Digital Computer Solution of Power-Flow Problems

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THE capabilities of automatic digital computers are receiving increasing attention in application to power system problems. This paper presents a method for solving on digital computers the power-flow problem which is probably the most frequently encountered type of problem in the field of power system network analysis. Digital solution of this class of problem can furnish a valuable tool to supplement the a-c network analyzer. In many system-planning studies, the network analyzer is still the best means for providing quickly and economically results of adequate accuracy. In some studies, such as analysis of losses and incremental losses, the network analyzer does not provide sufficient precision and, in such cases, the digital computer solution gains a distinct advantage.

In brief summary, the power-flow problem consists of imposing specified power input and voltage magnitude, or real and reactive power input conditions, at the terminals of a passive network. The desired solution will provide complete input and voltage information at the terminals and power flow in each branch of the network. A formal, closed form of solution for even very simple networks with such terminal conditions is almost hopelessly difficult, and some iterative process which converges on the solution is required. This feature is characteristic of analyzer solutions of the power-flow problem, where successive adjustment of loads and generators is required to converge on a simultaneous balance of all prescribed terminal conditions.

The digital solution of the power-flow problem as presented here follows four major steps:

1. Description of the network connections and impedances in the form of a list of parameters: The node basis of establishing a mathematical description of the network is used, and off-nominal transformer turns-ratio are treated rigorously. A part of the process in this first step is done manually, such as labelling the diagram, listing impedances, and forming a connection matrix. The major part of numerical computation can be mechanized on the computer.

2. Iterative solution for terminal voltages which satisfy the prescribed terminal conditions: This can be completely automatized so that the computer carries on the process to some prescribed degree of precision. In this part of the process, load and generator terminals must be handled differently, because at loads the known quantities are real and reactive power, while at generators the known quantities are voltage magnitude and power.

3. Computation of complete terminal information, i.e., real and reactive power, voltage magnitude, and voltage phase angle.

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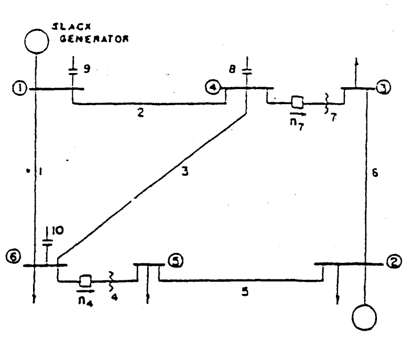

Fig. 1. Sample system

4. Computation of individual line flows. In the event that a solution for losses is all that is desired, this last step can be omitted, for with the precision provided by digital solution losses can be evaluated accurately by a summation of terminal inputs.

The details of the several steps in this plan of computation will be presented in the form of an example, with the use of a simple network. This computation has been organized for and worked out on the Purdue University electrodata computer. This is a medium-speed stored-program computer with a 4,000-word magnetic drum memory. The same method can, of course, be organized for other types of computers. The method has been prepared for the Purdue computer in a general program which will accept any network of a size up to 50 busses and 200 branches.

Network Connections and Impedances

Fig. 1 shows a 1-line diagram of the system used to explain and illustrate the power-flow computation. In this diagram the neutral bus is not shown, but it is considered as one of the nodes and is used as the reference for specification of node or bus voltages.

The first step in coding the network consists of labelling nodes and branches. One generator is selected to act as the slack machine, as in analyzer studies. Since the losses are unknown, power input cannot be specified at all terminals, and the slack machine serves to supply whatever output is necessary to balance other terminal inputs and losses. The nodes or busses are numbered serially starting with the slack generator as number 1, and numbering load and other generator busses in arbitrary order. The neutral bus, as the reference for voltages of the other nodes, is left until last and need not be assigned a number. Branches are numbered serially in arbitrary order. Fig. 1 indicates that the line-charging capacitance is lumped on busses at the line terminals, and each line-charging capacitor thus indicated is assigned a branch number. In Fig. 1, bus numbers are encircled and line or branch numbers are not.

Off-nominal turns-ratios are indicated on the diagram of Fig. 1 as n_4 and n_7. The notation used in Fig. 1 for such turns-ratios is explained pictorially in Fig. 2. The diagram in Fig. 2(A) implies the more detailed representation shown in Fig. 2(B). The ratio n can be greater than unity or less than unity, but in the following dicussion the 1-to-n ratio will be uniformly oriented with the n terminal connected to the branch representing the associated transformer leakage impedance. The description of the network is formulated first in a table of admittance coefficients as if all turns-ratios were unity, then changes in appropriate values are made, as a separate step, to represent effects of off-nominal turns-ratios.

