho}|u\rangle = \int P(v)\langle -u|v\rangle\langle v|u\rangle d^2v \quad = e^{-|u|^2}\int P(v)e^{-|v|^2}e^{uv^*-u^*v}\ d^2v$$

By Fourier integral, $P(v)e^{-|v|^2} = \dfrac{1}{\pi^2}\int e^{|u|^2}\underline{\langle -u|\hat{\rho}|u\rangle e^{-uv^*+u^*v}}d^2u$

characteristic function

More generally, there are other kinds of phase-space functions

Wigner representation $W(v) = \dfrac{1}{\pi^2}\int d^2u C_S(u,u^*)e^{-uv^*+u^*v}$

$$W_{\hat{\rho}}(X,P) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{iPQ}\left\langle X-\frac{Q}{2}\right|\hat{\rho}\left|X+\frac{Q}{2}\right\rangle \mathrm{d}Q$$

Examples of Phase-space function

Coherent states $\hat{\rho} = |v'\rangle\langle v'|$

$$\langle -u|\hat{\rho}|u\rangle = \langle -u|v'\rangle\langle v'|u\rangle = e^{-|u|^2-|v'|^2}e^{uv'^*-u^*v'}$$

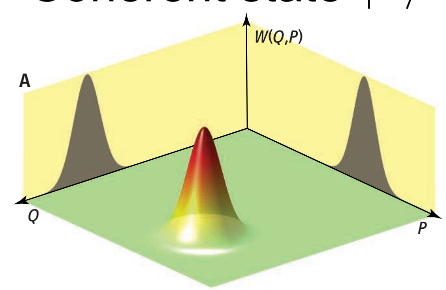
$$P(v)e^{-|v|^2} = \frac{1}{\pi^2}\int e^{-|v'|^2}e^{u(v'^*-v^*)-u^*(v'-v)}d^2u = e^{-|v'|^2}\delta^2(v-v')$$

$$\therefore P(v) = \delta^2(v-v')$$

Using optical equivalence theorem, $g^2(0) = 1$

As a alternative representation,

Coherent state $|\alpha\rangle$



Examples of Phase-space function

- Thermal light

$$P(v) = \frac{1}{\pi\langle n\rangle} e^{-|v|^2/\langle n\rangle}$$ mean photon number $\langle n\rangle$

$$g^{(2)}(0) = 2$$

- Fock state $|n\rangle$

$$P(v) = \frac{e^{|v|^2}}{n!}\frac{\partial^{2n}}{\partial v^{*n}\partial v^{n}}\delta^2(v)$$ sharper than delta function

$$g^{(2)}(0) = 1 - \frac{1}{n}$$

Single photon