The terminal currents and voltages can be completely specified by what will be referred to here as self- and mutual admittances. The self-admittance of node k, denoted by $Y_{kk}=G_{kk}+jB_{kk}$, is the sum of admittances of branches terminating on node k. The mutual admittance between nodes k and m, denoted by $Y_{km} = G_{km} + jB_{km}$, is the negative of the sum of admittances of branches which are connected between nodes k and m. Usually mutual admittances will consist of single branches, as in Fig. 1; however, parallel lines between two busses can be identified separately if desired. By this definition, all lines and transformers contribute to both self- and mutual admittance values, while capacitors or other constant impedances to neutral contribute only to self-admittances. The self- and mutual admittances are formed for each bus except the neutral bus. This process of forming the self- and mutual admittances can be carried out manually by inspection of the numbered connection diagram with the use of an adding machine. However, in a large network, the conversion of branch impedances to admittances and then the formation of self- and mutual admittances becomes a rather extensive computation. This can be mechanized on the computer, as described in a later section.

Of course, it is implied in the previous paragraph that a tabulation of branch impedances is available. Table I lists the impedances, in per-unit, associated with the network of Fig. 1. Each transformer impedance is specified on the voltage base associated with the bus opposite to the one connected to the turns-ratio symbol. For example, branch 7 impedance in Fig. 1 is expressed on the voltage base associated with bus 3, not bus 4. The turns-ratios n_4 and n_7 to be used in the solution of this problem are listed at the bottom of Table I. Branch admittances as computed from the given impedances are also shown in Table I.

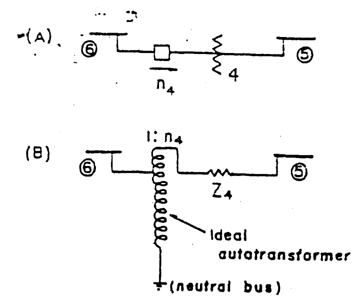

Fig. 2. Off-nominal turns-ratio representation

Terminal conditions specified for this network are listed in Table II. In the P and Q columns, input to the network is listed at each terminal, e.g., loads appear as negative numbers and generation as positive numbers. Bus 2 carries both a load and a generator, and the power quantity listed in Table II is the net generation, or the algebraic sum of the scheduled load power and generator power output. The generator reactive power loading will be obtained from the final solution by a suitable combination of the net reactive input to the system given by the solution and the scheduled load reactive power consumption. Synchronous condensers which are regulated to hold constant terminal voltage can be handled in the same way. Note that bus 4 has neither a load nor a generator, and this is indicated in Table II as a specification of zero impressed power quantities.

Treatment of Off-Nominal Turns-Ratios

In the node basis of formulating the network parameters, the presence of an ideal autotransformer in series with a branch influences only the one self-admittance of the node at which such transformer is connected and the one mutual admittance associated with the branch it is in. The transformer and associated turn-ratio of Fig. 2(B) can be represented by the equivalent π configuration of Fig. 3. Here the branch impedance is replaced by the corresponding admittance. In forming the self-admittance of mode (5), the sum of n_4Y_4 plus $(1-n_4)Y_4$ results in the original

Table I. Branch Impedances, Per Unit

Branch No.	Impedance R	Impedance X	Admittance G	Admittance B
1	0.123	0.518	0.433934	−1.827463
2	0.080	0.370	0.558260	−2.581096
3	0.007	0.407	0.554102	−2.324044
4	0.000	0.300	0	−3.333333
5	0.282	0.640	0.576541	−1.308462
6	0.723	1.050	0.444860	−0.646063
7	0.000	0.133	0	−7.518797
8	0.000	−34.100	0	0.029326
9	0.000	−29.500	0	0.033898
10	0.000	−28.500	0	0.035088

Turns-ratios: $n_4 = 1.0250$, $n_7 = 1.1000$.

value, namely Y_4. So the self-admittance of the node opposite the autotransformer is not affected. The sum of the two branches of the equivalent π which terminate on node 6 yields $n_4^2 Y_4$. And the mutual admittance between nodes 5 and 6 becomes $-n_4 Y_4$. Thus, the following rules can be formed for altering the set of self- and mutual admittances as obtained initially by ignoring off-nominal turns-ratios.

If branch j connected between nodes k and m has turn-ratio n_j at node k, add to Y_{kk} the quantity

$$(n_j^2 - 1)Y_j$$

and add to Y_{km} the quantity

$$-(n_j - 1)Y_j$$

This suggests a convenient procedure for changing the transformer taps after the complete list of self- and mutual admittances has been prepared and is stored in the computer memory.

Branch j, connected between nodes k and m, has a turns-ratio of n_j at node k. If it is desired to change this to ratio n_j'; add to Y_{kk} the quantity

$$[(n_j')^2 - n_j^2]Y_j \tag{1}$$

and add to Y_{km} the quantity

$$-[(n_j')^2 - n_j]Y_j \tag{2}$$

Calculations for inital settings of taps or off-nominal turns-ratios and for changing taps can be carried out by the computer, if desired, with the use of a small subroutine to accomplish the required operations.

Table II. Specified Terminal Conditions, Per Unit

Bus No.	|E|	δ	P	Q
1	1.05	0°		
2	1.10		0.50	
3			−0.55	−0.13
4			0	0
5			−0.30	−0.18
6			−0.50	−0.05

Tabulation of Self- and Mutual Admittances

A complete list of self- and mutual admittances for Fig. 1 is given in Table III, including the effects of the given off-nominal turns-ratios. Since the admittance matrix is symmetrical, this tabulation corresponds to a triangular matrix, listing mutual admittances only once. The terms are ordered in sequence as they appear in successive rows of the lower triangle matrix. The triangular matrix is suggested for the purpose of conserving memory space in the computer. For a large network, it would require almost twice as much storage space to use the entire square array which includes the mutual terms twice, although this would facilitate locating desired data in memory during computation. In the list of admittances Y_{km} of Table III, note that k, the first node number, is always less than or equal to the second number. In this listing of parameters, admittance Y_{km} will be found in a position given by the following number, counting down from the first entry

$$\frac{m(m-1)}{2} + k \tag{3}$$

where $k \leq m$. This relation can be used in the computer program to generate addresses of desired admittance data.

Iterative Method for Voltage Solution

This section will describe the basic principles of an iterative procedure for proceeding from a trial set of terminal voltages to converge upon a corrected set of voltages which satisfy prescribed terminal power inputs of the type given in Table II. Just how this method is implemented in an automatic computer will of course depend on the available order code and other characteristics of the machine.

Since most computers are designed to accept only real numbers, the computations involving complex number representation of a-c circuit quantities must be broken down into computation of respective real and imaginary parts. The present scheme handles complex numbers in $(a + jb)$ form and conversion of results to polar form, as for example voltage magnitudes and phase angles, is carried out at the completion of the iterative process.

The generator and load currents of a network, such as Fig. 1, are related to the terminal voltages by the following equations

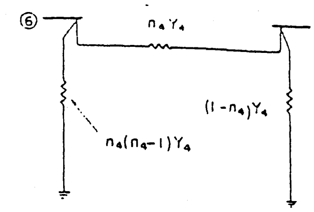

Fig. 3. Equivalent π circuit

$$I_k = \sum_{m=1}^{N} Y_{km}E_m = \sum_{m=1}^{N} (G_{km} + jB_{km})(e_m + jf_m) \tag{4}$$

where N is the total number of nodes, not counting the neutral or reference node. The real and imaginary parts of current I_k are given by

$$a_k = \sum_{m=1}^{N} (G_{km}e_m - B_{km}f_m) \tag{5}$$

$$b_k = \sum_{m=1}^{N} (G_{km}f_m + B_{km}e_m) \tag{6}$$

The scheme involves the following steps:

1. The process is started with a complete set of estimated voltages $E_k = e_k + jf_k$. The initial estimate for each bus voltage, except the slack generator, may be taken as $1.0 + j0$ for convenience. The slack generator voltage is known completely and the correct, known phasor value for this bus voltage is used throughout.

2. This initial set of approximate voltages is used in equation 4 to calculate the impressed current at bus 2. The resulting current I_2 is used with the original estimate of E_2 to compute the corresponding power input to the network at bus 2.

3. The scheduled power and the scheduled voltage magnitude or reactive power, depending on whether the bus in question is a generator or load bus, are used to obtain a correction in the estimate of E_2. This correction is obtained on the basis that

Table III. Self- and Mutual Admittances Including Off-Nominal Turns-Ratios

Nodes (k–m)	G_{km}	B_{km}
1–1	0.992203	−4.375561
1–2	0	0
2–2	1.021401	−1.954525
1–3	0	0
2–3	−0.444860	0.646063
3–3	0.444860	−8.164860
1–4	−0.558269	2.581096
2–4	0	0
3–4	0	8.270677
4–4	1.112371	−13.975358
1–5	0	0
2–5	−0.576541	1.308462
3–5	0	0
4–5	0	0
5–5	0.576541	−4.64170
1–6	−0.433934	1.827463
2–6	0	0
3–6	0	0
4–6	−0.554102	2.324044
5–6	0	3.410006
6–6	0.988036	−7.610402

Table IV. Voltage Solution

Bus No.	Initial Estimate e	Initial Estimate f	Iteration: First e	First f	11th e	11th f	54th e	54th f
2	1.0	0	1.104099	0.200945	1.009626	−0.029024	1.008111	−0.064443
3	1.0	0	1.077044	−0.044309	0.991523	−0.202254	0.975732	−0.221432
4	1.0	0	0.990435	−0.023908	0.927705	−0.145295	0.915942	−0.158809
5	1.0	0	0.984584	0.005765	0.906922	−0.176879	0.898044	−0.196412
6	1.0	0	0.984195	−0.065717	0.907647	−0.182189	0.898185	−0.194857

Table V. Solution for Terminal Conditions

Bus No.	E	Degrees	P	Q
1	1.050000	0	0.95220	0.43551
2	1.100000	− 3.35855	0.50000	0.18544
3	1.000542	−12.78007	−0.54909	−0.12009
4	0.929607	− 9.83632	0.00001	0.00001
5	0.919277	−12.33606	−0.30000	−0.18000
6	0.919079	−12.24036	−0.49999	−0.05000
Total Loss = 0.10223				

other bus voltages remain constant as the correction is made.

4. The corrected value of E_3 as well as the original estimates of other voltages is used in equation 4 to compute I_3. This current is used with E_3 to compute the power input.

5. Step 3 is repeated to compute a correction in the approximate value of E_3.

This process is continued through the list of busses to complete the first iteration. As each new corrected voltage is computed, it replaces the previous approximate value in the computation of the next bus current. The whole process is repeated over and over until all voltage corrections appearing in the process of traversing the list of busses from 2 to N become smaller than a predetermined precision index. When the precision index is satisfied, the resulting voltages are used to compute complete information on terminal conditions and, if desired, the flows in individual branches.

The feature of central importance in this process is the correction of each bus voltage in turn. This calculation of a voltage correction is different for the two types of terminal specifications, namely, generators and loads. This means that, in a completely automatic computation, some method must be used for identifying whether the generator or load type of calculation is to be performed for a given terminal.

In the case of either a load or a generator terminal, a simultaneous correction of two quantities is required. For example, at a load terminal, the computed current I with the approximate voltage E will produce, in general, both real and reactive power loading which differ from the scheduled values, and it is desired to determine a correction in both the real and imaginary parts of the voltage. Further, a change made in either the real or imaginary part of the voltage will alter both the real and reactive power. The scheme proposed here involves the solution of two simultaneous equations each time either a load or generator bus voltage correction is made.

Voltage Correction at Load Terminals

Consider terminal k, where the scheduled real and reactive power inputs P_{ks} and Q_{ks} are specified. The current $I_k = a_k + jb_k$ has been computed from equations 5 and 6 with the use of a set of approximate voltages including voltage $E_k = e_k + jf_k$. The real power and reactive power inputs corresponding to this voltage and current are

$$P_k = a_k e_k + b_k f_k \quad (7)$$

$$Q_k = a_k f_k - b_k e_k \quad (8)$$

Error quantities ΔP_k and ΔQ_k are defined as deviations from scheduled values

$$\Delta P_k = P_{ks} - P_k \quad (9)$$

$$\Delta Q_k = Q_{ks} - Q_k \quad (10)$$

Let a corrective voltage $\Delta E_k = \epsilon_k + j\xi_k$, be added to the estimated value E_k so that, with the other bus voltages held fixed, the new current together with the corrected voltage $E_k + \Delta E_k$ produce the scheduled real and reactive power

$$P_{ks} + jQ_{ks} = (E_k + \Delta E_k)(I_k + Y_{kk}\Delta E_k)^* \quad (11)$$

Upon expanding and separating real and imaginary parts, this can be rewritten as

$$\Delta P_k = \epsilon_k(e_k G_{kk} + f_k B_{kk} + a_k) + \xi_k(-e_k B_{kk} + f_k G_{kk} + b_k) + G_{kk}(\epsilon_k^2 + \xi_k^2) \quad (12)$$

$$\Delta Q_k = \epsilon_k(-e_k B_{kk} + f_k G_{kk} - b_k) + \xi_k(-e_k G_{kk} - f_k B_{kk} + a_k) - B_{kk}(\epsilon_k^2 + \xi_k^2) \quad (13)$$

Thus the complete equations for computing the voltage correction are second-order equations. With a reasonable initial estimate of bus voltages, satisfactory convergence is obtained by simply neglecting the second-degree terms and solving the resulting system of linear equations for ϵ_k and ξ_k. If an initial voltage estimate is drastically in error, say nearly 180 degrees out of phase with its true value, the method may converge upon a completely erroneous solution. In normal power-flow studies, however, voltages will seldom exceed a range of 90 degrees in angular position, and there would be little doubt, from inspection of results, if the second or unwanted solution were obtained.

Voltage Correction at Generator Terminals

Consider terminal m, where the scheduled power P_{ms} and the voltage magnitude E_{ms} are specified.

The current $I_m = a_m + jb_m$ has been computed from equations 5 and 6 with the use of a set of approximate voltages.

Using this current and the voltage estimate E_m, the following quantities can be obtained

$$P_m = (a_m e_m + b_m f_m) \quad (14)$$

$$|E_m|^2 = e_m^2 + f_m^2 \quad (15)$$

Let

$$\Delta P_m = P_{ms} - P_m \quad (16)$$

$$\Delta(|E_m|^2) = |E_{ms}|^2 - |E_m|^2 \quad (17)$$

Let $\Delta E_m = (\epsilon_m + j\xi_m)$ be the voltage correction which, when added to E_m, will produce the scheduled power and voltage magnitude, assuming that all other voltages remain fixed.

$$P_{ms} = R_e(E_m + \Delta E_m)(I_m + Y_{mm}\Delta E_m)^* \quad (18)$$

Expanding equation 18 (this is the same as equation 11)

$$\Delta P_{ms} = \epsilon_m(e_m G_{mm} + f_m B_{mm} + a_m) + \xi_m(-e_m B_{mm} + f_m G_{mm} + b_m) + (\epsilon_m^2 + \xi_m^2)G_{mm} \quad (19)$$

and from equation 17

$$\Delta(|E_m|^2) = |E_m + \Delta E_m|^2 - |E_m|^2 = 2e_m\epsilon_m + 2f_m\xi_m + (\epsilon_m^2 + \xi_m^2) \quad (20)$$

Upon discarding the second-degree terms, equations 19 and 20 become a system of linear simultaneous equations which can be solved for voltage corrections ϵ_m and ξ_m.

Table VI. Line-Flow Solution

Branch (k–m)	P_{km}	Q_{km}	P_{mk}	Q_{mk}
2–3	0.171753	−0.000088	−0.154127	0.025511
1–4	0.509128	0.270353	−0.485015	−0.158832
3–4	−0.395868	−0.155505	0.395860	0.179540
2–5	0.328248	0.185354	−0.295130	−0.110189
1–6	0.443069	0.202527	−0.416593	−0.091019
4–6	0.089159	0.004648	−0.088269	−0.000892
5–6	−0.004865	−0.070420	0.004865	0.071548

Table VII. Branch-Node Connection Matrix

Node No.	Branch Number 1	2	3	4	5	6	7	8	9	10
1	1	1	0	0	0	0	0	0	1	0
2	0	0	0	0	1	1	0	0	0	0
3	0	0	0	0	0	1	1	0	0	0
4	0	1	1	0	0	0	1	1	0	0
5	0	0	0	1	1	0	0	0	0	0
6	1	0	1	1	0	0	0	0	0	1

Voltage Solution of Sample System

The results of applying these techniques to the system of Fig. 1 and the terminal specifications of Table II are presented in Table IV. The voltage solution converged to a precision of $\pm 5 \times 10^{-6}$ in 45 iterations and to $\pm 5 \times 10^{-7}$ in 54 iterations. Time required per iteration was 25 seconds. In a large network it may be advantageous to eliminate passive nodes, i.e., busses at which no load or generator is connected. This would speed up convergence on specified inputs at load and generator busses but, at the same time, it would complicate computation of line flows, if they are required. A reduced set of self- and mutual admittance coefficients, eliminating passive node k, can be formed as follows.

Replace Y_{mn} for all values of m and n by

$$Y_{mn} - \frac{Y_{mk} Y_{kn}}{Y_{kk}} \qquad (21)$$

Considering the last column of Table IV as the final voltage solution, the results of computing complete terminal performance are given in Table V. If total loss is the item of interest, this table is regarded as the complete solution.

Computation of Line Flows

If line flows are desired in addition to the solution of terminal inputs, these may be obtained from the voltage solution; see Table IV. Assuming that each mutual admittance arises from a single branch in the original network (i.e., no parallel lines) then the real and reactive power flow in each branch not containing an off-nominal turns-ratio is obtained by the following

$$P_{km} + jQ_{km} = E_k(E_k - E_m)^*(-Y_{km})^* \qquad (22)$$

where P_{km} and Q_{km} are respectively the real and reactive power flow at node k and directed toward node m in the connecting branch.

In the event that the branch in question contains off-nominal turns-ratio n at either end, then the admittance Y_{km} existing in computer memory at the completion of the voltage solution includes the factor n. Hence, two cases must be considered:

If the turns-ratio appears at node k end of the branch connected between k and m

$$P_{km} + jQ_{km} = E_k(nE_k - E_m)^*(-Y_{km})^* \qquad (23)$$

If the turns-ratio appears at the node m end of the branch connected between k and m

$$P_{km} - jQ_{km} = E_k^*\left(\frac{1}{n}E_k - E_m\right)(-Y_{km}) \qquad (24)$$

The equations 22 through 24 can be separated easily into real and imaginary parts, suitable for implementing their evaluation by the computer. Line flows obtained in this manner for the system of Fig. 1 and the terminal conditions of Table II are listed in Table VI. These flows do not include correction for line-charging initially lumped on busses.

Computer Formation of Self- and Mutual Admittances

To automatize the formation of self- and mutual admittances, it is necessary to convert the geometry of the network connections into digital form as a list of numbers. This can be done by forming a table or matrix showing which branch numbers are connected to each node. For example, consider the network of Fig. 1 and the matrix of Table VII. In Table VII, as a row associated with a certain node is traversed, an entry of 1 means that one end of the branch identified by the corresponding column number terminates on that node. Conversely, a zero in row j and column k means that branch k is not connected to node j. All entries will be zero or unity. The self-admittance is formed by detecting nonzero entries in a given row and summing corresponding branch admittances. The mutual admittance between nodes j and k, for example, is determined by detecting coincidence of nonzero terms in rows j and k, and summing corresponding branch admittances with signs reversed.

A great saving in memory space required for storing the connection matrix can be accomplished by combining a segment of each row to form a single number. For example, if the computer accepts 10-digit numbers, the first row of Table VII can be stored as the single 10-digit number (1100000010). Then logical operations of shift, extract, and zero check can be utilized to program suitable inspection of the connection matrix. For a 200-branch network, a sequence of 20 10-digit words suffices to store one row of the matrix. The program for automatic formation of self- and mutual admittances must scan the connection matrix, detect nonzero entries, find the address of the correct branch conductance and susceptance, and store the sums in the correct positions in the list of self- and mutual coefficients.

Conclusion

1. For a large class of power-flow studies associated with planning system extensions, analyzer solutions have a number of advantages over digital solutions and, at the same time, provide adequate precision. For some studies, such as analysis of losses and incremental losses, the precision of analyzers is not adequate and digital computer methods can provide the solution.

2. Several papers[1,2] have described digital solution of power-flow problems, utilizing the mesh basis of formulating network equations. It appears that the node basis has distinct advantages in preliminary preparation of the problem for digital computation. Also the node basis readily permits rigorous representation of off-nominal turns-ratios, and turns-ratios can be changed without extensive recomputation of network parameters.

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Discussion

G. W. Stagg (American Gas and Electric Service Corporation, New York, N. Y.): The authors are to be complimented not only for their technical contribution but also for focussing attention again on a very new and important phase of power system engineering. In our opinion, the authors have appraised the situation correctly in both of their conclusions. However, it should be noted that the disadvantages of the digital computer solution exist in terms of the computers and techniques now available. It seems likely that the use of a faster and larger computer can reduce the cost of digital power-flow studies considerably. Moreover, it is apparent that rapid development of new techniques and computers will make the digital method more competitive even for studies in which its superior accuracy is not required.

Since in digital calculation all system data and the program are stored permanently on cards or tape, it requires only a few minutes to transfer these data into the computer to begin a calculation. Because of this, the following advantages of the digital method are foreseen:

1. The calculation of power flows necessary to a major system-planning study can be distributed over a period of weeks, allowing the engineer opportunity for frequent and careful appraisal of results.
2. The provisions for adequate periods of analysis will permit the more effective planning of future cases and will eliminate unnecessary and insignificant reading.
3. During the period required for analysis, the computer can be used for other power-flow studies or, for that matter, any other type of problems which arise in connection with electric utility business whether they be financial or engineering. In other words, the computer need not be idle while decisions are being made.
4. System planning can be made a continuous process rather than a periodic review. When new developments dictate, additional power-flow cases can be obtained readily to maintain up-to-date an analysis of present and future power-flow conditions.

The foregoing advantages result from the fact that the actual production of power flows by a digital computer does not require the attendance of an engineer. As a result, less engineering time and talent is spent in the mechanics of obtaining power flows, with a minimum disruption of normal engineering functions.

Modern trends in computer programming indicate that the execution of system-planning studies can be made automatic. It appears feasible that a large-scale digital computer can be programmed to follow the logical pattern which the engineers now use in evaluating a particular system plan. The computer could be programmed to make a detailed economic study of the new facilities proposed by the engineer, or it might go so far as to calculate on both an engineering and economic basis the additional facilities required for expected load conditions.

A few years ago the American Gas and Electric system required 1 or 2 weeks on the network analyzer to formulate adequate plans for future facilities for any one of its subtransmission systems. Today, studies of less than 4 to 8 weeks are rare. This has come about because of the need to extend studies to cover upward of 15 to 20 years to insure not only that our present-day investments in new facilities will tie in economically with our future needs but also to insure that we have adequate rights-of-way and station sites. Also, because of the large investment in system facilities now being required to meet load growth, a greater emphasis on economic analysis has been necessary. This requires the complete study of various alternate schemes before a final decision.

It seems imperative then that, if the electrical utility industry is to keep abreast of this increasingly complex problem, particularly in system planning work, new techniques and new tools must be developed to solve power flows. The digital computer offers a solution.

G. W. Bills (North American Aviation, Inc., Downey, Calif.): This excellent paper should give greater impetus to the changeover from analogue to digital solution of power system load flow studies. While it is true that a-c analyzers may have some advantages over digital computers, after 15 years' experience on analyzers and two on computers I believe the advantages of the latter are much greater than the former.

Once a general program is achieved, the setup time for a digital computer is less than for an analyzer, the running time per study should be less, and the automatic printout on a digital computer is certainly superior to hand logging. It is our hope at North American Aviation, Inc., to program and solve load flow studies on an International Business Machines *704* and then perhaps we can compare the analogue and digital methods as to cost per study.

It is gratifying that the authors agree that the nodal approach to the solution of load flow studies is superior to the mesh basis for most studies. The only problem in this type of analysis is that of convergence. At the Bonneville Power Administration this was achieved by the end of 1954 and, in the middle of 1955, the program was successfully run on an International Business Machines *650*.

What number of iterations are necessary when converging to 1×10^{-3}, the approximate accuracy of an a-c network analyzer? There is no doubt that the use of Summers' equivalent for transformers leads to an accurate and automatic method for representing transformers. This equivalent is also valid for phase-shifting transformers and 3-winding transformers.

Since the program can automatically be made to choose the correct transformer tap settings and maintain the correct bus loads and generator voltages and loading schedules, a digital computer can balance load flow studies as well or better than a competent a-c network analyzer operator.

The authors have made a definite contribution in presenting certain programming techniques for use in the automatic digital solution of load flow studies. I am sure this paper will not only be of interest to power system engineers but should aid them when programming digital studies.

REFERENCE

1. APPLICATIONS OF DIGITAL ANALYSIS TO POWER SYSTEM PROBLEMS. *Proceedings*, American Power Conference, Chicago, Ill., 1954.

E. E. George (Ebasco Services, Inc., New York, N. Y.): In view of the rapid recent developments in computer solution of problems of power system analysis, this paper is particularly welcome because it provides the following features which are apparently new or have never been published:

1. The procedure takes advantage of the high speed and large memory or storage capabilities of one of the new types of medium-size digital computers.
2. The method described requires no matrix inversion, thus eliminating the considerable time and expense which this item usually involves for large power systems. In some cases, matrix inversion is required for converting branch mutual impedances into branch mutual admittances, but this generally involves only a few lines.
3. It eliminates the necessity for manual construction of a connection matrix, because the computer can be programmed to develop and print this matrix.
4. It appears that the method proposed by the authors would not even require the construction of a system 1-line diagram. It certainly would not require the construction of a coded diagram, a "tree" diagram, a "track" diagram, or any other special diagram heretofore required for power-system studies on digital computers.
5. The method uses megawatt generation and scalar per-unit voltages at the generating plants rather than megawatts and megavars. The use of scalar voltages is much more realistic because load dispatchers normally employ them, and the a-c calculating board operator prefers them to generator megavar loadings.
6. The iteration of bus voltages is continuous instead of cyclic.

The foregoing advantages are largely caused by the use of a node method instead of the mesh method, or the node-pair method. Previous published descriptions of power-flow study methods have all utilized the mesh method. Manual construction of a connection matrix is relatively easy when using the mesh method because it uses a current matrix. The node-pair method involves a tree diagram and a voltage matrix, which is not always easy to construct even after considerable experience. In the node method, the assembly of the connection matrix and its associated matrix algebra are all done on the computer, the programming for which has already been developed for repeated future use. All three methods were originally developed by tensor analysis, but the use of either method requires no knowledge of tensors after the equations have been developed and the digital computer has been programmed.

It is rumored that the authors, subsequent to the preparation of this paper, have been able to devise an accelerating process which greatly reduces the number of iterations required. It is hoped that the authors will cover this in their closing discussion.

For those who have not followed digital computer developments heretofore, two or three of the technical terms taken from network theory as developed by mathematicians and communication engineers should perhaps be explained:

1. A node is the same as a bus.
2. A branch is a line, transformer, or other series impedance between two busses.
3. The slack machine is the regulating generator which controls frequency or tie-line loading, and which cannot be scheduled in megawatt output until the difference between total generation and total load plus loss is calculated, or measured by telemeters, or balanced by a frequency controller.

The authors have been rather modest in comparing the advantages of this method of digital computer solution with a-c calculating board solution. Among these advantages are the following:

1. No plugging diagram (or equivalent coded diagram and connection matrix) has to be prepared manually. One merely assigns numbers to each line and each bus, and allots these numbers to loads, generators, capacitances, etc. Tables may be used more easily than diagrams.
2. The results of digital computation are available in printed form and, on one competitive type of computer, these can be printed on a system diagram, thus eliminating the time and mistakes associated with the tedious process of transcribing (and checking) a-c calculating board readings onto a diagram suitable for reproduction.
3. It is now possible and practical to carry out almost any type of power system study on a digital computer, including transient stability, reduction to equivalent mesh, construction of star equivalent, calculation of short circuits, and determination of a loss formula.

If progress in digital computer application moves at the present accelerating pace for another year or two, this new tool for system analysis may become even more reliable than the well-known a-c calculating board.

C. A. MacArthur (Pennsylvania Power and Light Company, Allentown, Pa.): The authors of this paper are to be congratulated upon an excellent approach to this important problem. The approach has the simplicity and soundness that makes one ask why it wasn't thought of before.

There is only one point which must be protested. It is not a fault of this paper alone, but is common to a number of technical papers presented lately on the use of digital computers for engineering calculations. The point of protest concerns the degree of precision indicated in the numerical solutions.

Table IV indicates voltages calculated to seven significant figures and quite a point is made of the time required to attain this precision. However, this precision does not seem reasonable in light of the basic data from which the voltages were calculated. Table I shows the basic line impedances and Table II the specified bus loads and generations with the normal accuracy attainable. Line impedances are accurate to only 1 per cent or three significant figures. Bus load estimates are indicated to only two significant figures. These are normal accuracies because of the uncertainties involved. Impedances are affected by conductor temperature and number of wire transpositions. Estimating future bus loads is chancy, particularly if reactive loads are involved. Even today's bus loads are probably not known to better than two significant figures.

There is some justification for carrying along four or even five figures in the calculations to avoid stray errors and the need for rounding-off operations. However, to spend computer time to get a degree of precision in the solution better than 1 per cent is difficult to justify. Certainly to get better than four significant figures completely ignores the engineering assumptions made in the first place.

There are certain out-of-pocket costs, involved in attaining increased precision. In a problem involving 50 busses, extra time of 10 minutes might easily be required to increase accuracy of the results from four figures to seven. Considering that one system problem might entail a minimum of 20 studies, the increased precision would then cost 3 hours. At an hourly rental figure of $80 for a medium-capacity stored-program computer, this amounts to $240, a tidy little sum.

But aside from costs, it is questionable engineering practice to show precision that is not justified by the basic data and assumptions. It gives the management a false idea of system operation. It tends to hide the inherent errors and obscures the fact that load flow studies are only tools of limited usefulness because they cover only a limited number of conditions. A great deal of engineering judgment is needed to interpret the study results correctly and indicating unjustified precision only tends to obscure the basic facts.

These comments should not be considered as detracting from the value of the authors' work. Their approach is very ingenious and lends itself admirably to computer solution. Rather, it is hoped these comments will add to the value of the work by helping to stop the trend to excessive precision.

J. B. Ward and H. W. Hale: We appreciate the ideas and suggestions submitted in the discussions. Mr. Stagg suggests the possibility of extending the computer application to include economic evaluation of planned system extensions. Establishment of a mathematical model of the system planning process presents some imposing difficulties, but perhaps the techniques of operations research could be brought to bear to some advantage.

Both Mr. Bills and Mr. George refer to the possibility of digital computers displacing analyzers for load-flow calculations. We neither support nor dispute this view, for the matter is purely one of economics. At present there are aspects of convenience in analyzer studies such as spot checks during adjustment of a case, partial readings, and experimental establishment of tap settings and synchronous condenser loadings which are difficult to reproduce in digital solutions. However, if the same end results of analyzer studies can ultimately be achieved otherwise with less time and expense, then the analyzer may well be displaced.

The questions of accuracy of initial data and precision of results brought out by Mr. MacArthur are certainly pertinent to any engineering computation. In displaying results in the paper to five or six decimal places we do not imply that the voltages or flows are this accurate or this near to the true values that would appear on the actual system. In an extensive iterative computation such as this it is necessary to carry intermediate calculations to a much greater precision than contained in the input data, particularly if the result sought is a comparison of losses. Since calculation of line flows and source loadings involve differences of terminal voltages, it is necessary to carry the voltage solution to six places to assure three places in incremental loss evaluation. For other applications, the convergence need not be carried out this far.

As Mr. George points out, no direct matrix inversion is involved; however, the price paid for this is slower convergence. One might view the process as an iterative inversion of the node equation matrix, combined with iterative convergence to the specified power input at load and generator terminals. A method of accelerating convergence has been tried and seems very effective. It appears in the literature as "Aitkens Method."[1,2] Briefly, it involves storing the results of three successive normal iterations for each bus, and then utilizing these in a simple equation to produce a revised set of bus voltages. For example, let E_{k1}, E_{k2}, and E_{k3} denote the phasor value of bus k voltage for three successive iterations. Then a revised value E_{k4} is obtained by

$$E_{k4}=E_{k3}-\frac{(E_{k3}-E_{k2})^2}{E_{k3}-2E_{k2}+E_{k1}} \qquad (25)$$

This extrapolation is applied to each bus separately and requires a small fraction of the computing time of a normal iteration. In the example of the paper, application of this device twice during the process reduced the number of iterations from 54 to 18.

Although the necessary and sufficient conditions for convergence of the iterative method described in the paper are not known, it appears that the conditions prevailing in power system networks submit to solution by this method. It has been found that the usual star equivalent of three winding transformers can cause some difficulty if one of the legs of the equivalent contains a small negative reactance. The junction or star point of this equivalent representation has no physical reality, and the treatment of this junction as one of the busses in the network may cause failure to converge. This can be avoided by replacing the representation of the three winding transformers by an equivalent delta, in which the artificial junction does not appear.

References

1. Calculating Instruments and Machines. D. R. Hartree. University of Illinois Press Urbana, Ill., 1949, p. 117.

2. Principles of Numerical Analysis (book). A. S. Householder. McGraw-Hill Book Company, Inc., New York, N. Y., 1953, pp. 126–27.